Metaphysics
and the
Elements

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Outline-Foundation

Wednesday, July 5, 2023

General Facts

Every form of life, supports that life, by life support systems. A form of life is thus divided and categorized, taxonomy, by the number and quality of its life support systems.

The Common Notion, of all forms of life, is that in order to survive, it has to have a life support system which parses the environment to bring into the organism that which is good for survival, and keep out those things which are detrimental to its survival. Thus, evolution is in terms of life support systems and the ability of those systems to do its biologically defined job. Quantity and Quality.

A life support system then parses things, separating the good from the bad. Reference Plato; Bible; elementary biology. It uses the good, the goods, to maintain and promote its life, and it inhibits the evil for the very same purpose, to maintain and promote its own life. Therefore, next time you hear some fool claiming to be non-judgmental, then do not, in anywise, believe them, for if that were true, they would be dead. Claiming that it is a sin to do your biologically defined job is the claim of the mentally incompetent, or the deceitful liar.

As every life support system parses things to separate the good from the bad and since every life form has a number of these life support systems, a thing is parsed in accordance with various criteria of need.

If a life support system parses things for survival in order to do its job, it has a means of distinguishing what a thing is, and how to differentiate between different things.

Specific Facts.

The brain of a life form is a life support system. Its job is the overall management of sensory information. In advanced species, the brain evolves to produce what is called a mind which functions not on the perceptible level, but the intelligible, i.e., the management of behavior over time which is accomplished by the management of memory. Intelligence is the ability to acquire and manage memory to the advantage of survival. We have several names for this; judgment, rationality, sanity, and prophecy. Therefore, when you hear claims of man's subconscious, it defers to the biological and primitive functions of the brain, however, charlatans and quacks use this topic for deceitful purposes. They cannot teach you the conscious functions of the mind, so they pull the whole wool over your eyes to take advantage of you. The biggest lie is the implication that they know what they have first claimed is unknowable, however, even recognizing this feat of reasoning is beyond a vast majority of humans today. No life support system functions by that which it cannot even detect, or processes.

Fundamental to the management of memory are grammar systems, or in today's jargon, programming languages, even though Language is the wrong

term. Thus, in order to do our job, as an advanced species, is to learn everything about names and managing information using them and since a thing, qua thing, is not different from itself, then we have to learn information management, judgment, in accordance with what is intimated in the Bible, by Plato, by our own biology, and as demonstrated by the computer today, it is nothing more than binary recursion.

If one is not up to learning to do the job of a mind, it is probably because man is still evolving a functional mind. But one should realize, that since a mind has only one job to do, only one method of doing that job, and only one product to render, then the inability to do that job means that one is in the worst possible state of existence possible. You cannot do your job. A good or bad person has absolutely nothing to do with what people think of you, or how they treat you, in fact, if that were the measure, a rock, or tree would be your best of friends. We are good or evil as determined by biological fact, our ability to do our own work.

Therefore, all of my work concerns nothing but names, and how to use them in processing information so that we can do our own job.

Outline The Art of Names

Wednesday, July 5, 2023

A Thing

We are not a particular sense of the environment, we are designed to manage all the other senses of our environment, therefore, our first job is to grasp the notion not of any particular thing, but a thing in its universal expression so that we can apply it to all the particular things about us. During our evolution we have discovered exactly four methods of naming, via Common Grammar, Arithmetic, their combination Algebra and lastly, which was actually the first, Geometric. Each of the other grammar systems, translate via the written grammar, as a system of geometric figures.

Of those four methods, there is one which is exactly universal, the Geometric. The unit in Geometry is expressed as:

And we pair it in common grammar with one, and only one name; a line; not a straight line, not a line segment, not the shortest line, not an even line, not a rectilinear line, or an orange line. We defer to a simple one-to-one correspondence of names, one in Common Grammar, one in Geometry. We learn how to name its two parts using Common Grammar, Arithmetic and Algebra. And when we divide it, we do not say we intersect it, cut it, fall upon it, or mix our martini with it.

We learn how to differentiate the operations of equality when referring to the correlative versus the relative, i.e., when speaking of equality, we mean either arithmetic, geometric or their combination, and how it affords us Arithmetic Recursion based on the correlative, and Geometric Recursion based on the relative and the combination of these to perform complex operations. We will also learn how to pair these two types of recursion into a single figure, for, all of grammar is learning how to perform operations with names and how they are expressed in each particular system of Grammar. Geometry affords us a very powerful Grammar Matrix, by which all of the operations are performable in grammar can be performed. And, like a Mix Master, it can even be fitted with attachments to augment its abilities for more complex operations.

And we must learn to be mindful, as names are all we have, it often happens that someone is so simple they actually believe that a change of names, means we have changed the topic, however, there is no particular topic when learning how to apply universally, acrost all our systems of grammar, one and the very same set of operations.

Thus, a grammar book, sole purpose is not any particular use of the operations we learn, save in terms of binary recursion. How these are applied to acquire the same physical result is up to what are called the Applied Sciences, for the only Science a man is born to learn, is the Science of Judgment, the Science of Prophecy. The results of our learning indicates, for all the particular

things, what we may someday achieve by them, thus we learn the Art of Predictive Behavior, as Applied Science. And, being one and only one study, it is, as Plato noted, the only science which is over all the other particular sciences.

A tour de force in Philosophy, or in Grammar, has nothing to do, whatsoever, with the particular use of grammar, but with learning all we can about information management via words, words in Common Grammar, words in Arithmetic, words in Algebra and words in Geometry. One convention of names used in each particular grammar, is not the convention of names in the other, so in order not to become confused, one has to be mindful of how they start pairing names from the very first of their studies in formal grammar.

Our job is to regulate the behavior of not only our own body, but of the whole of our environment. Therefore, we start our study by learning how to pair our own hand to discover the whole of grammar with our very own hand. We learn to respect the Mark on the Hand and its relationship as Frontlets between our own eyes no matter if we fair well, or fair ill. There is one and only one, set of operations, and we learn to apply them in all four members of our Grammar Matrix in order to do our biologically defined job, so that we too, end up to tender the Garden of this, our Earth. The very same job we are made to do, not only according to the Bible, but according to Science.

Metaphysics

Monday, June 5, 2023

One can say that Metaphysics is the education required for learning the elements of mapping a good future. This education is a biological imperative, required for human survival. That man would eventually discover this fact was written about thousands of years ago, and therefore, impossible to be of human origins. A mind is designed to learn how to do its job, and therefore could not function until a cusp point in evolution, a time that mankind would be intelligent enough to recognize it; and as being designed to learn, it must first know the elements of a thing. Learning one thing affords one the ability to recursively apply that learning to learn everything. There is one, and only one path to that end; your ability to learn it depends on you: You have to learn exactly what one is, and how to use it to survive and this is an unescapable biological fact.

The first principle concerning purpose, reasoning can be stated thus: What may be predicated of any-thing is wholly determined by the definition of that thing. It is a statement of the Law of Identity over every system of grammar.

The thing of life, the first principles, which we should be of most interested in, is expressed by ourselves. In order to learn about ourself we have to become proficient in the ability to ask and answer questions, especially concerning ourselves. What are we, and why are we?

One, a thing, is a binary construct of a relative difference within limits and we learn all we can about it, as that one thing, the mastery over which, ensures survival of all life in creation. We are a life, and our job is the recursion of life through truth in judgement.

We are a mind whose job is to perform judgment concerning our every behavior such that that behavior maintains and promotes life. Life is a never-ending job, and we have a perfect method of doing that job: binary recursion. Reasoning is a process which cannot be seen, so we do and can learn, how to manage the perceptible via the invisible and quite remarkably, we are endowed to learn it from standards of behavior imposed upon our own hand, expressed via two very distinct grammatical tools, straightedge and compass. One part of a binary cannot be seen called the intelligible, but it is the part which manages the perceptible. Intelligence is proportional to the Principles of Language which are functionally resident in a mind as a Grammar Matrix derived from binary recursion; binary meaning the two parts of a thing, also known as the Intelligible Elements. Metaphysics is the elements of mapping the Intelligible, the invisible, Grammar Matrix which affords a sapient species life. We learn to map the future by learning how to map our very thoughts, our very behavior, by learning the elements of mapping the pure binary. You cannot map a future unless you learn to map the simplest of things, a pure binary. We learn the simple before we can learn to recursively apply it to construct the complex.

A brain is one of the life support systems of the body. One of its products is called Mind, we are that mind. As a mind is a biological product a mind has a

well-defined biologically determined job to perform and well-defined physically determined means of doing that job. As Plato stated, the obvious, our concern is doing our own work and no one can do it, if they are even ignorant of what they are, why they are, and how to do their biologically defined job. Metaphysics presents us the ability to learn the fundamentals of doing our own work. Our job is predictive behavior, the utilization of memory by which we can formulate recursive life supporting behavior. All of the fundamentals are capable of demonstrating Total Recall and application. Correct memory management is called judgment, and the application of that judgment affords us life.

It is a very simple fact, doing our own work as a mind and we live, and live more abundantly or not doing our biologically defined job and we die. Look around you, man is destroying the earth, dying is not a pleasant experience.

Language, whose function is creation, affords us the currency of our grammar matrix by which sapient behavior is exchanged for the recursion of life by true judgment.

All we can do, as a mind, is to read, using our body called perception, process what we have read in order to produce judgments in response to the environment, called conception and judgment, and write back to the body the results of those judgments in order to produce our behavior, called will. There is nothing mysterious about the general statement, however, most people do not know the particulars of information processing using a system of perceptible paradigms each of which shows proven mental operations and how by expressing them through our own behavior, we maintain and promote life, or again, our own work.

Our foundation is then judgment afforded by literacy which we use to regulate behavior so that we can have life, and have it more abundantly, and as our purpose and our abilities are well defined, we order and lay out those principles and call this foundation Metaphysics.

We process the memory of things and their interactions. In order to understand which part of a thing we can change; we have to comprehend what a thing is in the complete abstract. A thing is a binary construct of a relative within correlatives. A mind functions by parsing and ordering information. Parsing is the application of limits, boundaries, i.e., the correlative. As Aristotle noted, the verb, the relative is a given; it cannot be produced. One can say that we shape material to our advantage, so too the mind. Total recall is not about the particulars, it is about the methods of parsing information by which we gain advantage. Thus, in the following figure we can only add the correlatives:

The relative is a given. We do not create the material difference, a sculptor does not create the stone he shapes; he only shapes what is given; we sculpt our own behavior, by the definition of mind, and this is physically factual. By fact and by design we do not create anything except order by effecting judgment as to where a correlative is required. One can view what they see between the two correlatives is a record of memory, the memory of the motion of our own hand.

Judgment is the ability to apply correlatives in order to accomplish a given result, in the Universal, that result is, we get to live. Parroting gibberish is not judgment.

As we can see with our visible paradigm, one can also say that a thing is called a standard of behavior for as standard is to the correlative so too the relative is to behavior. Memory is then, a record of our own behavior, our parsing of sense perception. By biological fact, we, as mind, are responsible for the production of standards of life supporting behavior and the only method available to us is through binary recursion effected by asserting, or denying, correlatives to some given relative. By biological fact, we are craftsmen, designed to impose standards of behavior acrost our own environment, but we must first impose it upon ourselves. We craft grammar, i.e., judgments, and like any other product, are perceptible and from which we can learn and pass on to others, that learning. The parsing of a given produces perceptible things and we learning by parsing and expressing, pure binary recursion.

Grammar is then, no more than recorded memory used for playback in order to pass on that memory to our fellow man. This cannot be done, when man is still an illiterate savage.

As one may guess, all the mental operations performable are determined also as a function of binary recursion. This led Plato to formulate an observable fact, that there are two kinds of knowledge, the perceptible brought to us by sense perception, and the inborn ability to compare them for equality and inequality from which all operations upon information can be performed called the intelligible. Therefore, we have the visible, afforded by perception or the perceptible, and the invisible afforded by the intelligible, just like our binary an effect we notice when one relative ends and another begins. Metaphysics, grammar systems, are formulated to build on that ability, i.e., methods of effecting binary recursion. Thus, grammar systems are focused on how to denote a binary, and how to effect recursion with that given system of notation. Thus, it should have been obvious that a mind functions wholly by managing correlatives.

Common Notions

We discern limits when we observe a change in relatives. These changes are best discerned by the eye, followed by tactile sense. The senses are designed for elementary parsing of information. Change in odor, for smell, change in air pressure for sound, change in tactile qualities for touch, and changes in light. Thus, there are common notions, the ability to judge equal and not equal as greater and less in accordance with some sense. This ability is a given. As Aristotle noted, the loss of a sense perception is the loss of all the knowledge it affords us. Common Notions are all derived from this form of knowledge and that is why they are called common, they are common to all forms of life. The mind, then, is measured by how we can use this information for survival. And when someone is recognized not to be able to take advantage of this biological ability,

they are said not to have common sense. Common sense affords us common notions.

Thus, senses are designed for the elementary parsing of the perceptible. A mind is designed to predict the results of these changes and this is called rationalization, judgment, and even prophecy.

We learn intelligibly how to take advantage of sense perception by producing what has been called mathematical operations. These operations are also common to all members of our Grammar Matrix.

Thus, survival of the most fit, starts with sense perception followed by our ability to judge that information for being bad or good. Our function is designed to achieve the good, i.e., life. The next step in survival of the most fit is memory, how long can one use that information for the good? When one recognizes this third step, then one knows what the function of grammar systems are for, the recursion of memory, itself.

As we are a life support system, it is hard to imagine that people would call the good anything else than doing our own work to attain to it, life unless they are incredibly dim witted.

Common Notions then led to statements about equal, not equal, greater, less, and the results being developed into addition, subtraction, multiplication, and division, or four basic mathematical operations designed for predictive behavior. One will notice that these four are also a product of two squared. Two perceptible, times two intelligible. They are performed on both the relative and correlative meaning, arithmetic, and geometric recursion. In short, they are applicable to both parts of a thing, also known as the literal and the metaphorical. As the literal is to the correlative, i.e., noun, so too, the metaphorical is to the relative, i.e., verb.

How many methods of binary recursion are there for constructing grammar systems? Or, in the simple, how do we describe the two parts of a binary as an intelligible multiplied by the two parts of grammar which is perceptible and what is the result? What is two times two?

Grammar Matrix

Grammar is also a particular thing which consists of two parts and all we have to do is name those parts and establish them in a one-to-one correspondence with our universal binary.

Every system of grammar is composed of symbol sets, which are absolute, and methods of managing those symbols which is our relative behavior with them.

In Common Grammar we have symbols which are constructed with many motions of the hand, while the management of those symbols are also many. Common Grammar is optimized for non-organized information and when one does not learn how to order one's words, then one often spout words which are factually gibberish. One can see, then, why it is preferred by those who use it for the recursion of emotion. Most people only use Common Grammar by rote,

which, unfortunately, is the same way it is taught by those who may actually believe that they are speaking sense. Operations in Common Grammar and the application of words as to their being literal or metaphorical is never clearly distinguished.

Nothing is worse than a senseless teacher save, when one has no teacher at all. Thus, even a bad teacher is better than no teacher at all.

As by biological fact, grammar systems are used to teach the fundamental behaviors of our Universal Binary Unit, a grammar book which does not instruct you as to how to manage that behavior of a thing, as a universal, is not a grammar book at all.

As we name both relatives and correlatives in Common Grammar, then there is no method of discerning when one is used as the other except through memory. As Plato noted, it takes rigorous mental exercises to keep from making mistakes in Common Grammar and a retentive memory. We have to learn to instill in memory, nouns and verbs and how to use them, recursively in order to effect the basic management of fact. Part of that process involves dictionaries, dictionaries are memory, which cannot be produced by an illiterate culture such as the one we have today. Common Grammar demands an extensive system of dictionaries and socially preserved, in books, and in practice; however, the fundamentals that all grammar systems are composed of, are relatively simple, after all, we have two, and only two concepts to master. The invention of new words happens often because a dictionary is not available, or someone is too stupid to know how to use a dictionary or because a sociopath is playing a shell game with words. Yet there are no grammar books today, socially used, which teaches our biologically given Grammar Matrix, nor are there any dictionaries today, which are founded on a convention of naming and its preservation save basic weights and measures of perceptible goods.

In Arithmetic, again, we have symbols which are constructed with many motions of the hand, while the management of those symbols is constrained or ordered by what is called place value notation.

However, since the Arithmetic Naming Convention is set up to denote a unit standard which is literal, results may either be rational or irrational, meaning that Arithmetic cannot always produce an exact name for some result. This condition is named irrational, the inability to provide an exact name. The results of Arithmetic management is called a number. Arithmetic is then tailored to the noun.

In Algebra we use both symbol sets, where Common Grammar can afford it more control over the verb, thus, for the most part, eliminating the irrational, but not the irrational behavior of mathematicians. Algebra gives us names which are independent of the noun, thus it is better equipt than arithmetic for use in exact results; however, it also requires a more extensive system of dictionaries which explain the grammar of the Common Grammar symbols.

In Algebra, as we use both symbol sets, then one can always write the grammar without the limitations of Arithmetic irrationality. So, no, an Algebraic

name is not a number. The result of Algebraic management is called an equation, however, converting it to an Arithmetic number, the result cannot always be rational. Algebra is refined for the verb.

So far, we have started with common grammar, and then divided it into Arithmetic, i.e., recursion of the one, and Algebra, recursion of the many.

Geometry uses one symbol, the correlative, and we note the memory of the correlative called a loci, or line and again we use this to effect the memory of binary recursion by these loci, or lines. Each line is then a record of one relative, or many relatives, commonly called a line and a circle or curve. One can say that we have a binary and binary functions as the result of binary recursion.

Geometry can, as Plato noted, afford us visible paradigms, or Total Recall, of rational thought. And as all we can do is apply the correlative, a line, any line, is a correlative mapping. It can visually present us with things metaphorically, and afford us visual memory of the operations on them, factual operations over all things. The result of Geometric management is always exact, converting the result into any logical grammar is subject to that particular grammar's limitations. However, geometry also requires the greatest intelligence as it is wholly metaphorical; simple minds cannot see the metaphor at all or too dim wittedly.

We then start with a grammar; Common Grammar, whose naming convention does not stress the distinction between the noun and the verb, nor the literal and the metaphorical. To aid in reasoning we then develop Arithmetic aimed at the literal, while Algebra stresses the metaphorical but all of these retain symbol sets which are relative.

Geometry, the symbol set is arithmetic. With it we recognize only the application of the correlative, and its path displays the relative. This produces a grammar which is wholly metaphorical.

Every possible grammar is a method of utilizing binary recursion, be that as it may, binary recursion can only produce a binary result, and it is not possible for binary to contradict binary recursion; we contradict due to our level of illiteracy.

Summing all of this up, there are countless imbeciles who cannot multiply two times two, and claim that there are countless systems of grammar, for each of our four members, however, this only means, they do not even comprehend the notion of the unit, of one. Some of these people even construct new symbols, claiming that they are special, magical, but they are provably deluded.

The Logical Grammars, are then produced with many symbols, the Analogical Grammar of geometry, with only one, the correlative while the relative is recorded as a loci, or line. Therefore, in this stage of binary recursion, Geometry is the One over the many, Common Grammar, Arithmetic and Algebra, which agrees with the definition of mind, whose job is the regulation of behavior in order to maintain and promote life. We have made our way through the confusion of the many we were born into, into a single idea of our own behavior. One should also now see, why geometry, which actually is exhibited today as

interactive Geometry, becomes truly representative, for otherwise we would have to draw, as in the old days of cartoons, every step one frame at a time.

Straightedge and Compass

Like most of what Plato wrote, his words went right over people's head. Plato taught Dialectic, grammar in accordance with the two parts of a thing, correlative and relative, noun and verb. Another expression he used was the one and the many. And, if you did not know, it was Plato, not Euclid, who set the standard for the writing of geometry, and Plato set it, because he knew no other tools are required: the straightedge for producing the one, and the compass for producing the many of that particular unit, the rest is memory, which is a product of technology. Grammar is for the recording and preservation of memory, therefore, technology in making that memory, from stone, wood, paper, has evolved to the computer to record memory. Now, people are rather simple minded, starting with a line segment, we go on to produce segments proportional to it, we do not claim that a figure has to be made up of unit segments. We do, however claim, that all of computation is afforded by a standard unit, or our starting binary. Binary is the first principle of existence and we can state;

Language is Universal and Intelligible afforded by a binary construct, while Grammar is Particular and Perceptible afforded by the intelligence of a form of life. And in recognition of the evolution of intelligence; Psychology is commensurate with the principles of Language which are functionally resident in a mind as Grammar Systems. As a mind functions wholly by binary recursion, the only two parts of a thing, this produces a fact that there are four, and only four, possible systems of Grammar which we can name as: Common Grammar, Arithmetic, Algebra and Geometry, or the two intelligible parts of a thing, multiplied by the two behaviors of the hand. Evolution is the drive for the two elements, our binary, and in memory, evolution is a drive for the recursion of binary, or in short, that is how the universe, including life, becomes the conservation of the Universe. So, we not only have to be concerned with effecting our binary in grammar, but we effect it on media producing memory. Geometry is effected only by straightedge and compass, but do not confuse that element which changes over time, our ability to effect memory with them.

Another way to put the words 'the one and the many' is binary recursion, not curves which are produced by a circle. One can actually draw a circle with only straight lines. Or again, a simple segment implies both arithmetic and geometric recursion, or again, literal and metaphorical judgment, each of which are fully demonstrable in Geometry. The key is memory management which is the function of grammar to begin with. It is also a fact, and demonstrable, that a circle is simply the reciprocal of a straight line. Every possible grammar is a method of effecting binary recursion, and certainly, it has never mattered which two items one used to produce their result. The fact of the matter has always been, in geometry one has never needed more than these two tools, but it has never meant that a loci, produced during binary recursion had to be the result of only one kind of curved line. Binary recursion always produces a binary result,

and what every grammar uses to produce that result is only the application of the correlative which is not actually limited by the compass or straightedge. One is not limited by the tools, but in their intelligence on how to use them in relation to the ability to effect memory. Rock has evolved to become the computer, and this will evolve to become man as the functional memory of binary recursion. In information processing we can only parse information, we can neither create nor destroy it. We can, however, create and destroy its memory within us, and its availability through grammar for others.

If one does not understand grammar basics, they will find a lot of things impossible, that is what it means to be stupid. Therefore, the study of Metaphysics is not about the topics you find in a dictionary, those topics are only part of what one learns, Metaphysics is about biological fact, doing your own work in effecting binary recursion using a Grammar Matrix of Common Grammar, Arithmetic, Algebra and Geometry while using Geometry to construct visual paradigms in order to proof your comprehension and this process is to effect memory and that memory is designed to maintain and promote life by the ability to distinguish good from evil, life and death, relative and correlative. If you are looking to parrot words from works titled Metaphysics, ask yourself one question, which of them proved their words, and which of them showed how to prove yours? One can confuse themselves and others in the logical grammars, which only denotes the inability to keep one's agreement with random symbols, however, in geometry one can construct the very same things and visually examine any result. Geometry is the grammar of truth between body and mind, and it has the ability to standardize agreements in naming via logical systems of grammar, and the provable result of those agreements. The ability to give and keep those agreements are a measure of your intelligence, i.e., determine if you have a good brain or not.

Metaphysics is how to use the invisible to manage the visible, it is not about vain words. If one actually does understand this, then they can draw the map, they can demonstrate, via our Grammar Matrix, the same, concerning the same, in all four systems of grammar.

Evolution of Man

As man is defined by the Mind, it stands to reason that the really important evolution to comprehend is that of its ability to do its own work. Thus, our evolution is mirrored by our grammar systems. Psychology is commensurate with the principles of Language which are functionally resident in the mind as grammar systems. First there is Geometry, the behaviors of mankind codified for socially organized survival. Then Common Grammar, Arithmetic, and Algebra. Grammar started with Geometry, and it will be the last formalized. Only when they are first used, and continue to be used can one trace their individual evolution and their evolution relative to each other.

I recommend, that before you try to tackle my work, you read A Nation at Risk a companion file to this one. You might understand why, in the 50's a state I.Q. tester expecting a mentally challenged student as the school officials

informed him, he almost pissed his pants when I was tested. It was said that I was even smarter than the schools known genius. I am not bragging, but I do not know why people find simple ideas so hard other than evolution. I was not doing the work, because all the books I read I could see the flaws and they were extensive. I lived in a world of functionally illiterate people and that is scary, but, at the same time, I knew I was ignorant and it would take a long time to fix that problem on my own. An I.Q. tests potential, but without teachers, how could I develop it? How does one find their teachers if they do not even look? For a long time, I read. Eventually, I could see a hole in the mind of man. He could discern greater and less in terms of the body, pleasure and pain, but not of his own intelligence, the result is that he imagined that every intelligent form of life had some expression of his own mental limitations. Man could not see his own future. Our future can be understood as: What may be predicated of any thing is wholly determined by the definition of that thing, therefore, man is ignorant of what he is and thus why he is. A person claiming to know themselves, and not know their own work, is clearly simple and delusional.

I have been banned on almost every website, for my attempts to explain Metaphysics. I am shunned by organized educational, and even NASA websites because I can prove fact. I still live today among strangers in a very strange land. I was, however, asked a long time ago, to learn judgment and to help mankind in changing this world for one that is not strange at all, one that is a product which maintains the Garden of God. For the intelligible, God, the only power a mind can know, is binary recursion, from the perceptible to the intelligible. We simply learn to pass it on.

The time to go from one place to another is inversely proportional to the length of your legs so, we better get a move on.

I am One and We are Many

Sunday, July 2, 2023

I am one; one what? I am a mind, but a mind exists as one of the life support systems of the body. The body has many things it must experience, and the mind evolved to learn from those experiences and to build a shelter from those which are bad, and to achieve those which are good. All living things parses the environment to survive. The only common notion which all living things are born with, is that we must survive by the attainment of the good, the satisfaction of some physical need which aids in maintaining and promoting our life. From this common fact, we need more of this, and less of that in order to survive, a mind evolves, or learns to recursively comprehend the doing of just that, turning the physical more and less requirements of the body, into the intelligible methods of managing information often called mathematical operations, but it would be more correct to say, grammatical operations. We have but one common notion, the need to discern more of this, and less of that, however the more one evolves the more we are able to refine it into specific grammatical operations which is a form of recursion. Our common notion, the need to survive in terms of more of this, and less of that, is only one, and by recursion we turn it into the four basic operations expressible in grammar, and further refinement brings us a well-stocked tool-bag of operations we can perform on information in order to survive. The Common Notion all forms of life are born with, means that evolution is the process by which this Common Notion has been recursively developed into what is called rational thought, or again, predictive behavior. Predictive behavior is based on the Common Notion that a life form must parse the environment for its own nourishment, i.e., to maintain and promote its life.

And so, all life has one common notion, based on physical facts of survival; taxonomy, is then divided into the level which this common notion has been recursively developed.

We all start as something very simple, and one can say simple minded, and a species ends up which are composed of many individuals. What they all have in common, is afforded by evolution in terms of how well the one can manage its environment in order to perpetuate the physical existence of each member of the collective. So, I am one, and We are many, yet we all have something in common, in terms of the ability to do our biologically defined job, and as many we have diverse, radically diverse, situations to which it must be applied.

The mental operations a mind uses is independent of any particular situation, i.e., that although we all have diverse particular lives, what we have in common is the ability to recognize, organize, our life based on the common notion which has evolved into linguistic functions. Thus, our education will consist of two parts, one which is aimed at our specific environmental circumstance, and the other, the universal methods of managing those needs which are applicable to any circumstance.

As a mind, we are One, as individuals spread out over the earth, and even beyond, we are many.

We cannot unify the particulars of experience except by the same method evolution has taught us, through the intelligible, therefore, global education, learning how to recognize and use these intelligibles is required in terms of a fact, we are all individual humans. In short, an education in recognition of our evolution as a species and that keeps in our mind that we are all one, even though each is one of many.

Each individual has to learn to distinguish the one from the many, the noun from the verb, and recursively apply that ability, which is the fundamental ability of all forms of life, to the level applicable to our job as a life support system recognizing that it has to be applied over the whole globe. No individual can do that, it has to be achieved by a unified humanity, not the unity of gender, nor the unity of skin color, or eye color, or hair color, or geographical location, or how we dress, but in how we perform our biologically defined job through grammar systems. Grammar Systems are all evolved from a single common notion, our biological requirement to parse information for survival.

The common notion for every system of grammar, can be described many ways, but all it means is that we have, and are born with, the ability to parse information for survival. The evolution of parsing is the recursion of that one fact and it is expressible grammatically, however, not one result of that recursion is the first principle of either life, or grammar.

Learning the distinction between good and evil starts with the Common Notion which is achieved by parsing the environment for survival, and the recursion of that one Common Notion, by fact, ends in a unified species, a unified humanity when we become literate enough to do so.

Our mind is a biological book, but how we have to function can be preserved, and made socially available, by physical books, and virtual books simply called grammar books. Those books, however, have to all, work from the outline of evolution, the evolution of binary recursion and aimed at the recursion of a species, and of life, itself. Grammar, by demonstrable fact, has its origins in survival, a biological requirement for all life. Therefore, a cusp point in our evolution, is the production of such a grammar book. That book is based on our biological given, it is not bought or sold, but made available to every member of the species as a birthright.

This then defines our logistics for the recursion of our birthright, grammar. What you make of it, is what you have done with what you were born with. I am One, and We are Many, therefore, no one is the one, unless they are terribly confused.

Development of the Common Notion

Tuesday, July 4, 2023

Biologically we are born with one common notion which is developed as we mature into grammatical operations and whenever one meets with a systematized development of a grammar, then the degree that these are developed as a requirement for that particular work is outlined. If one desires to act like moron's as past Einstein's have done, that is on them, but if one becomes a sycophant of these people, that is on themselves. One's inability to standardized and organize their mind in accordance with the development of operations by which to process information is fundamentally a determinant of I.Q. no matter what anyone claims about a person's I.Q. Everyone of Einstein's so-called demonstrations were ridiculously childish and errant, yet here we are today with a crowd of gibbering sycophants, not one of which has shown that they can think their way out of a paper bag. I personally have never heard of any other work, defending lame arguments, using rigid rods, wind up clocks, and temperature variations, all perceptible arguments, when Euclid has some very fine intelligible arguments derived from observable figures, which are dismissed out of hand by the mentally lame. How many people even know that intelligible demonstrations do not contain sense perception data, but standards of binary recursion of the elements of a grammar system? This situation, tragic though it may be, is not up to a writer to rectify, as words, in of themselves have no meaning, and if one cannot follow what is right in front of their face, can words ever overcome this? Not in the least. Every fool, when they are too stupid to know how to apply what is known and given, claims that it is the fault of the grammar, and thus, one must become a magician and start blubbering nonsense.

We are born, as Plato noted, with only one common notion, derived from our life support systems, and if we are too stupid to develop the intelligible operations on information with that, then there is no helping that person.

So, a major portion of this Metaphysics is aimed at the development of our single common notion into provable methods of information management.

Nomenclature

Monday, July 3, 2023

Reviewing our situation for formalize the members of our Grammar Matrix, we have a unit, and that unit has to be named in the four members of our Grammar Matrix, and two distinct names for the operations one for each element of our thing, one for operations focusing on the noun, or arithmetic operations, and the other on the verb, or geometric operations. Arithmetic recursion goes from one, by addition, Geometric recursion from one to zero by division. Arithmetic recursion descends into Geometric, by subtraction, while geometric recursion ascends to arithmetic via multiplication. This transition always happens at the unit boundary, as each operational unit is specific to each element of a thing.

The names have to be consistently used for that particular member of the Matrix, not as we find in the Elements of Euclid; the result being, the same proposition is being demonstrated over and over again, for each time a particular synonym is used, and the names for our basic operations have to be consistent over the whole of the Matrix.

When we do this, we can sort the propositions first in accordance with the overall grammatical operation being exampled, and then do a separate write-up of the demonstration in each specific member of our grammar matrix.

As we have the Elements today, the grammar is virtually a mess. In fact, just the name of our grammatical unit, a straight-line, ends abruptly after Book 4. The proposition, by which book five can be demonstrated does not happen until book six, proposition ten.

Now, we can start to work.

The Unit

Friday, June 2, 2023

There are two concepts afforded by our unit which early Greeks had a lot of fun with. Is a thing motionless, or is a thing in motion? i.e., is a thing a noun or a verb. It may sound like nonsense, but it gives us two concepts, which afford us recursion, even the ability to go from one thought to another. One is called Arithmetic and the other Geometric. In short, are we effecting induction, i.e., adding and subtracting, or deduction, i.e., dividing and multiplying? i.e., produced by straightedge or by compass? Is everything One, or is everything Many? Trying to wrap your brain around the fact that we have two and only two concepts to master is very trying if you are unfamiliar with the definition of a thing, and have not the wit to manage it. Do we walk because our legs are in motion, or because a foot must be still upon the ground? Zeno became famous for amusing children with this fact. Calling binary recursion a paradox, is the judgment of a child. These historical facts do, however, tell us how young man is, and when it was his mind was becoming aware of the intelligible binary. Plato turned Zeno on his head when he used the argument to instill faith in his friends concerning immortality of the soul, from being able not to get anywhere to always be going somewhere.

In order to demonstrate our knowledge about any thing we first must acquire that knowledge. The ability to manage information is only possible after the acquisition of information and one's ability to demonstrate what they have learnt is a display of that person's intelligence. Even grammar must first be learnt and used prior to being able to organize it, sift and sort it, comprehend the common ideas among all of it, before actually taking up a serious study of it. Many who write dictionaries have constructed them too early in their life, and late in life, there is not enough time left to make such a product. The verb allows for the recording of the noun.

It seems that dictionary writers have never studied Plato, for if you look up the word thing in a dictionary, you do not get a definition, but a whole lot of gibberish, i.e., an idiot's display. Plato demonstrated this in his dialogs, ask someone to defined some intelligible thing, and the first reaction is often a heap of words involving particular things. The confusion is the inability to comprehend the distinction between the intelligible and the perceptible. However, if one has the least bit of common sense, they know that in terms of grammar, we must first have a convention of names, to learn how to manage them in relation to the parts of things learnt by, as Aristotle learnt from Plato, induction, i.e., perception and as everyone should have learnt as a child, by pairing intelligible names with perceptible objects. One must learn the evolution of grammar by using grammar for correcting it. This is done by finding first the most accurate, and applying it to the simpler grammars first to learn them.

Thus, one should realize, that we start with a convention of names and this involves teachers like Zeno, and Parmenides who constructed methods of

exercising the mind to aid in the ability to use our binary effectively and not become an idiot unable to see what idiots call paradoxes. Without the ability to distinguish between a noun and a verb, a mind cannot effect binary recursion at all. All those educated scholars who, criticized Plato for the dialog Parmenides, were only demonstrating their illiteracy of fundamentals. Learning definition, as it concerns only two concepts, demands the ability to clearly distinguish the relative from the correlative at all times, after all, a mind can only go astray when it cannot make the distinction, or misses one at some point in a chain of reasoning. In Parmenides, Plato even gave you a hint, follow the arguments using a simple geometric figure.

Synonyms for 'a thing' are 'a unit' and even 'a group.' So, let us provide for ourself a perceptible paradigm for all of these synonyms.

Therefore, by grammatical and biological fact, if someone wanted to know the definition of a thing, the simplest thing to show anyone is the simplest thing which we can produce with our own hand. And, as learnt from Plato, establish our relationships so that they are clear in our mind, viz. correlative is to the noun as relative is to verb; or again, correlative : noun :: relative : verb. And, repeating as Aristotle was taught, we have the matter, i.e. verb, the form, i.e. correlative, and the combination of these, the concrete whole. Thus, we find the definition of a thing in Aristotle, yet nobody seems to know it.

Many current drawing programs emphasize the fact of the limits, i.e., something which looks like Whimpie's barbells. Suffice it to say, in Geometry we define a thing by showing how to construct an example of the simplest possible thing. It is called also a line-segment, however, when you use a particular name for a particular thing, people often never make it intelligible and their mind tends to spin off into the ozone layer of mythology. One cannot impose any behavior upon the given figure which would make it simpler. During the evolution of the brain, the ability to correlate information is often impossible for that mind.

An arithmetic line segment, i.e., straight line, is then the First Principle thing in geometry.

Now, there is a bit of a complicated metaphor for the above figure suggested perhaps by the Bible, consider it a walking-cane used to aid the blind for guiding one's steps. If you think about it, the mind is blind, the information reaching the mind is at the behest of other life support systems. A mind parses information, and once the information is parsed, it can be ordered, i.e., organized. So, our First Principle thing in Geometry is a walking-cane for the blind and we have to first construct it, pick it up, and learn how to wield it for walking and for defense.

What does the above mean? Well, by biological fact, we are a form of life and every form of life is composed of a number of life support systems. A life support system is that system of a living organism which must acquire things from the environment so that it can process those things for products which maintain and promote its life. A mind differs in one respect to the other life support systems, for each of them take advantage of particular things in the environment, where

a mind must learn intelligible things exampled by everything. Thus, our body is feeding the brain, information expecting, in return, for its contribution to life. A brain is encased in a shell, and a stupid person is no better than a turtle without the contents of its shell.

How many parts of a thing are there?

How do we count the parts? Is the limit at one end different from the limit at the other? And is that difference the difference physically between them? i.e., are they different only by geographical location? Is common grammar, for example, limited to speaking about geographical locations? Or is arithmetic, and algebra so limited? But if that is what you may claim, did anyone ask for a difference of geographical location, or a different part of a thing? Is the difference between the limits of the line, any particular difference whatsoever? Does the word difference differ from itself? If the difference between terms is predicable of either term, what then would you call that difference? The gist is, is the perceptible difference the same as the intelligible difference? Perceptibly we focus on the wrong topic in regard to the question, intelligibly we do not get this distraction. So, one must learn that the limit of a thing is not the thing of which it is a limit, nor is the relative of a thing, relative to the thing of which it is a relative. Many people in the past have considered geometry something they make out of what is perceptible, literal. Geometry is wholly metaphorical. If one asserted that the difference belonged to either relative, then one would be saying that there are factually two relatives, which is not true.

The relative difference between terms is not predicable of either term.

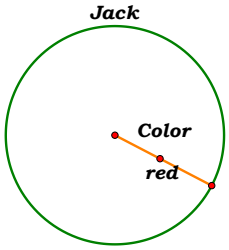
Thus, people differentiate themselves not by what they are, but by some chance relative difference such as color, geographical location, beliefs, gender, religion, etc., but none of those things are defining characteristics of Man. Such claims are made by the mentally dysfunctional. A government which makes laws aimed at religion, gender, ethnic origins, etc., is not a fit government for man.

A predicate are the two parts of a thing, grouped by the recursion of a correlative acting as a container for that group, i.e., Noun equals noun and verb: Viz., Jack is a man. It is called a definitive sentence. The definitive sentence is a unit of recursion in accordance with particular applications. A definitive sentence denotes a container, which may contain non-definitive data, or that which we learn about that thing. These relative differences, which are put into that container, cannot be predicated of that container. The grouping is done intelligibly, something no other sense of the body can do. This means that a definitive sentence is a geometric ratio, or again, metaphorical. The other type of sentence is literal, used for denoting synonyms, or members of a class. Jack is red, is not the same as Jack is a color. A literal statement is not definitive, it is arithmetic. Jack is red translates into Jack + red and then we have to ask, how was this addition made? By blushing? At any rate, Jack is now a data set, which does not, in anywise, change the definition of Jack as a man, red is not a defining characteristic of a man, nor is gender, beliefs, etc. A literal sentence, equates

either two nouns or two verbs or both. Jack is red, is divisible into A man is A color. Fast is good. All of this should be obvious, however we have to learn how to parse in common grammar, and we should learn first using Arithmetic, Algebra and Geometry for it will take time to apply these notions into sentence parsing.

I can draw Jack is a man.

But now, the relative differences to what designates man, is a verb. Jack is a man, Noun equals noun + verb, and man is a verb. How do I add such particular relative differences to Jack such that I do not change the relative difference of Jack as measured by the standard of man? If I say Jack is red, I can draw that too.



And I am simply creating a synonym, the complexion of Jack is an example of the color red. The problem, currently with common grammar is that generally common grammar consists of terms which are not well defined but contextually defined. What is clear, there is no inference between the two logical statements as shown by the analogic. What should be clear, the first example is arithmetic, while the second geometric. Geometric, as we can see by the second example, is a proportion. Thus, in recursion, we can keep track of both.

A line, our binary, becomes universally expressed, in a circle. An idiot can repeat a rule, but shows no ability to use it.

Now, if one has read Plato's Theaetetus, then one should have learnt that these two parts of a thing are also called a thing's elements and that the limit is not the relative, nor the relative a limit. Stop, go, stop, and stop is not go, and go is not stop. Although every child is assumed to have learnt this, most PhD holders demonstrate that they have not otherwise they would know that it is not possible to have more than two parts of speech as Plato tried to teach them, and as our own biology tries to teach us. For me, public education was very scary. Grown-ups, one and all, teaching gibberish, obviously mad. The fact of the matter is, in order to get an education, in a safe environment, one has to study in their own home. When one finds they are living in a world of mentally dysfunctional people, people they have to rely on, then one has to learn to anchor themselves. I was taught how to do that in a dream. I was shown that I had the ability to do it, even though it is hard. Fear is the mind killer, but then so is any

emotion when not comprehended. You have to change people's mind and that is done by education, but first one must become educated.

As exampled by the computer, which can also process all information using binary recursion and produce any product with that ability, Grammar is binary, as Plato noted, composed of only relatives and correlatives, nouns and verbs, etc. So, we need to use that to come to an understanding of grammar systems. Given that grammar systems are things, then it is composed of just two parts, relatives and correlatives. Which part is correlative and which part relative?

Reviewing, the correlative part of grammar systems are its symbol sets. The relative part of grammar systems is the methods by which we use the symbols to effect binary recursion. In common grammar, also, we have many behaviors of the hand to construct the symbols and we have many behaviors by which to assign those symbols to name things.

In arithmetic, we have also, many symbols but the behavior imposed upon them is rooted in place value notation. Names are ordered. However, in arithmetic the subject of any particular computation is one thing; often not even named.

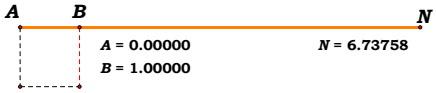
Algebra combines common grammar and arithmetic, taking symbols from both so that the grammar can effectively denote many interacting things.

Geometry on the other hand is established by a one-to-one correspondence between relative and correlative. It is also the only grammar for, as Plato noted, producing universal perceptible paradigms, working examples.

And therefore, two multiplied by two, produces for us a Grammar Matrix: Common Grammar, Arithmetic, Algebra and Geometry, put into Biblical metaphor, the four corners of the earth, or four methods of effecting standards of judgment. With this knowledge, we should be able to figure out, because it is generally unsorted, that Common Grammar requires the most knowledge and work to comprehend, and therefore our preliminary work will be learning Arithmetic, Algebra and Geometry first, as Arithmetic and Algebra are much easier, because simpler, than Common Grammar. Many people believe that it is just the opposite because they have become habitualized into not thinking about what they are saying, parroting is not thought. One cannot judge the difficulty of a thing, when they factually are ignorant of it.

It is not possible to be literate in Common Grammar, but not in the other members of our Grammar Matrix. The operations performable upon things are identical in every member of our Grammar Matrix, and if we do not know them, we cannot apply them, we are not rational because we can glue words together independent of factual operational mechanics. Every system of sentence parsing that I have seen to date, indicates that those who constructed them were simple minded and illiterate. Everything that they learnt in arithmetic, algebra and geometry was simply not used, but they deferred to the mist in their own mind. So, let us learn the paring of Arithmetic, Algebra, and Geometry to help us anchor our mind upon the ground so that even if we were one of these hot air balloons, we would never let the winds take us away.

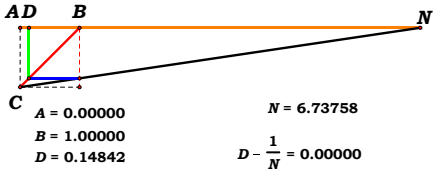
I will show the Four Corners of the intelligible earth, a way to manage information using geometry.



AB is the One, while AN is the Many. Notice the equation between Algebra and Geometry, keeping in mind that Algebra is a combination of Common Grammar and Arithmetic, which means that we have a grammar matrix visualizable in geometry. It is not a number line, but a Grammar Matrix and I will be showing, in the following pages, how to use it for Arithmetic and Geometric recursion of our binary. In geometry, the computations are always exact, and there is no processing time, i.e., output is concurrent with input, meaning faster than light travel as it is independent of speed.

So, how do we map the equality between the unit, AB and the result AN? AB is the One, while AN is the many. AC is our method of showing the relationship between them.

And we map the relationship like this:

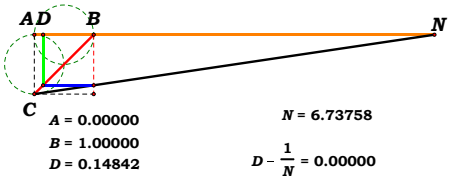


For centuries, Geometer's have tried to comprehend this relationship between the one and the many, and have failed. Yet, take a look at it, it really is simple, is it not? With a single line, BC, we show the relationship over the whole of grammar, the idea is simple, its application is never ending. Therefore, the intelligible is not complicated, it is using it for each particular application which is often difficult.

So, consider, in computation, that AB is a correlative, while AN is a relative. AB is a Noun, while AN is a Verb, therefore, even in arithmetic, the numerator is the noun, and the denominator, our verb. We have two, and only two, categories of names, and we must take a moment to identify which is which, when we think. Our Universal Binary, denotes this division between a limit, and the material difference within those limits. Think of it as a stick, and taking up another, we are rubbing them together to make fire, to turn on the light.

However, the Grammar Matrix, denoted above, has yet to clearly denote the distinction between arithmetic recursion and geometric recursion. We can

recursively use the Unit in two ways, as denoted above. I will make an addition, which may in time, become clear to you.



Performing the same mathematics with the circle, as with a line, is a bit more difficult to comprehend, but hopefully we will get there, step by step.

So far, we have our bare binary, a method of effecting it arithmetically, by straightedge, and geometrically, by compass, and some media to record the memory of their interaction. Until simple minds can comprehend the intelligible of Geometry, they can only repeat, straightedge and compass as tools, forgetting that media or memory, is our most important tool.

Descriptions.

Pairing the Arithmetic Unit and the Geometric Unit

$EO := 1 \quad D := .33444 \quad EF := D \quad EG := 2 \cdot EO$

$FG := EG - EF \quad CF := \sqrt{EF \cdot FG} \quad AC := 2 \cdot CF$

$FO := EO - EF \quad AB := 2 \cdot FO \quad N := \frac{1}{EF}$

$CJ := \sqrt{AC^2 + FO^2} \quad BC := EG$

$AC^2 + BC^2 - \left(2 \cdot CJ^2 + \frac{AB^2}{2}\right) = 0$

Definitions.

$EO - 1 = 0 \quad D - D = 0 \quad EF - D = 0$

$EG - 2 = 0 \quad FG - (2 - D) = 0$

$CF - \sqrt{D \cdot (2 - D)} = 0 \quad AC - 2 \cdot \sqrt{D \cdot (2 - D)} = 0$

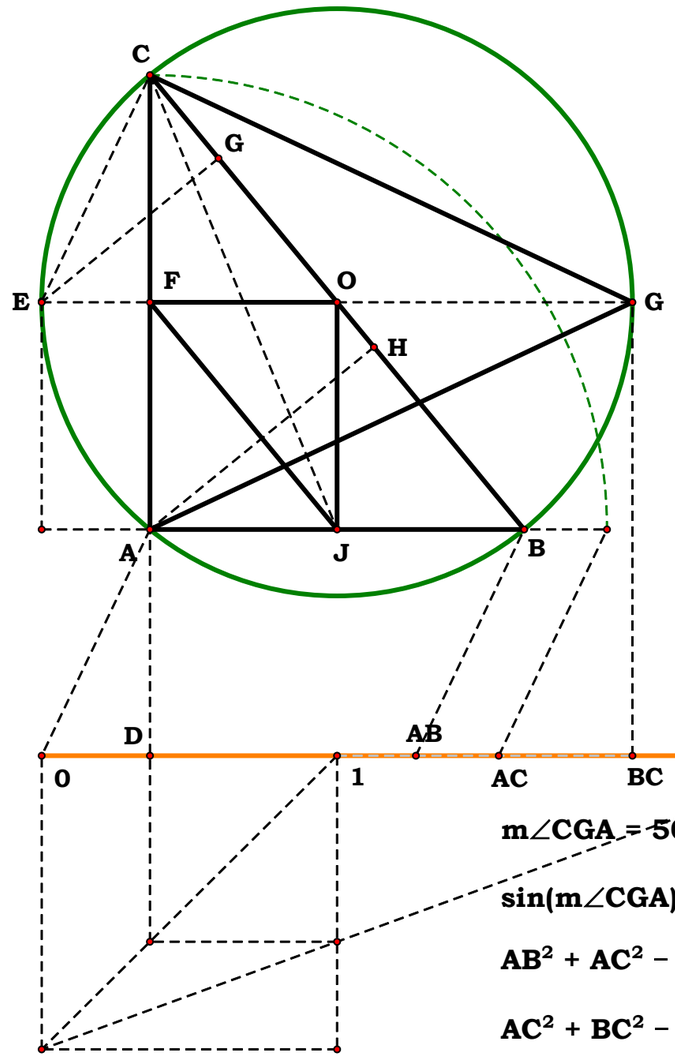
$FO - (1 - D) = 0 \quad AB - (2 - 2 \cdot D) = 0$

$AB - \left(2 - \frac{2}{N}\right) = 0 \quad N - \frac{1}{EF} = 0 \quad AB^2 + AC^2 - 2^2 = 0$

$CJ - \sqrt{6 \cdot D - 3 \cdot D^2 + 1} = 0$

$AC^2 + BC^2 - \frac{4 \cdot \left(N^2 + 2 \cdot N - 1\right)}{N^2} = 0 \quad AC^2 + BC^2 - \left(8 \cdot D - 4 \cdot D^2 + 4\right) = 0$

One to One



0 = 0.00000
D = 0.36648
AB = 1.26703
1 = 1.00000
AC = 1.54746
N = 2.72863
BC = 2.00000
CJ = 1.67212

m∠ACB = 39.30997°
m∠ABC = 50.69003°
m∠BAC = 90.00000°

$\sin(m\angle ACB) - \frac{AB}{2} = 0.00000$

$\sin(m\angle ABC) - \frac{AC}{2} = 0.00000$

$\sin(m\angle BAC) - \frac{BC}{2} = 0.00000$

m∠CGA = 50.69003°

$\sin(m\angle CGA) - \frac{AC}{BC} = 0.00000$

$AB^2 + AC^2 - 2^2 = 0.00000$

$AC^2 + BC^2 - \left(2 \cdot CJ^2 + \frac{AB^2}{2}\right) = 0.00000$

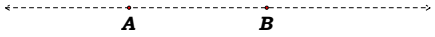
Recursion

Our first recursion, known as complete induction and deduction, of our binary actually takes place by our ability to put it into one system of grammar, the grammar of our own behavior with our own hand: Geometry.

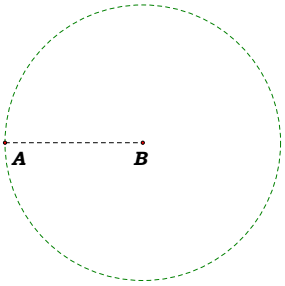
Recursion is afforded by Complete Induction and Deduction. Induction and Deduction are effected by both noun and verb producing; addition, subtraction, multiplication and division. The recursion of these will yield even more specific tools.

Starting with Geometry, we noted our second example of recursion, writing our First Principle Thing realizing that it is a memory stick of the application of correlatives:

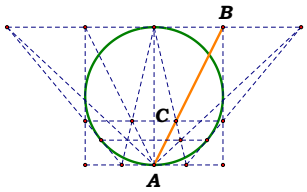
Recursion is two kinds, explicit, for example, we can always make our FPT. The second is implicit, i.e., we could choose to have made the FPT any size we wished, i.e., we could have stopped our hand anywhere in construction, and we can place any number of correlatives within that same difference. The first method, is also called induction, establishing the perceptible. The second is called Deduction, using what we have to proceed with reasoning, again, managing the given data. Let us examine more sophisticated methods of recursion using our FPT.



What I am going to write are possible methods of recursion, both effecting deduction and induction. One of the items a compass affords us is memory not only of our FPT but also immediate recursion from either correlative which denotes that unit becomes also a container.

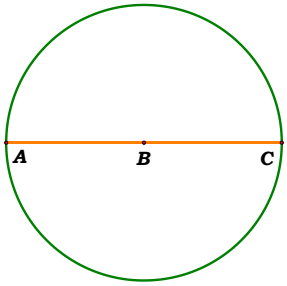


We can also produce what is called the circle at least two different ways using just lines, and writing it as a function of lines. In other words, we can learn about the universe of discourse, by exhaustive use of the unit, itself. So, we can denote it first, or learn of it last. One way is with a unit line, the other is produced with an infinite line, as a reciprocal. One can say that beyond small and great, is the circle. A circle is the reciprocal of the line. Or again, the verb is the reciprocal of the noun, and the noun is the reciprocal of the verb.

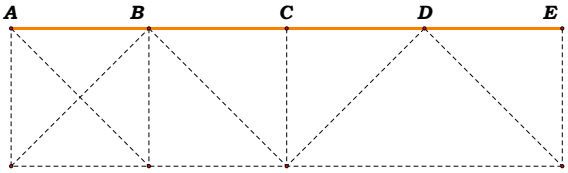


Perhaps it is well that this was not discovered first, as there would be those claiming that the only tool used in geometry is a straightedge, without paper. So, no, a compass is not just for drawing circles, nor are straight lines the only possible loci for the assertion of correlatives. And since the equation for the figure can also produce an ellipse, so no, an ellipse is not produced by cutting through an ice cream cone even if it is the right flavor or not. And as you can see, all of geometry is effected by binary recursion, not the tools which aid us in writing in the grammar. I need a break. Do I say I squared a circle, or bounced around a square cage? At any rate, do not show stupidity claiming that a geometric construction is Euclidean, unless by choice, or the product of two tools made optional by a digital environment. Or, I can say the circle exists a priori in the realm of the unenlightened flat screen; if I had a tire pump to inflate myself. Or provably, I walked straight on forever, and eventually found myself back home again.

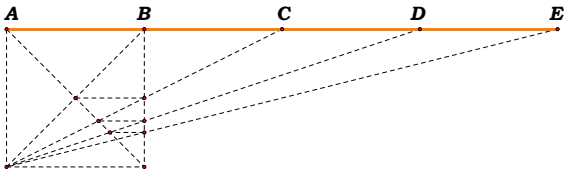
Another way to produce recursion, using the compass is in this wise. This produces arithmetic induction.



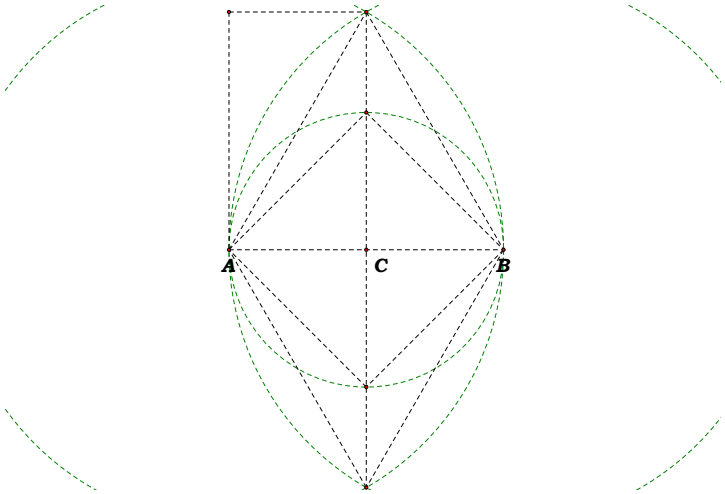
Demonstrated with just a straightedge



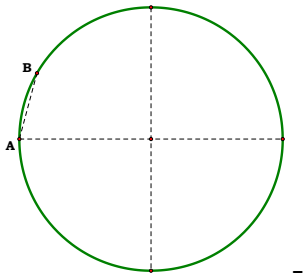
An example of using straight lines and proportion



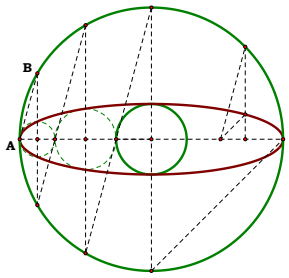
Recursion by deduction leads to an interesting figure:



The figure presents us with all the fundamental topics of Euclid's presentation of the elements. However, something is missing, and apparently has been missing since the dawn of geometry, for when I once read an algebra book, the author claimed that exponential notation was not geometrically demonstrable, that it was a pure conceptual abstraction, which, frankly, is impossible, at least to me, for early on in my investigations, I was demonstrating it.

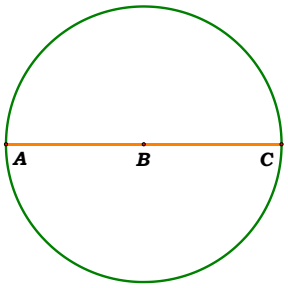


With only a straightedge, divide the circle by AB.

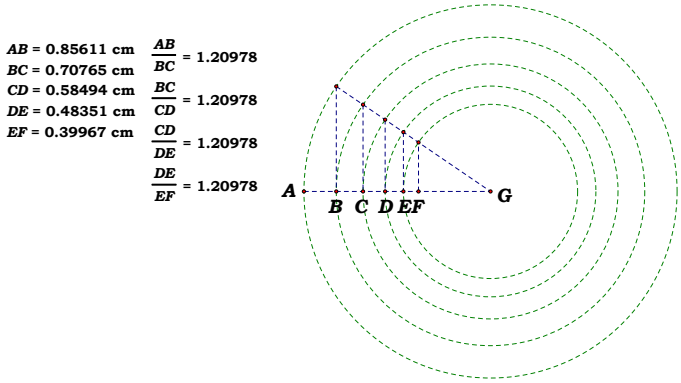


I not only divided it arithmetically, but geometrically as well, and only using a straightedge. Thus, we see, before our eyes, that working with angles is only the product of a more advanced method of effecting arithmetic by the hand. With the straightedge, I a made a tool which produced the ellipse using the proportion expressed by the angle of AB. And we see the diameter, divided by equal angles, which is really advanced arithmetic. Imagine that, dividing a straight line by equal angles.

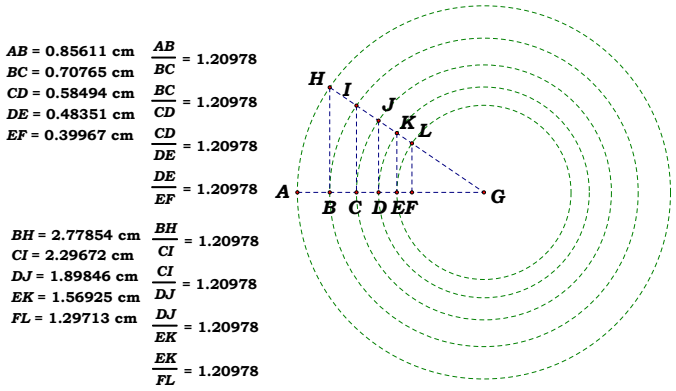
So far, in the above forms of recursion, we find one missing, for all of the results are actually arithmetic recursion. What is the mistake? How do we show the geometric recursion of the circle?



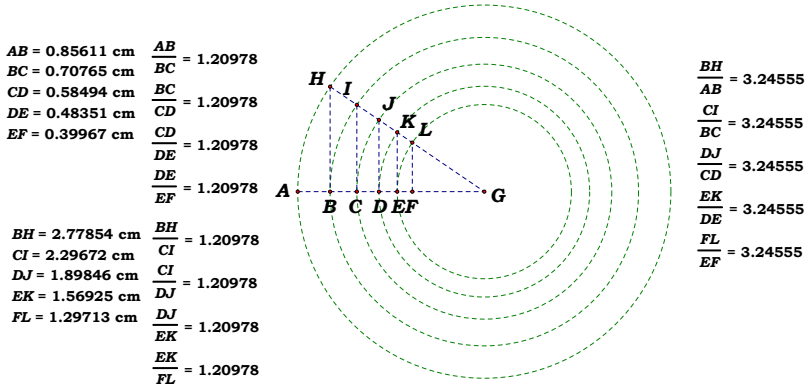
The above figure shows how to use the circle for arithmetic induction producing an arithmetic progression, and we can recursively apply the circle in that manner, but what constitutes the recursion of the circle deductively, geometrically? And since it has apparently been a thorn in geometer's side for so long, let me show it step by step where, given a circle we are going to add another line within the circle and learn geometric recursion.



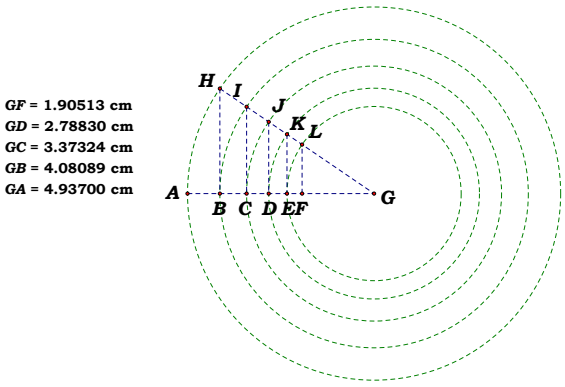
We find we have another kind of unit called a geometric ratio, where in primitive times it was called an angle, and it was in the opening book of Euclid, defined as a ratio, i.e. geometric, not arithmetic, which readers quickly forgot. In the Elements of Euclid, it is defined as an inclination between two lines, i.e. a ratio which is not what people consider an angle to be today, where it is something mystical emanating from a not thing, a single and lonely correlative called a vertex or point. Current so-called teaching, does not account for the hole in the idea, the missing second correlative.



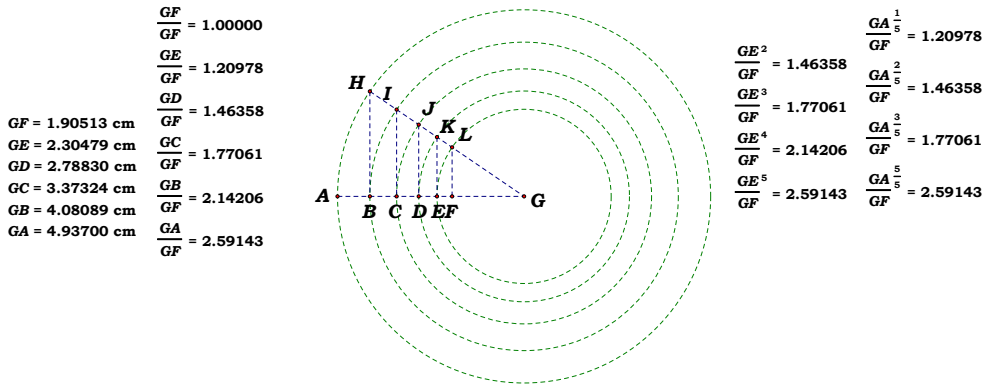
And, if we compare the lines vertically, they are in the exact same ratio.



The inclination actually turns out to be 3.24555 in this example and it has absolutely nothing to do with the antiquated concept of degree which is just a an agreed upon number by illiterates. Once we have our Geometric unit of progression, we can pair it to an arithmetic progression. Let us take what remains, FG and call it a unit.

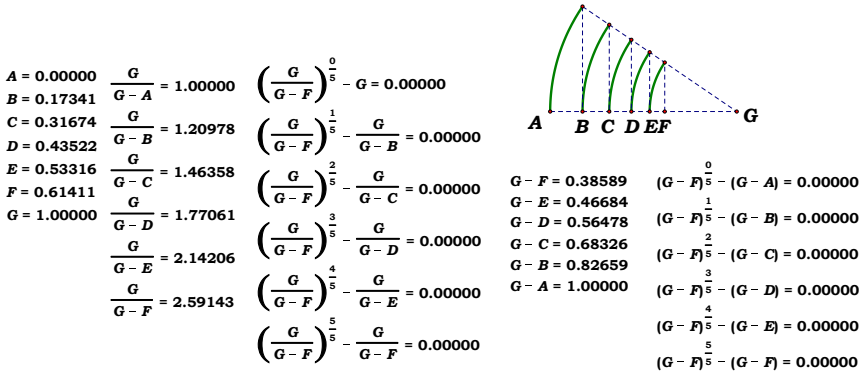


Using any system of measure, we desire; we note the differences. If GF is our unit, we convert the series into that unit like this.



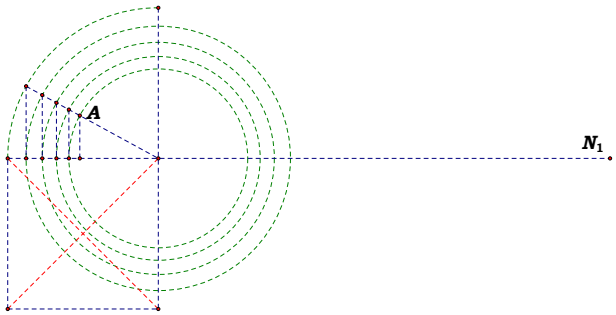
Thus, the only difference between powers and roots, is the direction one is reading from. So, all the properties of exponents are easily demonstrable in geometry, the only question which is at the top of the list is, how does one pair an arithmetic progression with a geometric progression, by way of each's unit?

Lastly we conclude:

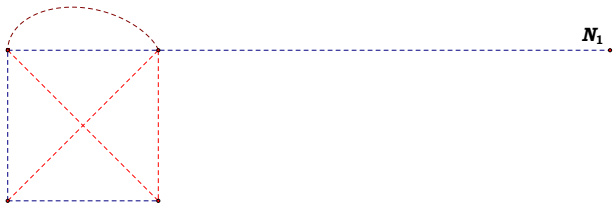


You find today, in elementary Algebra, the explicit, or implicit statements that exponential manipulation is not possible in geometry, that it is a product of inexplicable human genius. So, what does it mean when scientist all over the world believe that they can predict the birth, life and death of the Universe, but cannot predict the behavior of a simple straightedge and compass, the product of a simple line and circle? It is called schizophrenia, even megalomania. Parroting, mixed with mythology, is symptomatic of a dysfunctional mind. Am I Superman, waiting for his outfit to be returned from the cleaners, so I can go out and buy a ticket to board a flying rock to other dimensions? Or am I just another ape scratching his head?

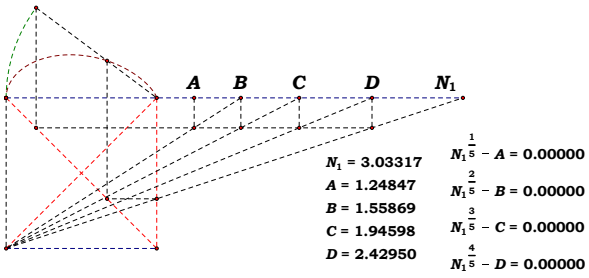
As geometry is that grammar by which one can demonstrate Total Recall by Total Memory, how do we produce that memory for exponential progressions?



I first decide how many geometric units I want to record; I will stay with five; the only point I need to record then is A.

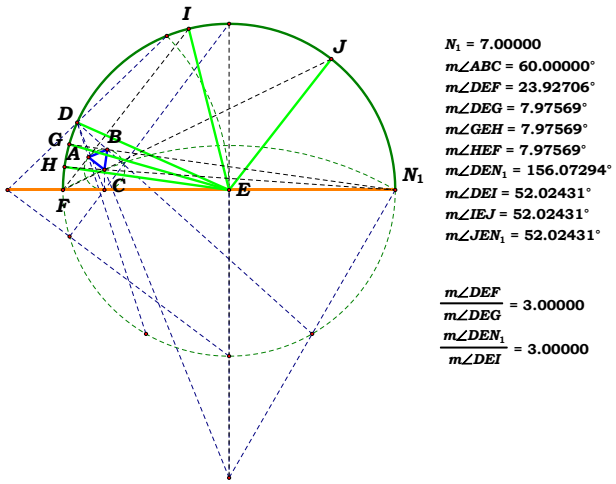


Ignoring everything else but the result, we can now use it to form a fifth exponential series with any N as such. Notice it is not a power or a root, but a fifth in a series.



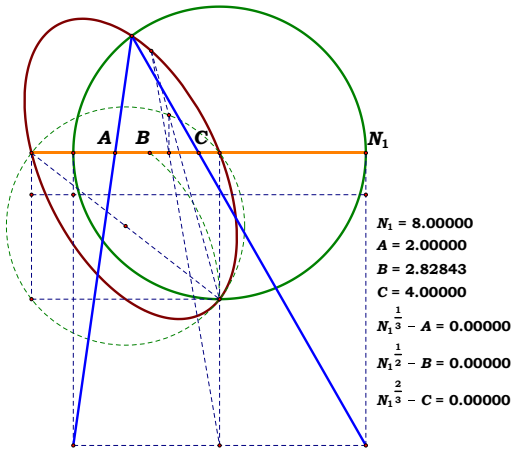
Thus, any geometric series can be applied to any arithmetic. Any claim that I can record and recall or some correlatives by some custom, making a thing different from itself, is just absurd.

In all of this, I am not saying that there are no methods of applying the division of the circumference arithmetically: It actually can be done if one is willing to take the time to work out each figure. However, again, what you find in literature is the simple minded conchoid, not the relationship between actual induction and deduction of the unit, as in the following figure.



However, it too depends upon proportion, but not a good figure for explaining elementary recursion. One can, however, also use memory to find 1 degree, or a portion of it. What use it has, I can not say but one can spend a lot of time slicing and dicing circles.

And I have also shown a rude solution to the Delian problem which again, is not elementary:



Nor perhaps is it very useful as a topic of Metaphysics at this time.

Now, if one is able to wrap their brain around the elementary ideas of Metaphysics, actually demonstrated, the next step is rather mundane, demonstrating simple mathematical operations with our Grammar Matrix.

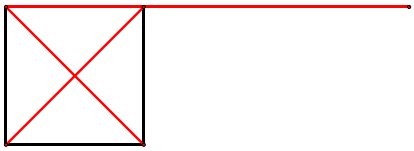
Grammatical Operations

Are derived from Common Notions afforded by sense perception

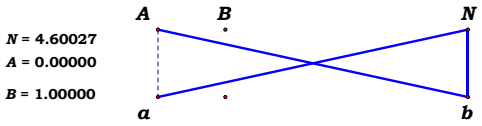
The Reciprocal

June 3, 2023

Nomenclature

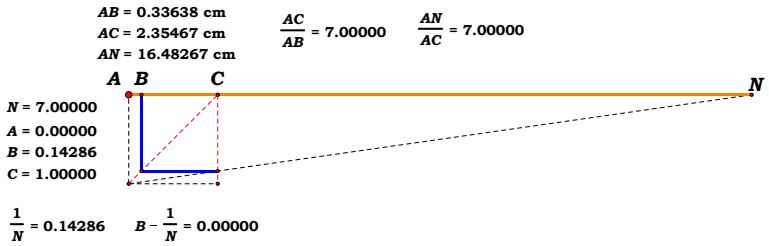


The above is called our grammar matrix and it is as a pot which we prepare all of our food in. The next is showing how to use that pot. When I was a kid, I hung around my mother a lot, and my father when he got home from work also. My goal was to watch and learn. I watched and learnt because some day, I wanted a wife and family of my own. Unfortunately, I learnt how to cook, clean and do laundry from my mother, sew and mechanics from my father, and how to avoid wannabe intellectuals from our schools, but never found a wife who learnt much of anything or any clear idea of what they wanted, nor any force of will or truth in them. All of the materials I acquired for learning about things, taking them apart and making things, I acquired by dumpster diving, perceptible and intelligible.

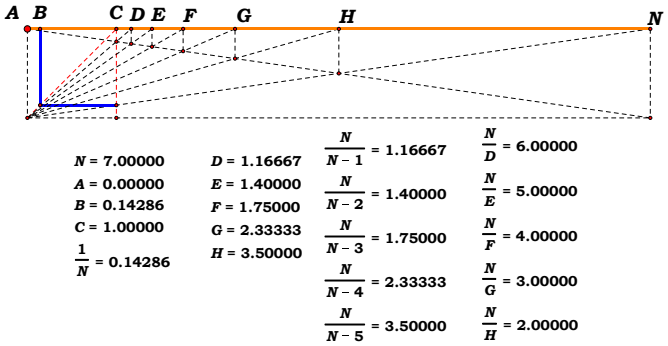


If we are working with any correlative N, we have immediately at our disposal, what I call operational tails, aN, Ab, and Nb which we use to perform operations with. As we go along, we will put them to use. Notice that two operational tails, Ab, and aN, are useful in the box, unit box, and Nb is outside the box. So, we can think in the box, or outside of the box. Ain't that just yuppy? When using any correlative, there are ways of indicating its influence by the use of operational tails. Each correlative actually has a constructable web, which we eventually call upon in our demonstrations. So, every correlative is not only a point, it is either a starting point or a point of termination of correlatives we apply and remember. Learning is using the in-betweens to communicate with. However, we cannot just assert a random point and claim some operation by it. It has to follow by using the givens.

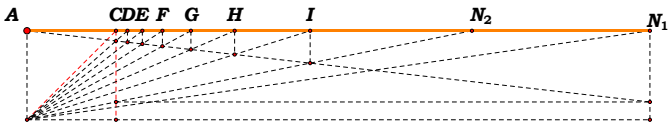
Let us now use our operational tails starting with the creation of a reciprocal. Given any-thing, show its reciprocal.



AC : AB :: AN : AC, which is another way of denoting a reciprocal and it is one way of asserting what the unit is in any situation where the points are not marked. Logically, If AC is to AB as AN is to AC, then AC is the unit relative to A, B, C and N. The reciprocal is commonly called a fraction. Notice, when you use the figure, that the result is concurrent with any change in N. Let us add a tail and with it effect recursion of our result.



Suddenly we enter the realm of Algebraic operations. Now, how would I apply this result using that recursion to divide any other difference whatsoever? Let us draft N_2 upgrading N to N_1.



$N_1 = 7.00000$
$N_2 = 5.00000$
$A = 0.00000$
$C = 1.00000$
$N_1 \cdot N_2 = 35.00000$
$N_2 - 1 = 4.00000$

$D = 1.12903$
$E = 1.29630$
$F = 1.52174$
$G = 1.84211$
$H = 2.33333$
$I = 3.18182$

$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - (N_2 - 1)} = 1.12903$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)} = 1.29630$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)} = 1.52174$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)} = 1.84211$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)} = 2.33333$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)} = 3.18182$

$\frac{N_1}{D} = 6.20000$
$\frac{N_1}{E} = 5.40000$
$\frac{N_1}{F} = 4.60000$
$\frac{N_1}{G} = 3.80000$
$\frac{N_1}{H} = 3.00000$
$\frac{N_1}{I} = 2.20000$

$\frac{(N_1 \cdot N_2) - (N_2 - 1)}{N_2} = 6.20000$
$\frac{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)}{N_2} = 5.40000$
$\frac{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)}{N_2} = 4.60000$
$\frac{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)}{N_2} = 3.80000$
$\frac{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)}{N_2} = 3.00000$
$\frac{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)}{N_2} = 2.20000$

$\frac{N_2}{D} = 4.42857$
$\frac{N_2}{E} = 3.85714$
$\frac{N_2}{F} = 3.28571$
$\frac{N_2}{G} = 2.71429$
$\frac{N_2}{H} = 2.14286$
$\frac{N_2}{I} = 1.57143$

$\frac{(N_1 \cdot N_2) - (N_2 - 1)}{N_1} = 4.42857$
$\frac{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)}{N_1} = 3.85714$
$\frac{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)}{N_1} = 3.28571$
$\frac{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)}{N_1} = 2.71429$
$\frac{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)}{N_1} = 2.14286$
$\frac{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)}{N_1} = 1.57143$

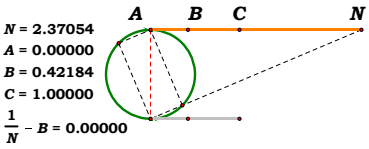
We see that we can proportion any result of N_1 by using its unit tail set in proportion by N_2. The result is, a lot more information to consider than we had with the simple reciprocal.

Well, just the reciprocal puts our little Grammar Matrix, to the test. With a couple of basic moves, we are deep into Algebraic series and we are not limited at all by what is considered irrational in Arithmetic, in fact, irrational only means the inability to apply an exact name in that convention of naming. Let us examine

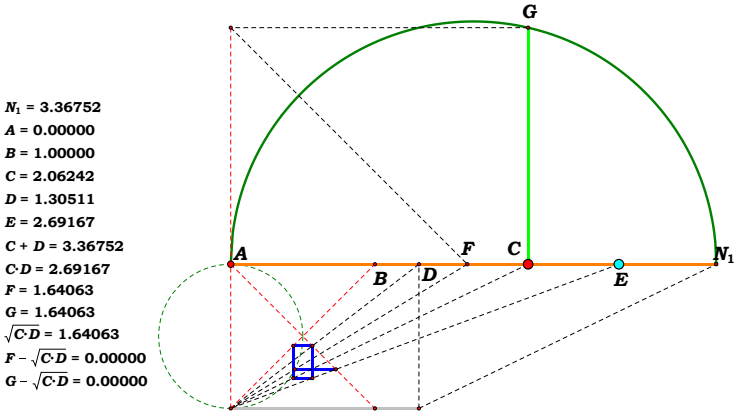
the simple operations in of themselves as it is clear, our grammar matrix tells us they can be performed.

The Circle

The easiest way to draw a circle with a straight line is, of course with the reciprocal like so:



In order to make any size circle, it is probably a good idea to know the equation for a circle, which one can acquire from the Elements, or one can get the wrong equation from the Internet.



Therefore, one can draw a unit circle with straight lines and produce a proportion of it to any desired size simply as a function of a reciprocal, a reciprocal of the entire range of possible magnitudes; i.e., from the infinitely small to the infinitely large, all compacted in the unit also known as Complete Induction and Deduction. The phrase Complete Induction and Deduction confuses people as they interpret complete in terms of something perceptible, when it infers the intelligible, meaning it can always, under any circumstance, be demonstrated for any-thing. Would not want anyone floating off into the ozone layer of so-called Transfinite Mysticism.

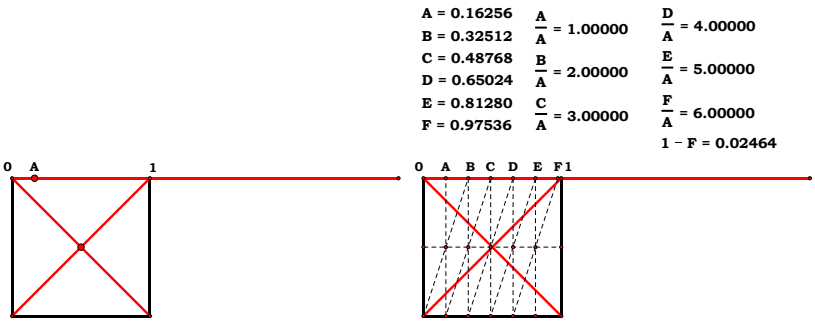
The circle, which became producible by the compass, has now been demonstrated to be a function of Complete Induction and Deduction of binary recursion as the reciprocal of a unit. The circle is not the only product of binary recursion, so it is a sign of feeble-mindedness to claim that geometry does not function completely with all curves.

Keep in mind also, that the reciprocal of the correlative is the relative, and therefore, there is absolutely no prohibition against any demonstrable correlative

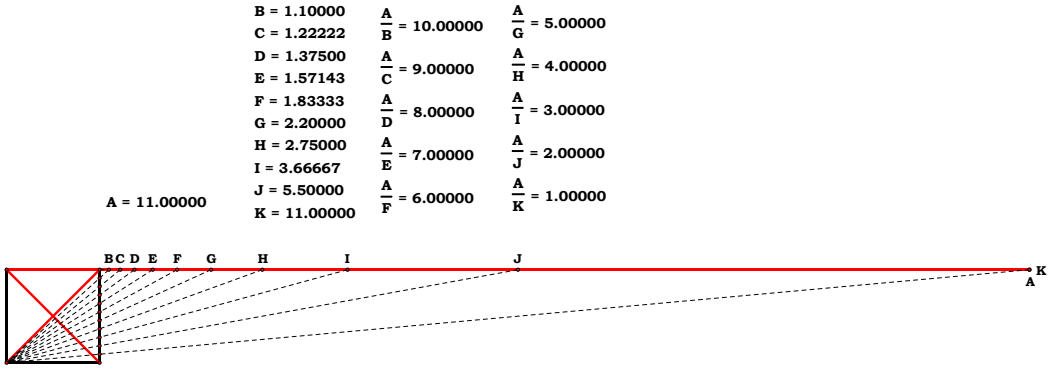
path. What this means is that in my Basic Analog Grammar Dictionary Sampler, are solutions to very complex problems just waiting to be applied.

What should be clear by now, is that just like in the Elements, the circle is the product of the recursion of a simple segment, it is not the product of a compass, a compass is only used for memory, just like the paper we draw on, real or virtual. You can believe all the wannabe geometer's in the world about the compass, or you can believe your own eyes by a production with your own hand.

Given, AB, divide the unit by it.



Notice that we can divide a circle in the exact same method as a straight line and with, relatively, the same result.



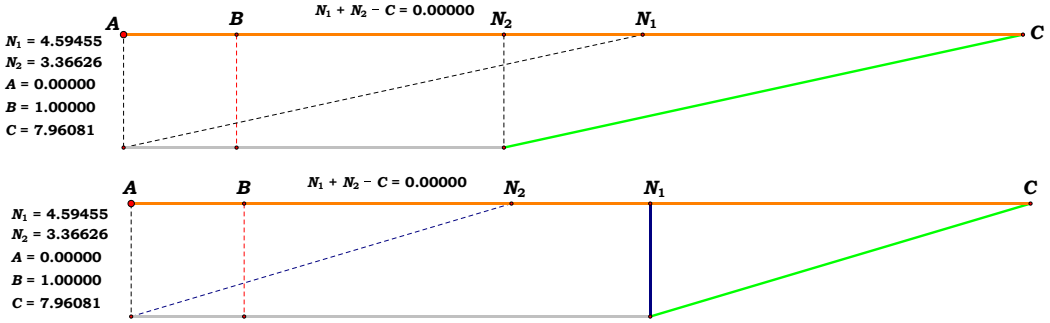
What one can do on one side of a pan, allows one to do something wonderful on the other.

Addition

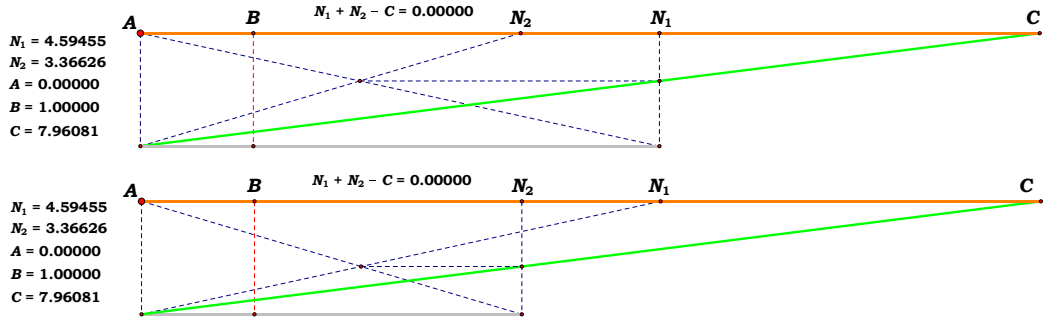
Tuesday, June 6, 2023

We effectively have two operational tails, one is used arithmetically and the other geometrically, thus we can add any two values using each one, and the result will be identical. Because the result is identical, we tend to think of Addition as a single operation, when it is in fact performed in two different concepts of a unit, principally as a noun, or principally as a verb.

I will, therefore example the arithmetic method of addition first, the unit will be constant. Arithmetic addition requires both operational tails, and to demonstrate how plastic the figure is, I will demonstrate the two possible configurations for them.



The following two figures will demonstrate addition using proportion, or geometrically. Here the main operational tails form a ratio which is projected to the unit tail and from there projected to acquire the result.



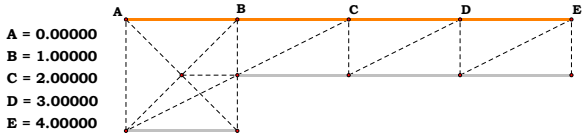
In addition, the order to the terms is irrelevant. This is not true of subtraction.

Addition

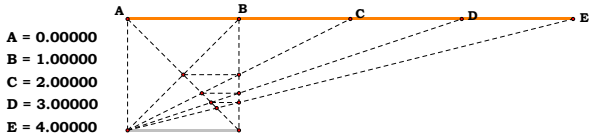
Construct the Number 4

It has to be a minimal requirement to add one to one.

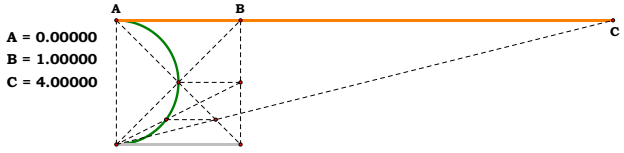
Now, if one has studied Euclid, one should find this easy, under the heading of equal parallelograms.



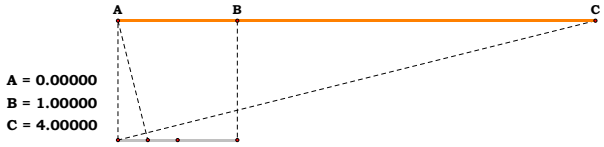
or



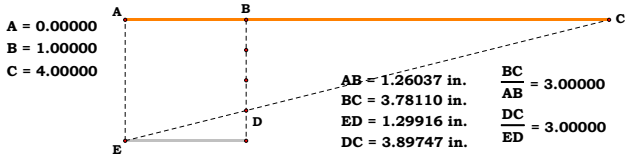
or as two squared.



Or the reciprocal



most rudimentary by dividing our original one

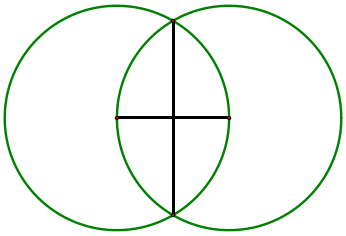


As AB is to BC, so too ED is to DC, kind of thingy. Three to One means one has cut 4, into two parts, a unit, and the multiple of that unit.

Although it may not be apparent, any angle greater than 0 and less than 90, can be standardized in accordance with an arithmetic progression.

Or again, any relative difference, or verb, be it expressed by a synonym like angle, magnitude, number, etc., is capable of being set into standards of units.

Therefore, Euclid's first figure, if you were able to draw it,



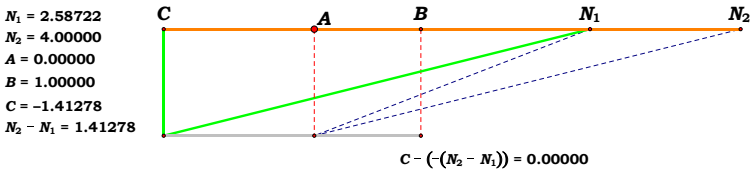
implies that you are able to do the whole of mathematics using geometry providing you can add one to one, physically and mentally. In that figure, you factually have expressed, the Universe of Discourse both arithmetically and geometrically, the whole foundation of proportion. And so, the same way you divide a line, you can also divide a circle.

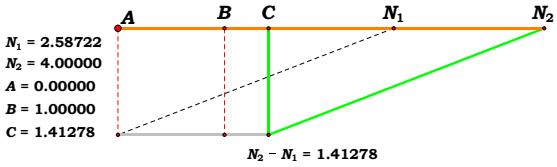
So, some people can look at a thing, and simply repeat it, and others can look at the very same thing and discover the whole of elementary mathematics, or binary recursion.

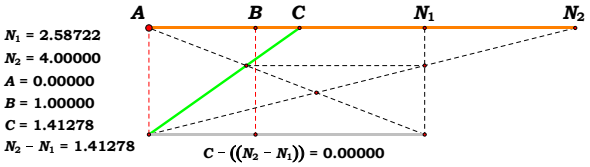
Subtraction

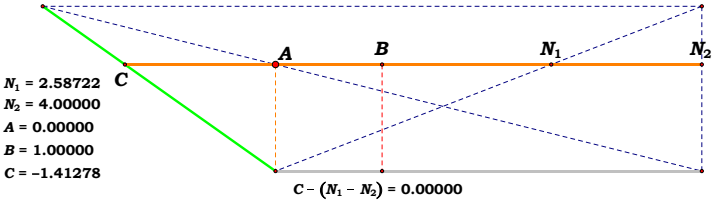
Wednesday, June 7, 2023

Again, the first two equations are Arithmetic and the second two Geometric.





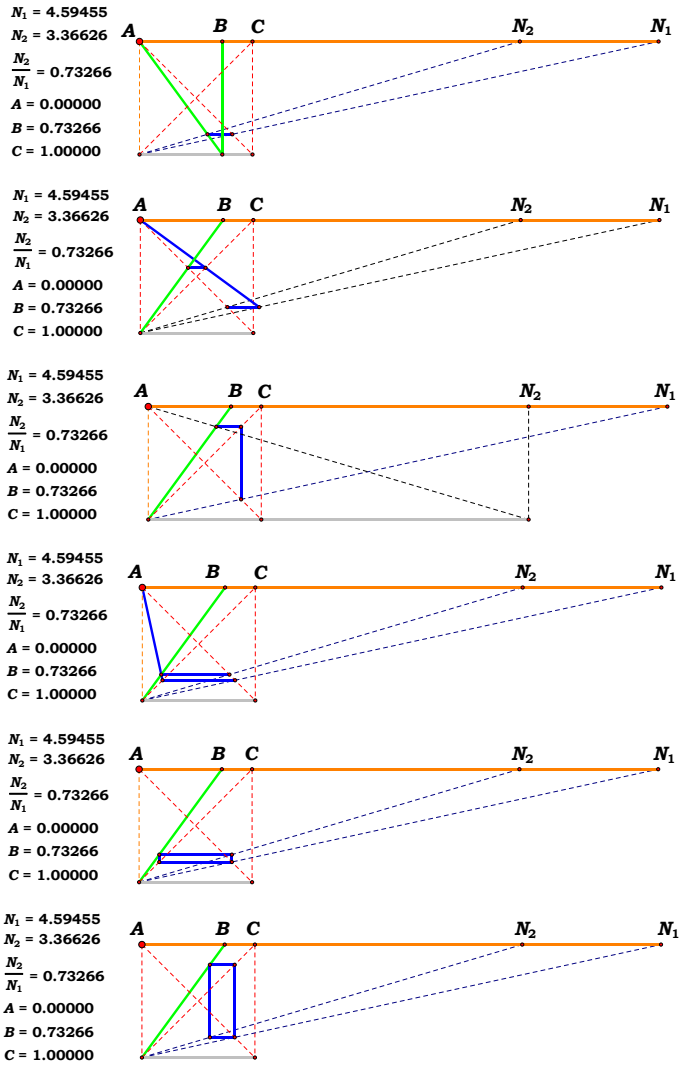




Division

Wednesday, June 7, 2023

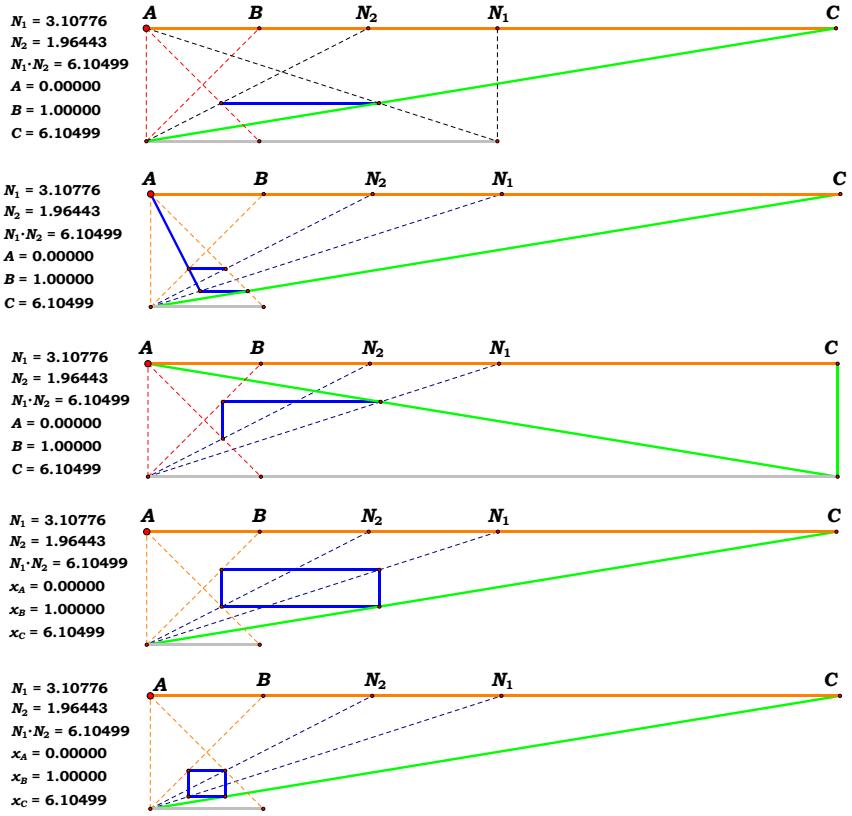
For division, I rounded up six plates. I imagine them in pairs, but as all these plates, I do not imagine that I have discovered all that is doable.



Multiplication

Thursday, June 8, 2023

I only keep five methods of multiplication handy. The reason for only five, is I believe one plate actually turns out to be redundant. The retention of the plate is then just too boring to tolerate.

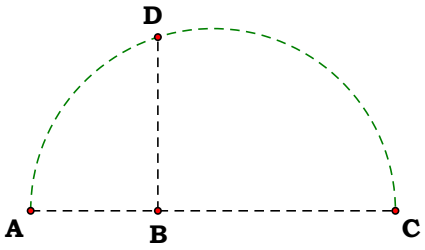


Of these four mathematical operations, one can choose which method for any particular is their fav, or keep their work spicy.

Exponential Series.

Thursday, July 6, 2023

What is the origin of Exponential Series? Exponential series originates from the fact that the compass, the circle, is the product of multiplication and division, it can be constructed both ways. Euclid gives a standard figure as;

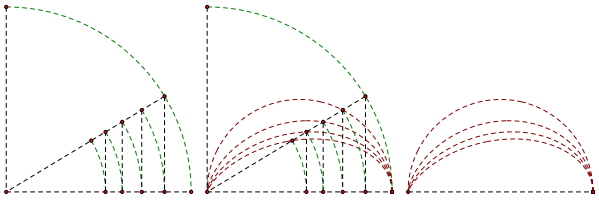


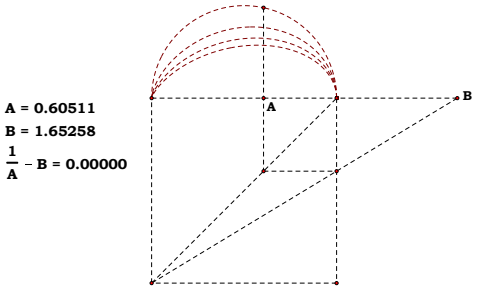
B = 0.34875

D = 0.47657

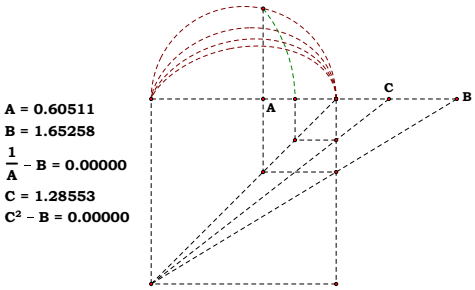
$\sqrt{B \cdot (1 - B)} - D = 0.00000$

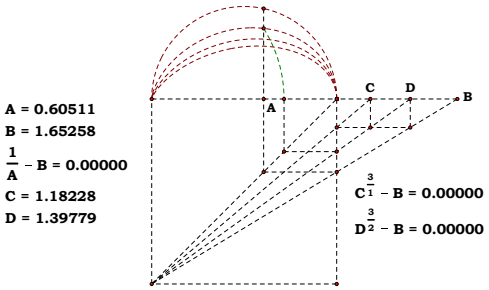
So, we are given the straightedge, by which to produce any unit, and the compass which provides us all of our basic mathematical operations. Therefore, with the given intelligence, we should be able to figure it out; the circle is the reciprocal of the unit which implies, it does the math. So, let us show how to use what is given to show the math.

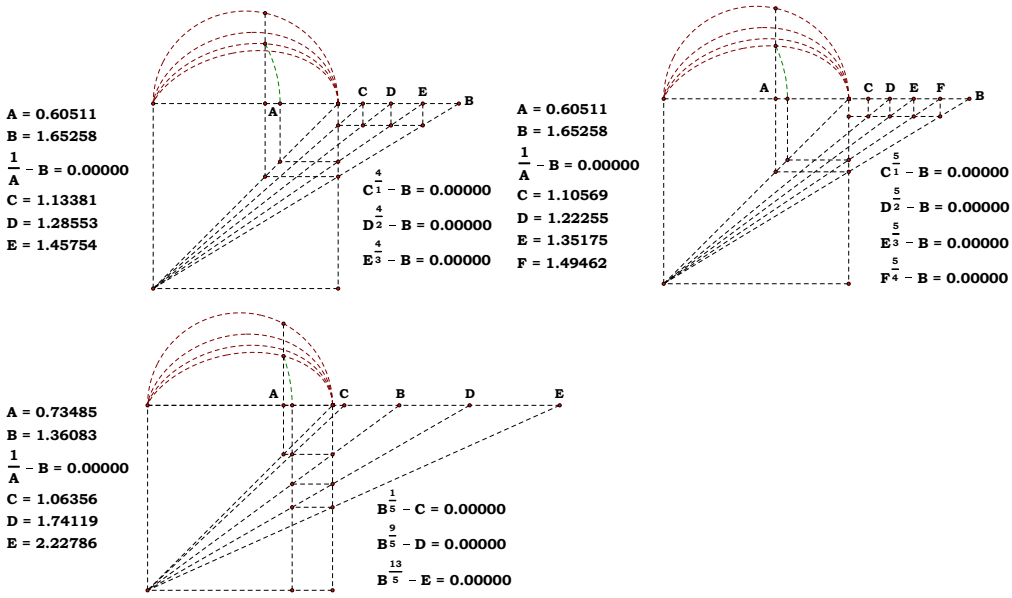




Our first reciprocal constructed the simple circle, each succeeding one, the continuation of the exponential series, or continued proportion.





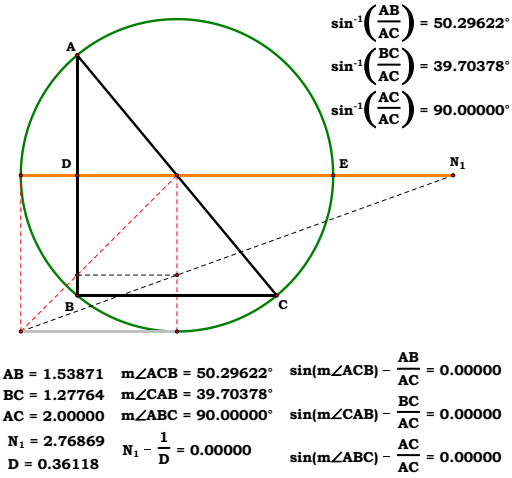


What we call mathematical operations, or grammatical operations, are simply the result of the definition of a thing as a binary construct. The question to be answered, is how long does someone have to hang on to a tool, and yet be ignorant of what it means? As every grammar is the product of binary recursion, the a tool which is not a tool for constructing the binary, is used for the recursion of it, in anyway it can.

The Angler

Thursday, July 6, 2023

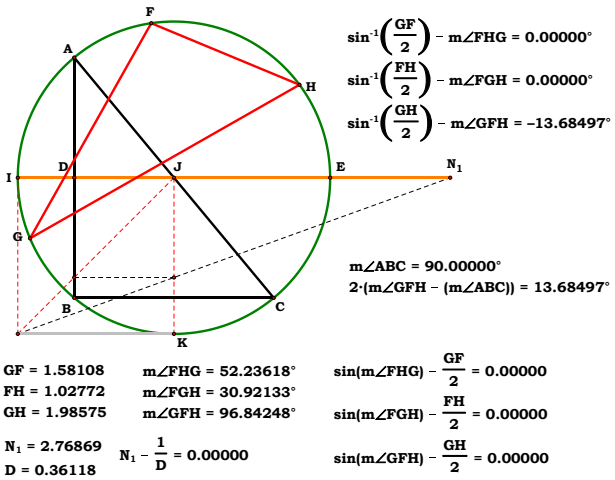
Ever heard of anyone setting the unit into a one-to-one correspondence with the angle? I mean, why would Euclid define these things if people are too stupid to use them?



Ever heard all those stories told in all the Trigonometry Books, how one can only work with angles using a right triangle, and one needs a basket full of function names to do it?

In binary recursion, one only has two states for the unit, the unit, and its reciprocal.

Well, it looks like a programmer for a function did not check his work. That arcsine falls short when the angle goes over 90. An arcsine is supposed to function with the sine, one-to-one. Looks like that if you actually believe the mythology of Trigonometry, you tend to stop thinking, or checking your work. If one dispels the mythology of Trigonometry, the entire field should just vanish. Sine functions are fine though and not a right angle in the bunch. We all make mistakes, but a mistake that has lasted this long? Really? Maybe it is true, that mankind will be illiterate for a long period of his evolution, and then suddenly, he will wake up.



One time, working in a factory, I was known by many as “The Professor.” I do not know where that came from, but some guy came up to me with a problem his daughter was asked to solve as it could only be solved using Trigonometry, or so her teacher said. I quickly solved it using simple geometry and algebra. Her father told me that I had to be wrong, that it was not solvable the way I did it. Well, the girl handed in the paper, and found the answer was correct, and I was informed that this stranger that asked me to solve it had apparently made a mistake. Trigonometry is a mythological science, built up on claptrap nonsense.

Long before I had ever read a word of Plato’s, I made the Socratic choice, educate myself.

Three Pieces of Paper

John Clark

John 312

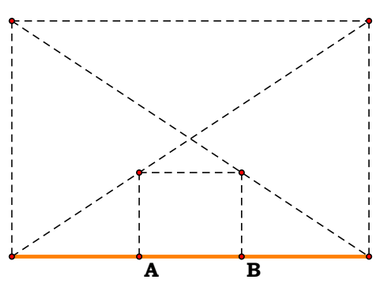
Three Pieces of Paper

Saturday, June 17, 2023

I have been putting off this project for over twenty-two years simply because I did not believe, and rightly so, that I could be happy with the results until I had a better understanding. So, here it is, as one can divide a straight line into any number of equal segments, so too, one can divide a circle into an equal number of segments, the results are arithmetic, however, it turns out to be advanced arithmetic, a very neglected advanced form of arithmetic.

One can better understand the difference between Arithmetic Progression and Geometric Progression by way of the sum of the extremes of a progression, what does it take to achieve 1?

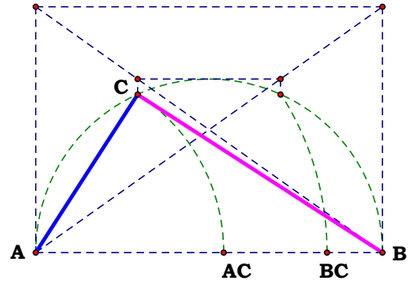
In terms of Arithmetic Progression, the sum is straight forward, adding result to result, and the product, if one has the extremes, simply the sum. Where a Geometric Progression, the sum of the squares are one, like so.



A = 0.35658
B = 0.64342
A + B = 1.00000

AC = 0.54219
BC = 0.84025

$AC^2 + BC^2 = 1.00000$



When it comes to a static figure, such as a circle divided an equal number of times, it will mean that the distances along the circumference are equal, but the ratio produced on the diameter is an ellipse, i.e., one ratio which will produce exactly the same angle regardless of size onto the diameter. Thus, the mathematics required, although being arithmetic, means a very complete study of the ellipse and circle.

And one will notice a difference from dynamic figures in the static figure division by angle, every name of everything in the figure resolves to arithmetic equations, numbers, and operations on them. Therefore, in writing up the division of a circle by angles, the only symbols in the whole work are arithmetic. The recursion of the unit is both arithmetic and geometric, and the operations in the one have a way of being expressed in the other without the mystic bs of contemporary and past scholars.

Archimedean Paper Trisector

Descriptions.

$AR := .75220 \qquad CR := 2 - AR \qquad ER := \sqrt{AR \cdot CR}$

$AE := \sqrt{AR^2 + ER^2} \qquad AQ := \frac{AE}{2} \qquad QS := \frac{ER}{2}$

$AS := \frac{AR}{2} \qquad BS := 1 - AS \qquad BQ := \sqrt{BS^2 + QS^2}$

$BT := \frac{BS}{BQ} \qquad PT := \frac{QS}{BQ} \qquad CT := 1 + BT \qquad RU := \frac{CT \cdot ER}{PT} \qquad BR := 1 - AR$

$BU := RU - BR \qquad BO := \frac{1}{BU} \qquad BF := BO \qquad BV := \frac{BR}{BU} \qquad OV := ER \cdot BO \qquad CV := 1 + BV \qquad EO := 1 - BO \qquad NO := EO \qquad NV := \sqrt{NO^2 - OV^2} \qquad CN := CV - NV \qquad CN - NO = 0$

$CE := 2 \cdot BQ \qquad CM := CN \qquad MW := \frac{ER \cdot CM}{CE} \qquad MW - OV = 0$

Definitions.

$CR - (2 - AR) = 0 \qquad ER - \sqrt{2 \cdot AR - AR^2} = 0 \qquad AE - \sqrt{2} \cdot \sqrt{AR} = 0 \qquad AQ - \frac{\sqrt{2} \cdot \sqrt{AR}}{2} = 0 \qquad QS - \frac{\sqrt{2 \cdot AR - AR^2}}{2} = 0 \qquad AS - \frac{AR}{2} = 0 \qquad BS - \frac{(2 - AR)}{2} = 0 \qquad BQ - \frac{\sqrt{2 - AR}}{\sqrt{2}} = 0$

$BT - BQ = 0 \qquad PT - \frac{\sqrt{2} \cdot \sqrt{2 \cdot AR - AR^2}}{2 \cdot \sqrt{2 - AR}} = 0 \qquad CT - \frac{2 + \sqrt{2} \cdot \sqrt{2 - AR}}{2} = 0 \qquad RU - \sqrt{2 - AR} \cdot \left(\sqrt{2 - AR} + \sqrt{2}\right) = 0 \qquad BR - (1 - AR) \qquad BU - \left(\sqrt{2} \cdot \sqrt{2 - AR} + 1\right)$

$BO - \frac{1}{\sqrt{2} \cdot \sqrt{2 - AR} + 1} = 0 \qquad BF - BO = 0 \qquad BV - \frac{(AR - 1) \cdot \left(\sqrt{2} \cdot \sqrt{2 - AR} - 1\right)}{2 \cdot AR - 3} = 0 \qquad OV - \frac{\sqrt{2} \cdot \sqrt{2 \cdot AR - AR^2}}{2 \cdot \left(\sqrt{2 - AR} + \frac{\sqrt{2}}{2}\right)} = 0 \qquad CV - \frac{\left(AR \cdot \sqrt{2 - AR} - \sqrt{2 - AR}\right) \cdot \sqrt{2} + AR - 2}{2 \cdot AR - 3} = 0$

$A = 0.00000$

$B = 1.00000$

$C = 2.00000$

$D = 2.41421$

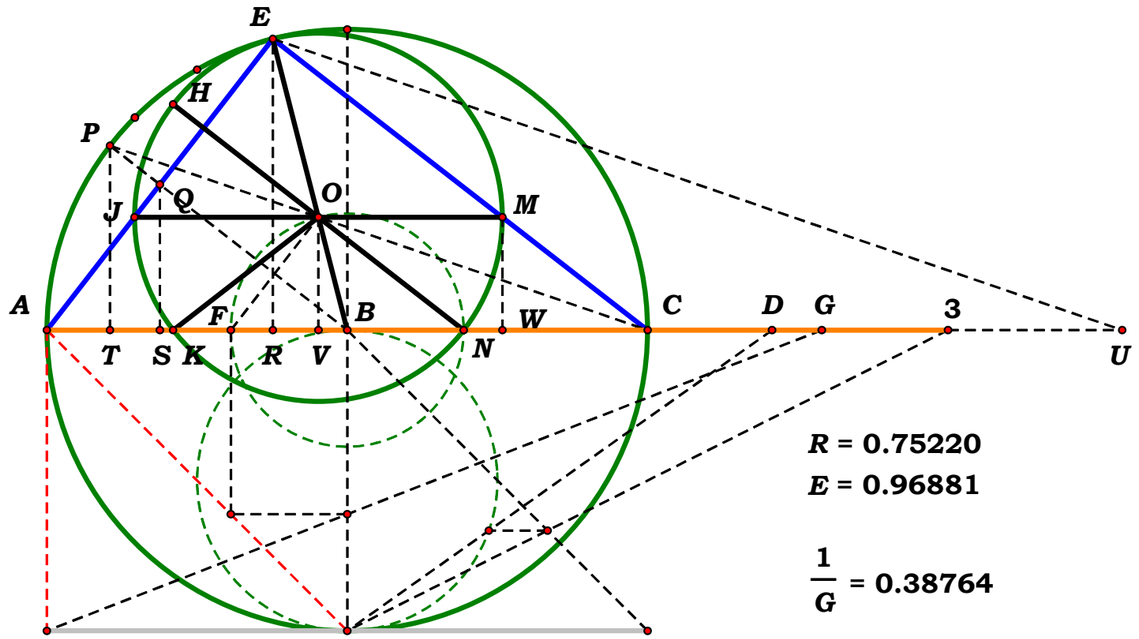
$G = 2.57974$

$G - D = 0.16553$

$m\angle EOH = 37.82643°$

$3 \cdot m\angle EOH = 113.47929°$

$\frac{3 \cdot m\angle EOH}{m\angle EOH} = 3.00000$



$R = 0.75220$

$E = 0.96881$

$\frac{1}{G} = 0.38764$

$$EO - \frac{2\cdot AR + \sqrt{4 - 2\cdot AR} - 4}{2\cdot AR - 3} \qquad NO - EO = 0$$

$$NV - \frac{(2 - AR)}{\sqrt{2\cdot\sqrt{2}\cdot\sqrt{2 - AR} - 2\cdot AR + 5}} = 0 \qquad CN - NO = 0$$

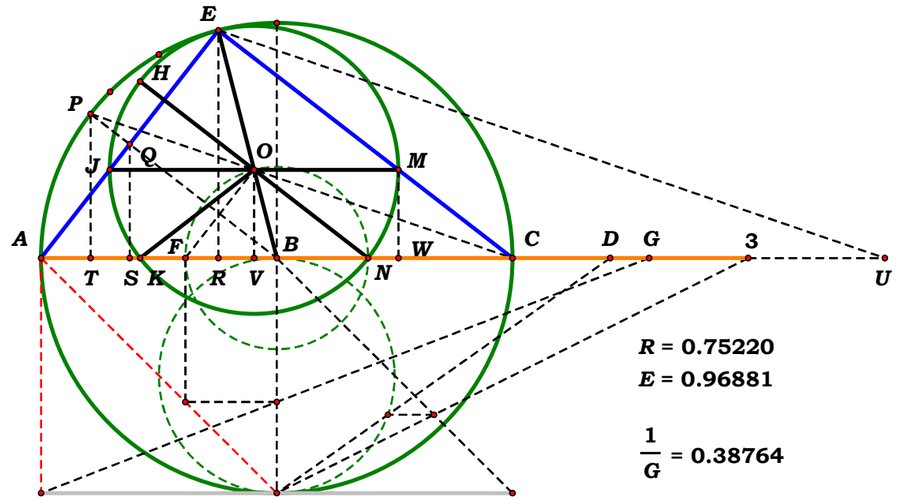
$$CN - \left(\frac{AR - \sqrt{2}\cdot\sqrt{2 - AR} + \sqrt{2}\cdot AR\cdot\sqrt{2 - AR} - 2}{2\cdot AR - 3} + \frac{AR - 2}{\sqrt{2\cdot\sqrt{2}\cdot\sqrt{2 - AR} - 2\cdot AR + 5}}\right) = 0$$

$$CE - \sqrt{2}\cdot\sqrt{2 - AR} = 0 \qquad CM - CN = 0 \qquad MW - \frac{\sqrt{2}\cdot\sqrt{2\cdot AR - AR^2}}{2\cdot\left(\sqrt{2 - AR} + \frac{\sqrt{2}}{2}\right)} = 0 \qquad MW - OV = 0$$

$$AR - \frac{(BF + 1)\cdot(3\cdot BF - 1)}{2\cdot BF^2} = 0$$

$$G := \frac{1}{BF} \qquad AR - \frac{(G + 1)\cdot(3 - G)}{2} = 0$$

$A = 0.00000$
$B = 1.00000$
$C = 2.00000$
$D = 2.41421$
$G = 2.57974$
$G - D = 0.16553$
$m\angle EOH = 37.82643°$
$3\cdot m\angle EOH = 113.47929°$
$\frac{3\cdot m\angle EOH}{m\angle EOH} = 3.00000$



$R = 0.75220$
$E = 0.96881$

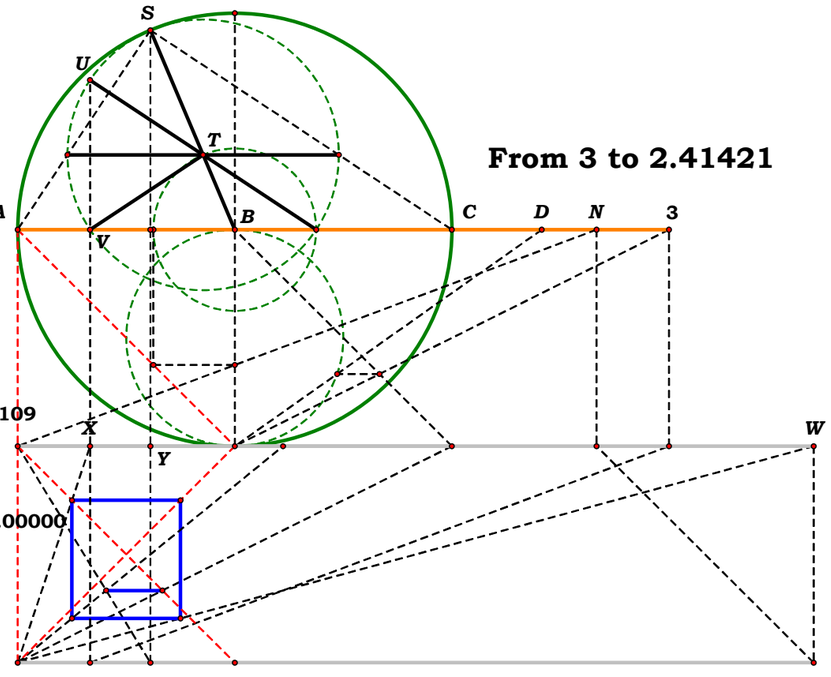
$\frac{1}{G} = 0.38764$

$m\angle STU = 33.55681°$
$m\angle STV = 100.67042°$
$\frac{m\angle STV}{m\angle STU} = 3.00000$

$N = 2.66668$
$W = 3.66668$
$(N + 1) = 3.66668$
$W - (N + 1) = 0.00000$
$X = 0.33332$
$(3 - N) = 0.33332$
$X - (3 - N) = 0.00000$
$Y = 0.61109$
$\frac{(N + 1)\cdot(3 - N)}{2} = 0.61109$

$Y - \frac{(N + 1)\cdot(3 - N)}{2} = 0.00000$

$\frac{1}{N} = 0.37500$



From 3 to 2.41421

042897 On Trisection

Descriptions.

$N := 1.69426 \quad AB := 1 \quad AC := \frac{1}{N}$

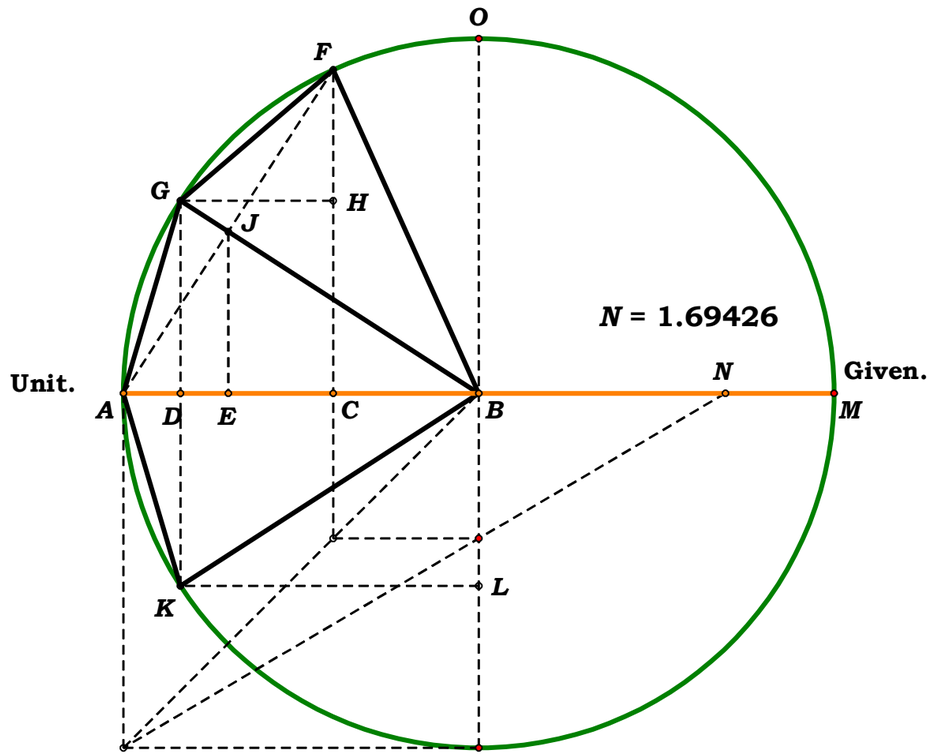
$AM := 2 \cdot AB \quad CF := \sqrt{AC \cdot (AM - AC)}$

$AF := \sqrt{AC^2 + CF^2} \quad AJ := \frac{AF}{2} \quad AE := \frac{AC}{2}$

$EJ := \frac{CF}{2} \quad BE := AB - AE \quad BJ := \sqrt{BE^2 + EJ^2}$

$GJ := AB - BJ \quad BD := \frac{BE}{BJ} \quad DG := \frac{EJ \cdot BD}{BE}$

$FH := CF - DG$



Definitions.

$AB - 1 = 0 \quad AC - \frac{1}{N} = 0 \quad AM - 2 \cdot AB = 0 \quad CF - \frac{\sqrt{2 \cdot N - 1}}{N} = 0 \quad AF - \frac{\sqrt{2}}{\sqrt{N}} = 0$

$AJ - \frac{\sqrt{2}}{2 \cdot \sqrt{N}} = 0 \quad AE - \frac{1}{2 \cdot N} = 0 \quad EJ - \frac{\sqrt{2 \cdot N - 1}}{2 \cdot N} = 0 \quad BE - \frac{2 \cdot N - 1}{2 \cdot N} = 0$

$BJ - \frac{\sqrt{4 \cdot N - 2}}{2 \cdot \sqrt{N}} = 0 \quad GJ - \frac{2 \cdot \sqrt{N} - \sqrt{2} \cdot \sqrt{2 \cdot N - 1}}{2 \cdot \sqrt{N}} = 0 \quad BD - \frac{\sqrt{4 \cdot N - 2}}{2 \cdot \sqrt{N}} = 0 \quad DG - \frac{\sqrt{2}}{2 \cdot \sqrt{N}} = 0$

$FH - \frac{2 \cdot \sqrt{2 \cdot N - 1} - \sqrt{2} \cdot \sqrt{N}}{2 \cdot N} = 0$

Man in the Middle

Descriptions.

$AN := 3.53349 \quad AC := 1 \quad AB := \frac{1}{\sqrt{AN}} \quad EF := AC - AB$

$CF := \sqrt{1 - EF^2} \quad AF := AC - CF \quad CG := CF \quad AM := 2 \cdot AC$

$GH := EF \cdot CG \quad JK := 2 \cdot GH \quad KM := \frac{CF \cdot JK}{EF} \quad AK := AM - KM$

Definitions.

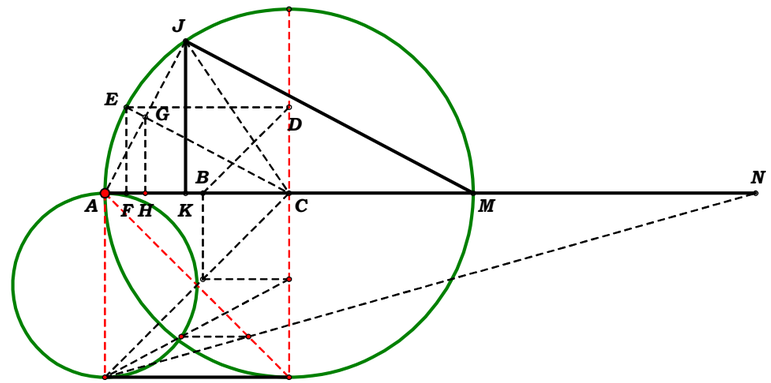
$AN - AN = 0 \quad AC - 1 = 0 \quad AB - \frac{1}{\sqrt{AN}} = 0$

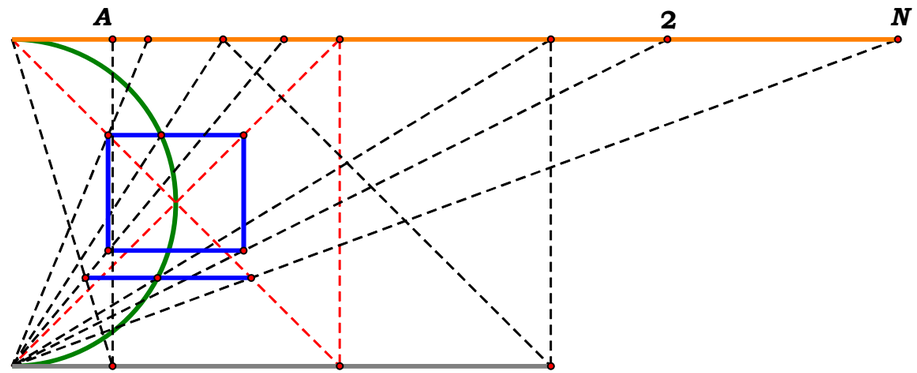
$EF - \frac{\sqrt{AN} - 1}{\sqrt{AN}} = 0 \quad CF - \frac{\sqrt{2 \cdot \sqrt{AN} - 1}}{\sqrt{AN}} = 0$

$AF - \frac{\sqrt{AN} - \sqrt{2 \cdot \sqrt{AN} - 1}}{\sqrt{AN}} = 0 \quad CG - CF = 0 \quad AM - 2 = 0 \quad GH - \frac{\sqrt{2 \cdot \sqrt{AN} - 1} \cdot \left(\sqrt{AN} - 1\right)}{AN} = 0$

$JK - \frac{2 \cdot \sqrt{2 \cdot \sqrt{AN} - 1} \cdot \left(\sqrt{AN} - 1\right)}{AN} = 0 \quad KM - \frac{2 \cdot \left(2 \cdot \sqrt{AN} - 1\right)}{AN} = 0 \quad AK - \frac{2 \cdot \left(\sqrt{AN} - 1\right)^2}{AN} = 0$

$N = 3.53349$
$A = 0.00000$
$B = 0.53198$
$C = 1.00000$
$\frac{1}{\sqrt{N}} = 0.53198$
$D = 0.46802$
$G = 0.41360$
$K = 0.43808$





$N = 2.70326$

$\frac{2 \cdot \left(\sqrt{N} - 1\right)^2}{N} = 0.30699$

$A = 0.30699$

$A - \frac{2 \cdot \left(\sqrt{N} - 1\right)^2}{N} = 0.00000$

On Trisection 042897

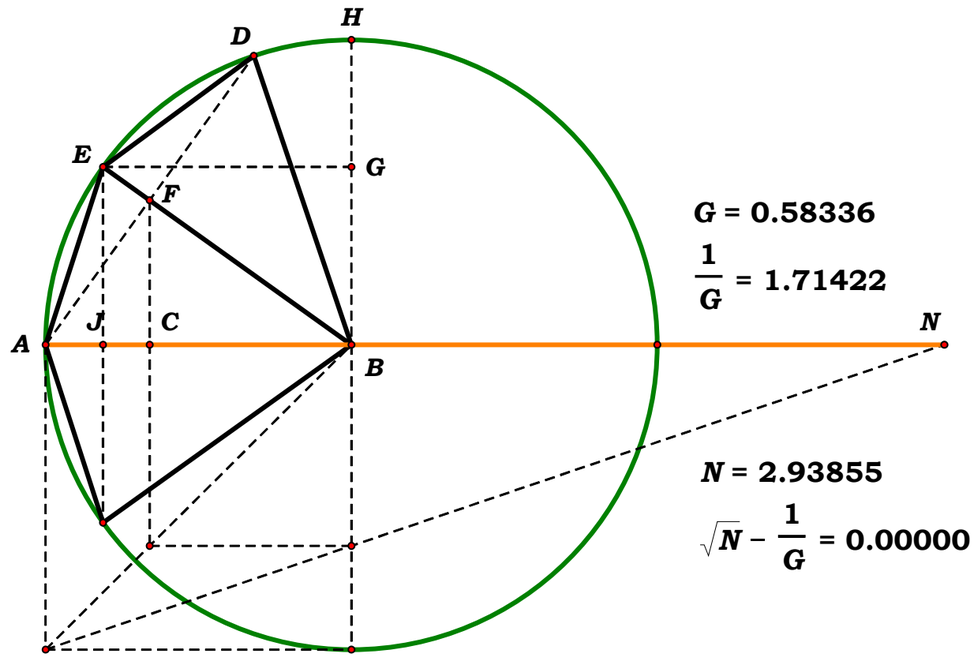
Descriptions.

$BG := .58336 \quad AB := 1 \quad GH := AB - BG$

$EG := \sqrt{GH \cdot (2 \cdot AB - GH)} \quad AJ := AB - EG$

$BF := AB - AJ \quad BJ := AB - AJ \quad BC := BJ \cdot BF$

$AC := AB - BC \quad AN := \frac{1}{AC} \quad \frac{1}{BG} - \sqrt{AN} = 0$



$G = 0.58336$

$\frac{1}{G} = 1.71422$

$N = 2.93855$

$\sqrt{N} - \frac{1}{G} = 0.00000$

Definitions.

$BG - BG = 0 \quad AB - 1 = 0 \quad GH - (1 - BG) = 0 \quad EG - \sqrt{\left(1 - BG^2\right)} = 0$

$AJ - \left(1 - \sqrt{1 - BG^2}\right) \quad BF - \left(\sqrt{1 - BG^2}\right) \quad BJ - \sqrt{1 - BG^2} = 0 \quad BC - \left(1 - BG^2\right)$

$AC - BG^2 \quad AN - \frac{1}{BG^2} = 0 \quad \frac{1}{BG} - \sqrt{AN} = 0$

Trisection and Cube Roots

Descriptions.

$AB := 1 \quad AN := 7 \quad BN := AN - AB \quad AC := AN^{\frac{1}{3}} \quad AE := AN^{\frac{2}{3}}$

$CE := AE - AC \quad CD := \frac{CE}{2} \quad AD := AC + CD \quad AG := CD$

$DG := \sqrt{AD^2 + AG^2} \quad CM := DG \quad MO := AG \quad JM := \frac{MO^2}{DG}$

$JO := \sqrt{MO^2 - JM^2} \quad RS := AG + 2 \cdot JO \quad AS := DG - 2 \cdot JM$

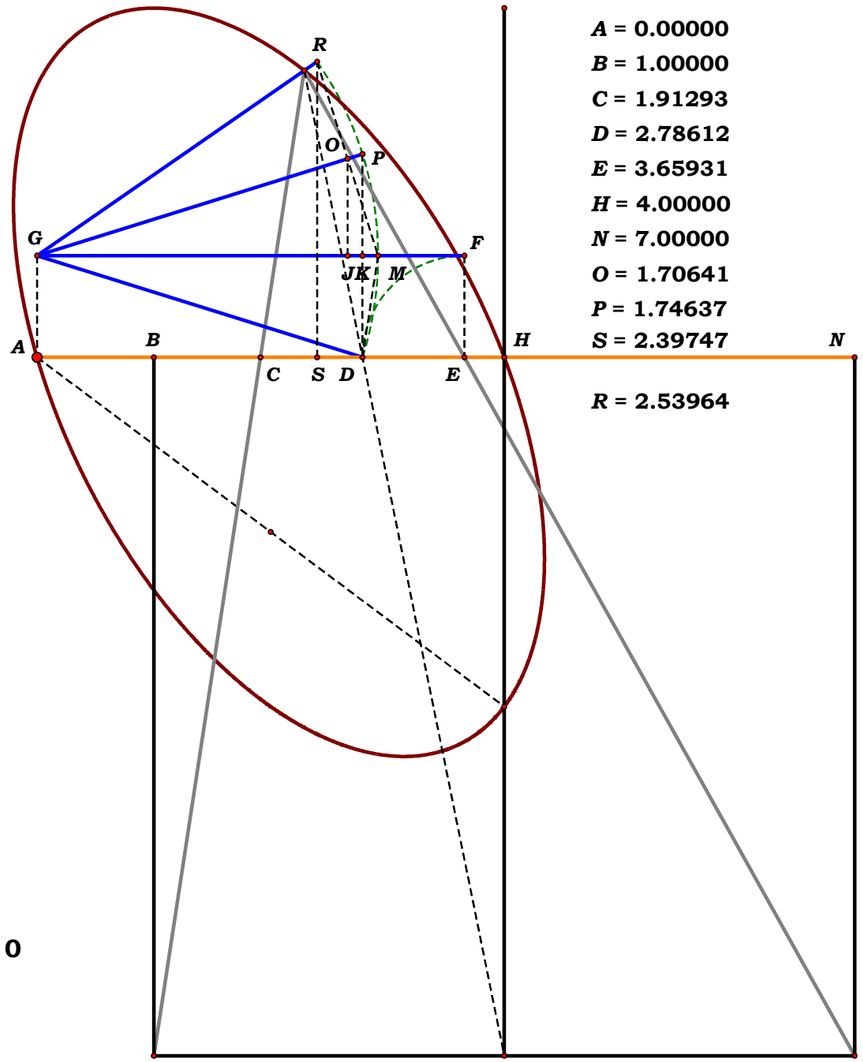
Definitions.

$AB - 1 = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad AC - AN^{\frac{1}{3}} = 0$

$AE - AN^{\frac{2}{3}} = 0 \quad CE - \left(AN^{\frac{2}{3}} - AN^{\frac{1}{3}}\right) = 0 \quad CD - \frac{AN^{\frac{2}{3}} - AN^{\frac{1}{3}}}{2} = 0$

$AD - \frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{1}{3}} + 1\right)}{2} = 0 \quad AG - CD = 0 \quad DG - \frac{\sqrt{AN^{\frac{2}{3}} \cdot \left(AN^{\frac{2}{3}} + 1\right)}}{\sqrt{2}} = 0$

$CM - DG = 0 \quad MO - AG = 0$

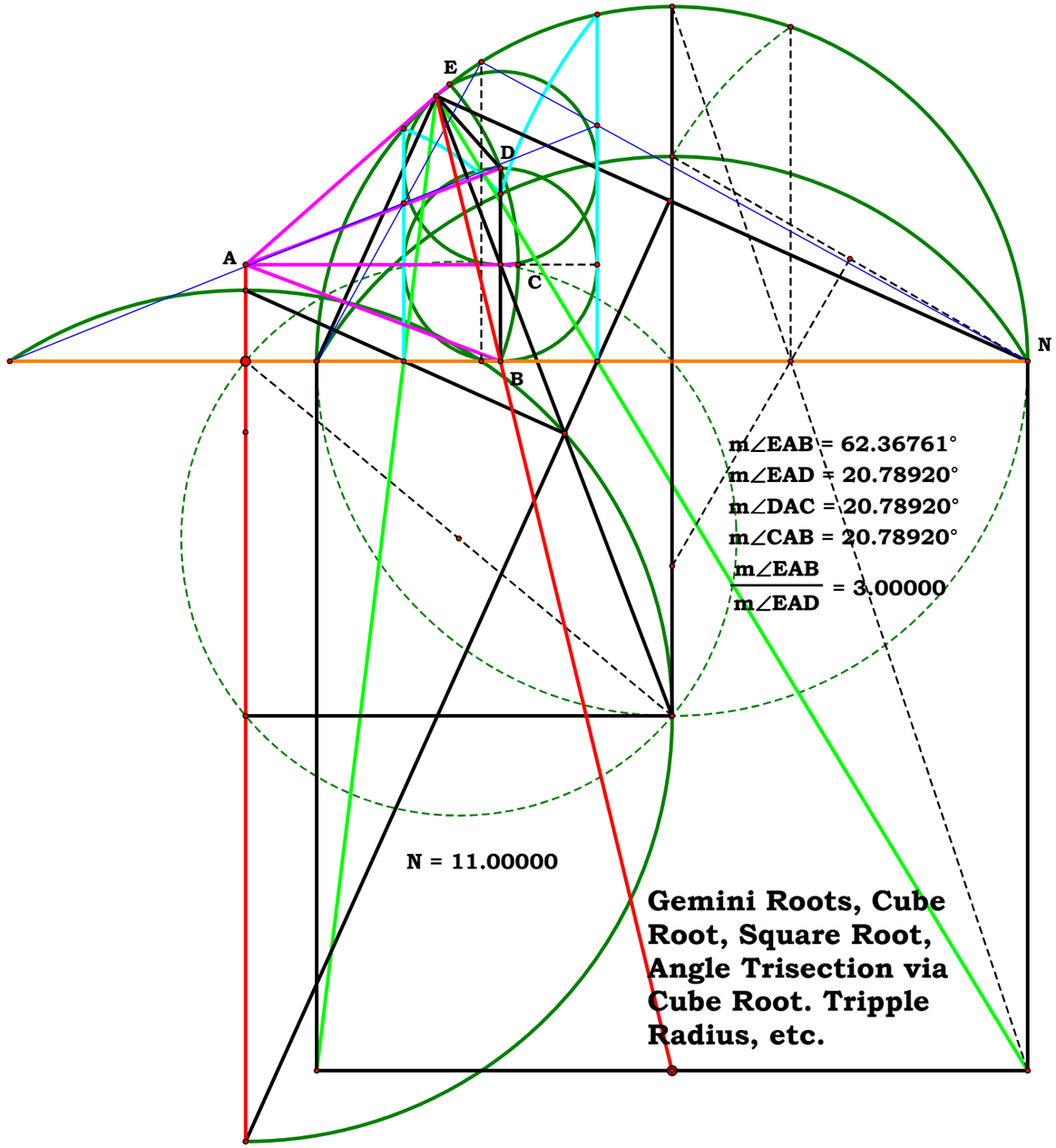


$A = 0.00000$
$B = 1.00000$
$C = 1.91293$
$D = 2.78612$
$E = 3.65931$
$H = 4.00000$
$N = 7.00000$
$O = 1.70641$
$P = 1.74637$
$S = 2.39747$
$R = 2.53964$

$JM - \frac{\sqrt{2} \cdot AN^{\frac{2}{3}} \cdot \left(AN^{\frac{1}{3}} - 1\right)^2}{4 \cdot \sqrt{AN^{\frac{2}{3}} + AN^{\frac{4}{3}}}} = 0 \quad JO - \frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{2}{3}} - 1\right)}{2 \cdot \sqrt{2 \cdot \left(AN^{\frac{2}{3}} + 1\right)}} = 0 \quad RS - \left[\frac{AN^{\frac{1}{3}} \cdot \left(AN^{\frac{1}{3}} - 1\right) \cdot \left(\sqrt{2} \cdot AN^{\frac{1}{3}} + \sqrt{AN^{\frac{2}{3}} + 1 + \sqrt{2}}\right)}{2 \cdot \sqrt{AN^{\frac{2}{3}} + 1}}\right] = 0 \quad AS - \frac{\sqrt{2} \cdot AN}{\sqrt{AN^{\frac{2}{3}} + AN^{\frac{4}{3}}}} = 0$

Trisection and Cube Roots

One has to be careful with drawing this for point E is not on the circle, so in drawingit, one might want to hid the circle it is almost on top of.



Paper Roots

Unit.

Given.

Descriptions.

$AN := 5 \quad AD := \sqrt{AN} \quad AB := 1 \quad BN := AN - AB$

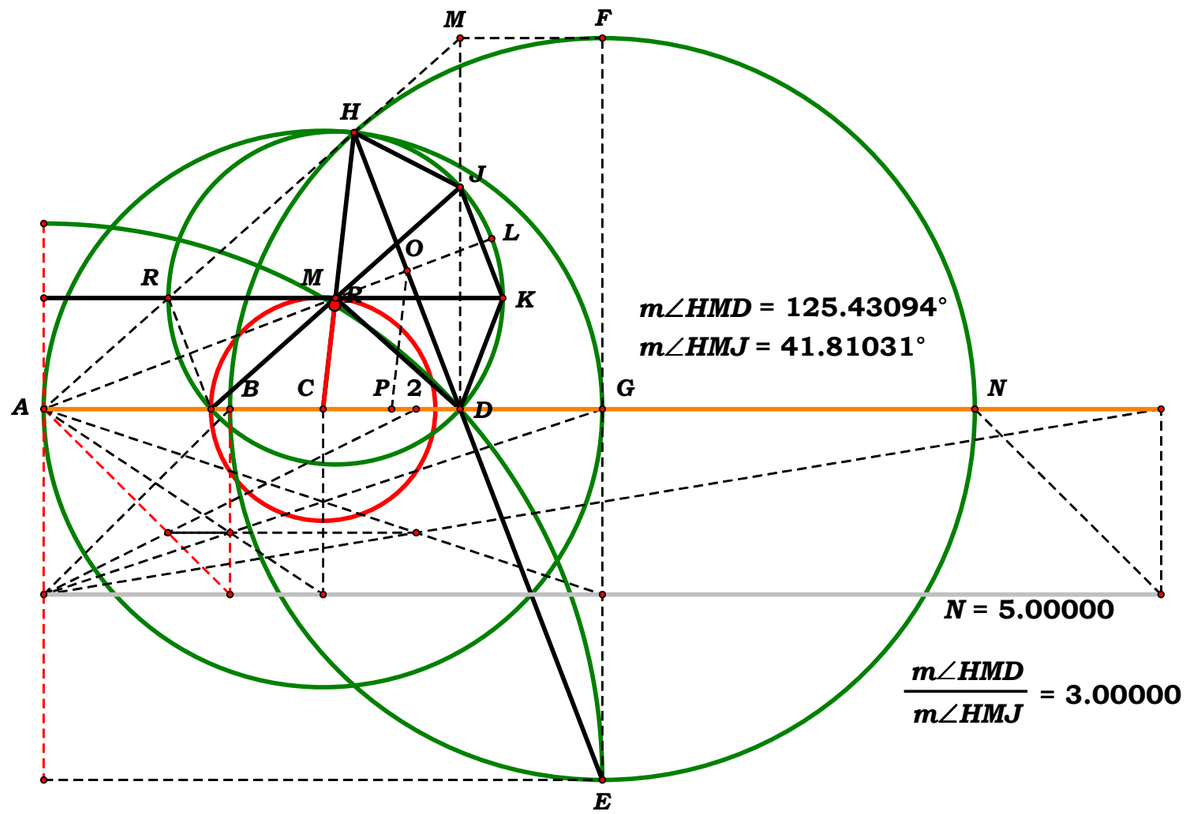
$BG := \frac{BN}{2} \quad AG := AB + BG \quad AC := \frac{AG}{2} \quad CH := AC$

$DG := AG - AD \quad DE := \sqrt{DG^2 + BG^2} \quad EH := \frac{BG \cdot BN}{DE}$

$DH := EH - DE \quad DO := \frac{DH}{2} \quad OP := \frac{CH \cdot DO}{DH}$

$CD := AD - AC \quad CP := \frac{CD}{2} \quad AP := AC + CP \quad CM := \frac{OP \cdot AC}{AP}$

$AH := AD \quad AR := \frac{AH \cdot CM}{CH} \quad HM := CH - CM \quad HM - AR = 0$



Definitions.

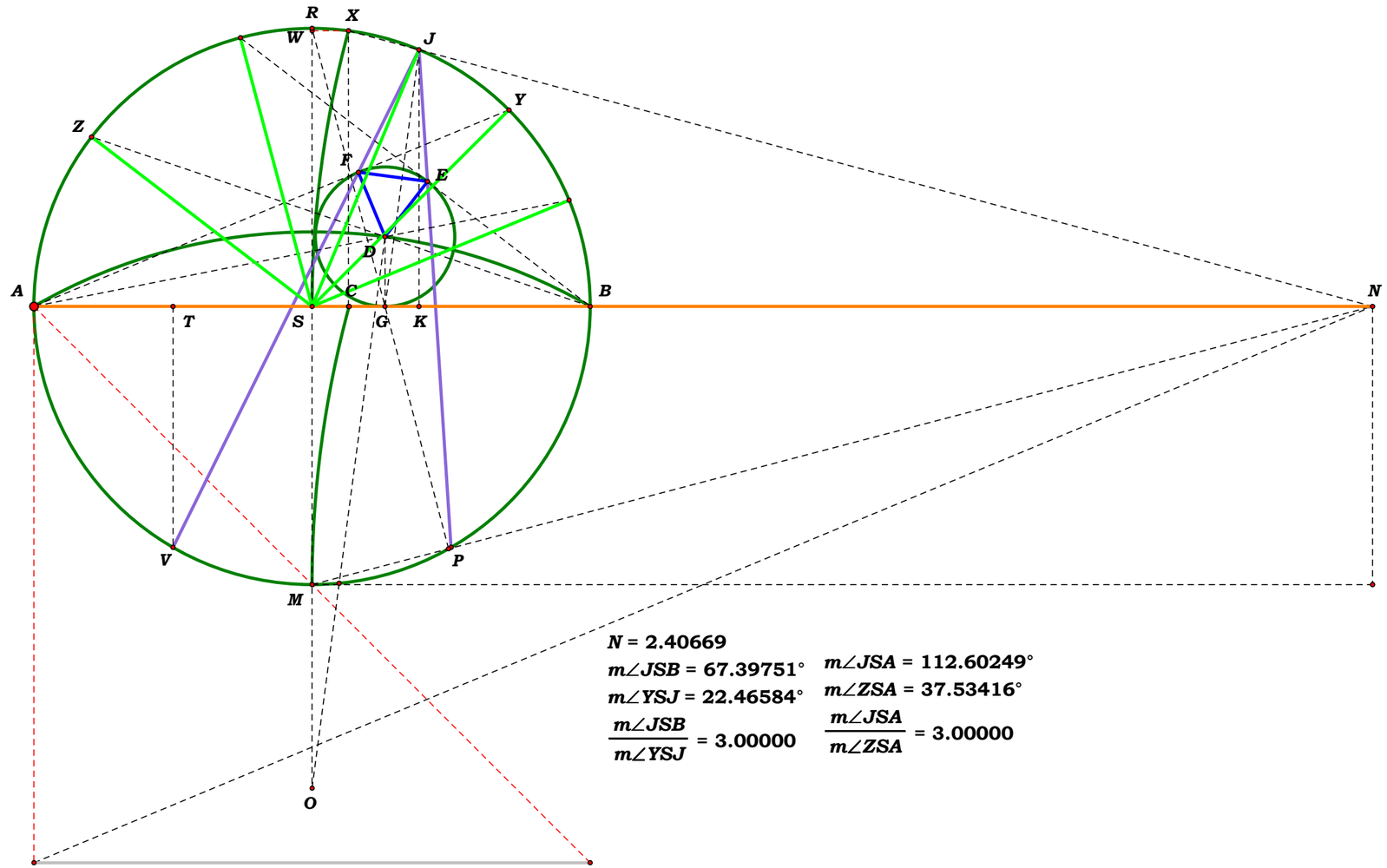
$AN - AN = 0 \quad AD - \sqrt{AN} = 0 \quad AB - 1 = 0 \quad BN - (AN - 1) = 0 \quad BG - \frac{AN - 1}{2} = 0 \quad AG - \frac{AN + 1}{2} = 0 \quad AC - \frac{AN + 1}{4} = 0 \quad CH - AC = 0$

$DG - \frac{\left(\sqrt{AN} - 1\right)^2}{2} = 0 \quad DE - \frac{\left(\sqrt{AN} - 1\right) \cdot \sqrt{(AN + 1)}}{\sqrt{2}} = 0 \quad EH - \frac{\sqrt{2} \cdot \left(\sqrt{AN} - 1\right) \cdot \left(\sqrt{AN} + 1\right)^2}{2 \cdot \sqrt{AN + 1}} = 0 \quad DH - \frac{\sqrt{2} \cdot \sqrt{AN} \cdot \left(\sqrt{AN} - 1\right)}{\sqrt{AN + 1}} = 0$

$DO - \frac{\sqrt{2} \cdot \sqrt{AN} \cdot \left(\sqrt{AN} - 1\right)}{2 \cdot \sqrt{AN + 1}} = 0 \quad OP - \frac{AN + 1}{8} = 0 \quad CD - \frac{\left(4 \cdot \sqrt{AN} - AN - 1\right)}{4} = 0 \quad CP - \frac{\left(4 \cdot \sqrt{AN} - AN - 1\right)}{8} = 0 \quad AP - \frac{AN + 4 \cdot \sqrt{AN} + 1}{8} = 0$

$CM - \frac{(AN + 1)^2}{4 \cdot \left(AN + 4 \cdot \sqrt{AN} + 1\right)} = 0 \quad AH - AD = 0 \quad AR - \frac{\sqrt{AN} \cdot (AN + 1)}{AN + 4 \cdot \sqrt{AN} + 1} = 0 \quad HM - \frac{\sqrt{AN} \cdot (AN + 1)}{AN + 4 \cdot \sqrt{AN} + 1} = 0 \quad HM - AR = 0$

Angle Trisection



$N = 2.40669$

$m\angle JSB = 67.39751°$ $m\angle JSA = 112.60249°$

$m\angle YSJ = 22.46584°$ $m\angle ZSA = 37.53416°$

$\frac{m\angle JSB}{m\angle YSJ} = 3.00000$ $\frac{m\angle JSA}{m\angle ZSA} = 3.00000$

Angle Trisection

Unit.

Given.

Descriptions.

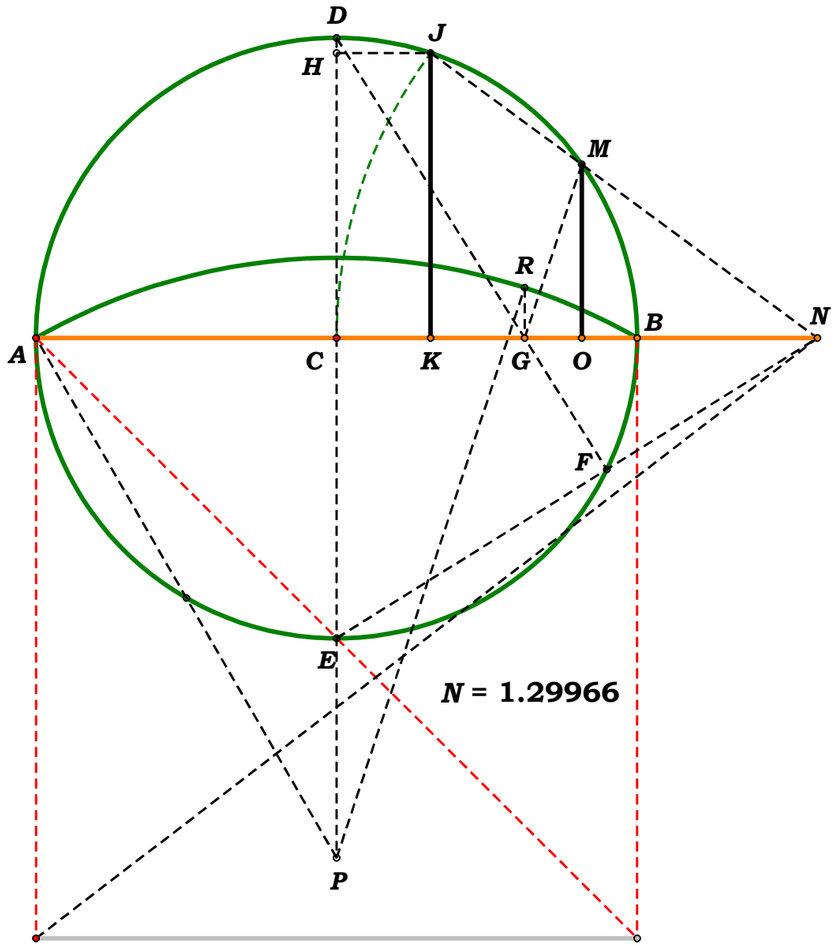
$AB := 1 \quad AN := 1.29966 \quad AC := \frac{AB}{2} \quad CN := AN - AC$

$CG := \frac{AC^2}{CN} \quad S_1 := CN \quad S_2 := S_1 \quad S_3 := AC$

$JK := \frac{\sqrt{(S_1 + S_2 - S_3)\cdot(S_1 - S_2 + S_3)\cdot(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2\cdot S_1}$

$KN := \sqrt{CN^2 - JK^2} \quad CK := CN - KN \quad MN := CN - 2\cdot CK$

$NO := \frac{KN\cdot MN}{CN}$



Definitions.

$AB - AB = 0 \quad AN - AN = 0 \quad AC - \frac{1}{2} = 0 \quad CN - \frac{2\cdot AN - 1}{2} = 0$

$CG - \frac{1}{2\cdot(2\cdot AN - 1)} = 0 \quad S_1 - CN = 0 \quad S_2 - S_1 = 0 \quad S_3 - AC = 0$

$JK - \frac{\sqrt{(4\cdot AN - 1)\cdot(4\cdot AN - 3)}}{4\cdot(2\cdot AN - 1)} = 0 \quad KN - \frac{\left(8\cdot AN^2 - 8\cdot AN + 1\right)}{4\cdot(2\cdot AN - 1)} = 0 \quad CK - \frac{1}{4\cdot(2\cdot AN - 1)} = 0 \quad MN - \frac{2\cdot AN\cdot(AN - 1)}{2\cdot AN - 1} = 0$

$NO - \frac{AN\cdot(AN - 1)\cdot\left(8\cdot AN^2 - 8\cdot AN + 1\right)}{(2\cdot AN - 1)^3} = 0$

Which Mathcad says is solvable but perhaps too much for my pc. Or, contrary to popular belief, any angle can be trisected. So, not only can you draw it, you can, if you desire, do it with algebra because you can even draw the equations with geometry.

Three Quarter Boot

Descriptions.

$AB := 1 \quad AN := 3 \quad AC := \frac{1}{AN} \quad BC := AB - AC \quad CD := \frac{BC}{2}$

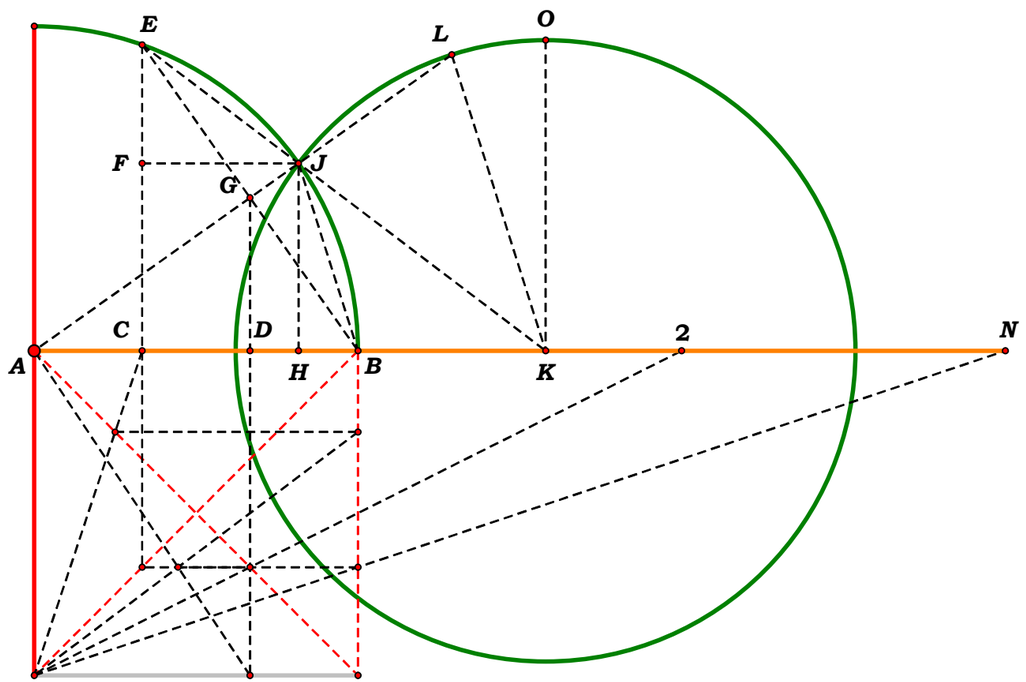
$CE := \sqrt{BC \cdot (2 \cdot AB - BC)} \quad DG := \frac{CE}{2} \quad AD := AC + CD$

$AG := \sqrt{AD^2 + DG^2} \quad AH := \frac{AD}{AG} \quad HJ := \frac{DG}{AG}$

$EF := CE - HJ \quad FJ := AH - AC \quad CK := \frac{FJ \cdot CE}{EF} \quad AK := CK + AC$

$EJ := \sqrt{EF^2 + FJ^2} \quad HK := AK - AH \quad JK := \frac{EJ \cdot HJ}{EF} \quad JK = 0.957058$

$A = 0.00000$
$N = 3.00000$
$B = 1.00000$
$C = 0.33333$
$D = 0.66667$
$E = 0.94281$
$F = 0.57735$
$G = 0.47140$
$H = 0.81650$
$J = 0.57735$
$K = 1.57980$
$O = 0.95706$



Definitions.

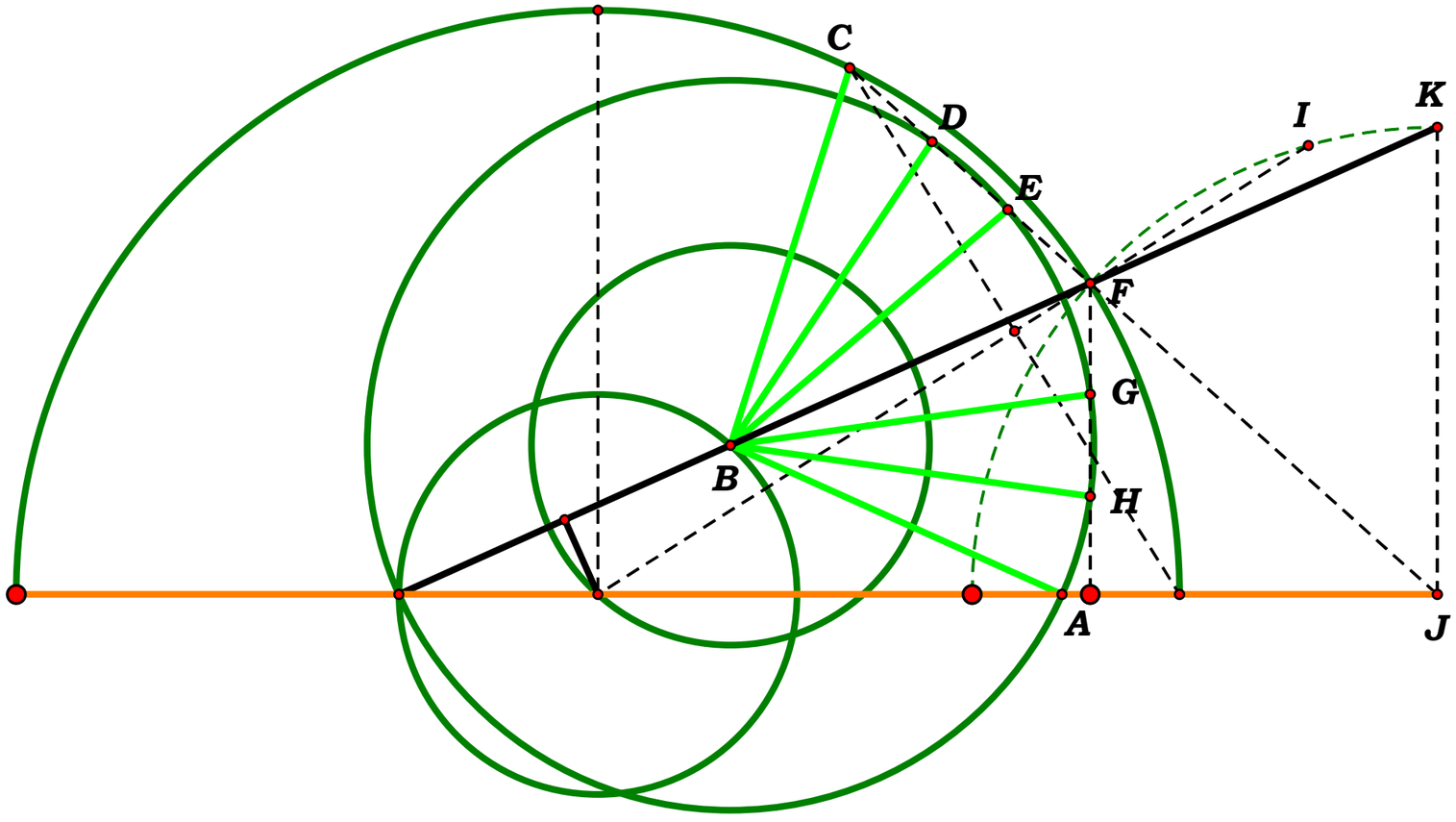
$AB - AB = 0 \quad AN - AN = 0 \quad AC - \frac{1}{AN} = 0 \quad BC - \frac{AN - 1}{AN} = 0 \quad CD - \frac{AN - 1}{2 \cdot AN} = 0 \quad CE - \frac{\sqrt{(AN - 1) \cdot (AN + 1)}}{AN} = 0 \quad DG - \frac{\sqrt{(AN - 1) \cdot (AN + 1)}}{2 \cdot AN} = 0 \quad AD - \frac{AN + 1}{2 \cdot AN} = 0$

$AG - \frac{\sqrt{AN + 1}}{\sqrt{2 \cdot AN}} = 0 \quad AH - \frac{\sqrt{2} \cdot \sqrt{AN + 1}}{2 \cdot \sqrt{AN}} = 0 \quad HJ - \frac{\sqrt{2} \cdot \sqrt{AN^2 - 1}}{2 \cdot \sqrt{AN} \cdot \sqrt{AN + 1}} = 0 \quad EF - \frac{\sqrt{AN^2 - 1} \cdot \left(2 \cdot \sqrt{AN + 1} - \sqrt{2} \cdot \sqrt{AN}\right)}{2 \cdot AN \cdot \sqrt{AN + 1}} = 0 \quad FJ - \frac{\sqrt{2} \cdot \sqrt{AN} \cdot \sqrt{AN + 1} - 2}{2 \cdot AN} = 0$

$CK - \frac{\sqrt{AN + 1} \cdot \sqrt{(AN - 1) \cdot (AN + 1)} \cdot \left(\sqrt{2} \cdot \sqrt{AN} \cdot \sqrt{AN + 1} - 2\right)}{AN \cdot \left(2 \cdot \sqrt{AN + 1} - \sqrt{2} \cdot \sqrt{AN}\right) \cdot \sqrt{AN^2 - 1}} = 0 \quad AK - \frac{\sqrt{2 \cdot AN}}{2 \cdot \sqrt{AN + 1} - \sqrt{2 \cdot AN}} = 0 \quad EJ - \frac{\sqrt{2 \cdot AN^{\frac{3}{2}} + 2 \cdot AN^{\frac{5}{2}} + \sqrt{2} \cdot \sqrt{AN + 1} - \sqrt{2} \cdot (AN + 1)^{\frac{3}{2}} - \sqrt{2} \cdot AN^2 \cdot \sqrt{AN + 1}}}{\sqrt{AN^{\frac{3}{2}} \cdot (AN + 1)}} = 0$

$HK - \frac{\sqrt{AN} \cdot \sqrt{AN + 1} - \sqrt{2}}{\sqrt{AN} \cdot \left(2 \cdot \sqrt{AN + 1} - \sqrt{2} \cdot \sqrt{AN}\right)} = 0 \quad JK - \frac{\sqrt{2} \cdot \sqrt{AN} \cdot \sqrt{\sqrt{2 \cdot AN + 2} + 2 \cdot AN^{\frac{3}{2}} + 2 \cdot AN^{\frac{5}{2}} - AN^2 \cdot \sqrt{2 \cdot AN + 2} - \sqrt{2} \cdot (AN + 1)^{\frac{3}{2}}}}{\left(2 \cdot \sqrt{AN + 1} - \sqrt{2 \cdot AN}\right) \cdot \sqrt{AN^{\frac{3}{2}} \cdot (AN + 1)}} = 0$

Foot Note to Three Quarter Boot



$m\angle ABC = 96.52156°$
$m\angle CBD = 16.08693°$
$m\angle DBE = 16.08693°$
$m\angle EBF = 16.08693°$
$m\angle FBG = 16.08693°$
$m\angle GBH = 16.08693°$
$m\angle HBA = 16.08693°$
$m\angle IJK = 16.08693°$
$\frac{m\angle ABC}{m\angle CBD} = 6.00000$

042401

Descriptions.

$AB := 1 \quad AN := 4 \quad BN := AN - AB \quad BF := \frac{BN}{2}$

$AF := AB + BF \quad AK := AF \quad FK := BF \quad AE := \frac{2 \cdot AK^2 - FK^2}{2 \cdot AF}$

See Pythagoras Revisted in the Delian Quest for AE.

$AJ := AE \quad JK := AK - AJ \quad HJ := JK \quad AH := AK - 2 \cdot JK$

$AC := \frac{AE \cdot AH}{AK} \quad CE := AE - AC \quad BE := AE - AB \quad EN := BN - BE$

$EK := \sqrt{BE \cdot EN} \quad CH := \frac{EK \cdot AH}{AK} \quad DE := \frac{CE \cdot EK}{EK + CH} \quad DF := 2 \cdot DE \quad HM := \sqrt{CE^2 + (EK + CH)^2}$

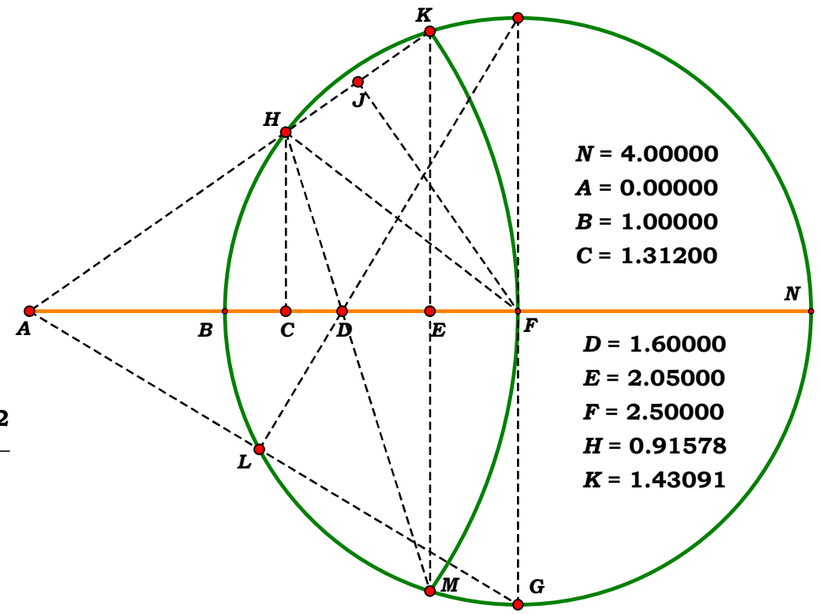
Definitions.

$AB - AB = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad BF - \frac{AN - 1}{2} = 0 \quad AF - \frac{AN + 1}{2} = 0 \quad AK - AF = 0 \quad FK - BF = 0 \quad AE - \frac{AN^2 + 6 \cdot AN + 1}{4 \cdot (AN + 1)} = 0$

$AJ - AE = 0 \quad JK - \frac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad HJ - JK = 0 \quad AH - \frac{2 \cdot AN}{AN + 1} = 0 \quad AC - \frac{AN \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{(AN + 1)^3} = 0 \quad CE - \frac{\left(AN^2 + 6 \cdot AN + 1\right) \cdot (AN - 1)^2}{4 \cdot (AN + 1)^3} = 0$

$BE - \frac{(AN + 3) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EN - \frac{(3 \cdot AN + 1) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EK - \frac{(AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(4 \cdot AN + 4)} = 0 \quad CH - \frac{AN \cdot (AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(AN + 1)^3} = 0$

$DE - \frac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad DF - \frac{(AN - 1)^2}{2 \cdot (AN + 1)} = 0 \quad HM - \frac{(AN - 1) \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{2 \cdot (AN + 1)^2} = 0$



One should recognise the core of the figure for angle trisection. This is a simple write-up of relationships.

And, as the figure is wholly expressible algebraically, one can solve for more than one point by which the figure can be solved and reproduced for any angle one wishes to trisect, as I said before.

Unit.

Given.

What is the name of EK in Algebra?

What is the Name 042501

Descriptions.

$AB := 1 \quad AN := 3 \quad BN := AN - AB \quad CE := \frac{BN}{2}$

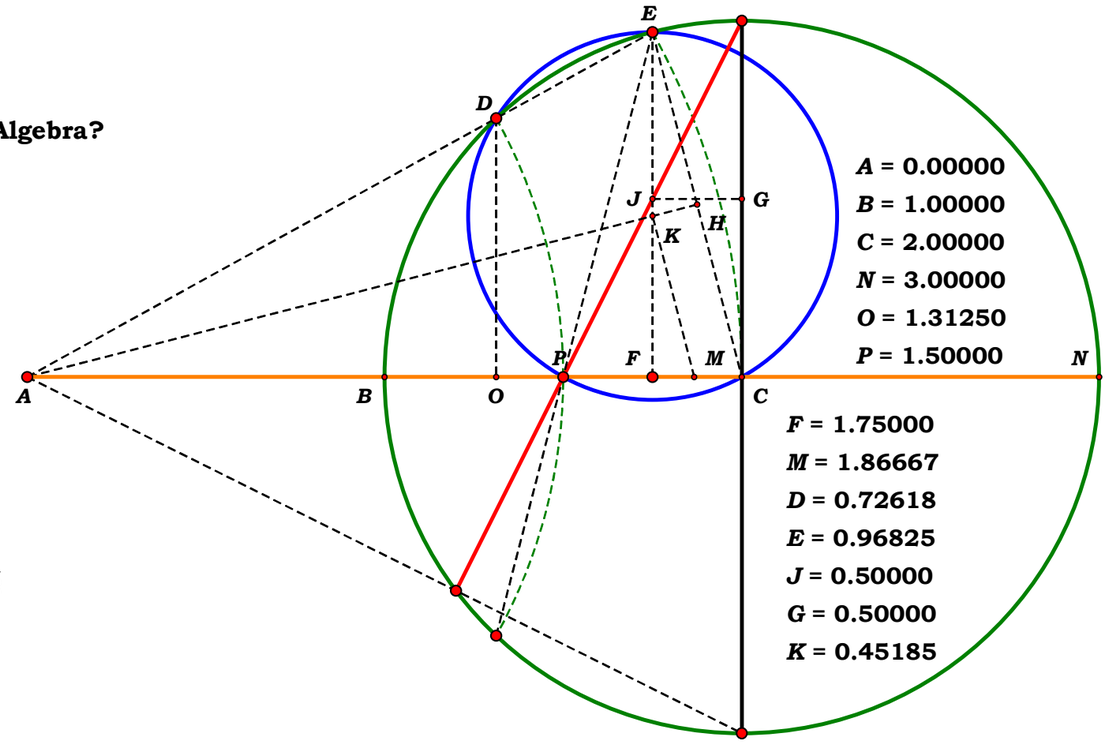
$AC := AB + CE \quad AE := AC \quad AF := \frac{2 \cdot AE^2 - CE^2}{2 \cdot AC}$

$CF := AC - AF \quad CG := \frac{BN}{4} \quad PJ := \sqrt{CG^2 + CF^2}$

$BF := AF - AB \quad FN := BN - BF \quad EF := \sqrt{BF \cdot FN} \quad AH := \sqrt{AC^2 - CG^2}$

$FK := \frac{CG \cdot AF}{AH} \quad EK := EF - FK \quad EK - \sqrt{FK^2 + CF^2} = 0$

$CP := \frac{CE^2}{AC} \quad 2 \cdot CF - CP = 0$



Definitions.

$AB - 1 = 0 \quad AN - AN = 0 \quad BN - (AN - 1) = 0 \quad CE - \frac{AN - 1}{2} = 0 \quad AC - \frac{AN + 1}{2} = 0 \quad AE - AC = 0$

$AF - \frac{AN^2 + 6 \cdot AN + 1}{4 \cdot (AN + 1)} = 0 \quad CF - \frac{(AN - 1)^2}{4 \cdot (AN + 1)} = 0 \quad CG - \frac{BN}{4} = 0 \quad PJ - \sqrt{CG^2 + CF^2} = 0 \quad BF - \frac{(AN + 3) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0$

$FN - \frac{(3 \cdot AN + 1) \cdot (AN - 1)}{4 \cdot (AN + 1)} = 0 \quad EF - \frac{(AN - 1) \cdot \sqrt{(AN + 3) \cdot (3 \cdot AN + 1)}}{(4 \cdot AN + 4)} = 0 \quad AH - \frac{\sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}}{4} = 0 \quad FK - \frac{(AN - 1) \cdot \left(AN^2 + 6 \cdot AN + 1\right)}{4 \cdot (AN + 1) \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0$

$EK - \frac{AN^2 - 1}{2 \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0 \quad EK - \frac{(AN - 1) \cdot (AN + 1)}{2 \cdot \sqrt{3 \cdot AN^2 + 10 \cdot AN + 3}} = 0 \quad CP - \frac{(AN - 1)^2}{2 \cdot (AN + 1)} = 0 \quad 2 \cdot CF - CP = 0$

Now, as in one of Superman's Super Villans, get him to say his name backwards.

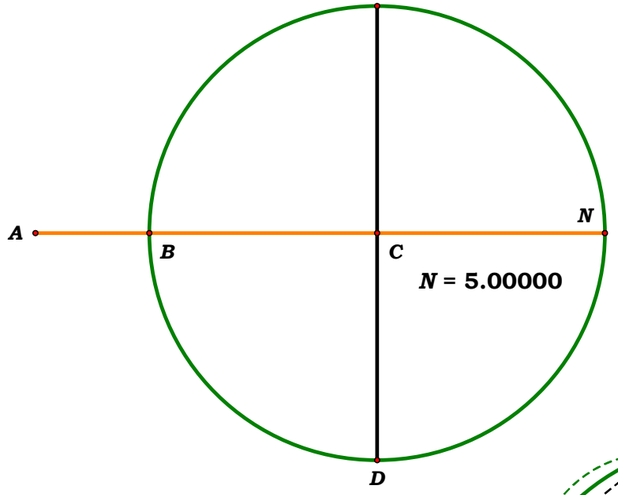
Descriptions.

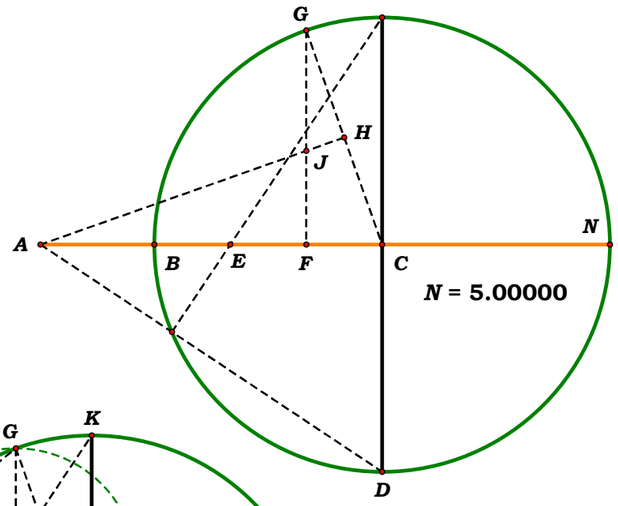
$ab := 1 \quad an := 5 \quad bn := an - ab \quad bc := \frac{bn}{2}$

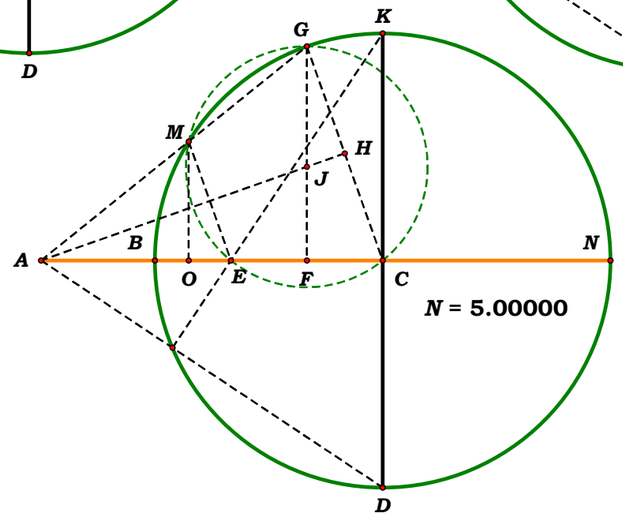
$ac := an - bc \quad ad := \sqrt{ac^2 + bc^2} \quad ce := \frac{bc^2}{ac}$

$cf := \frac{ce}{2} \quad bf := bc - cf \quad fn := bc + cf \quad fg := \sqrt{bf \cdot fn}$

$ae := ac - ce \quad af := ac - cf \quad ao := \frac{af \cdot ae}{ac} \quad mo := \frac{fg \cdot ae}{ac}$







Definitions.

$ab - 1 = 0 \quad an - an = 0 \quad bn - (an - 1) = 0 \quad bc - \frac{an - 1}{2} = 0$

$ac - \frac{an + 1}{2} = 0 \quad ad - \frac{\sqrt{2} \cdot \sqrt{an^2 + 1}}{2} = 0 \quad ce - \frac{(an - 1)^2}{2 \cdot (an + 1)} = 0$

$cf - \frac{(an - 1)^2}{4 \cdot (an + 1)} = 0 \quad bf - \frac{(an + 3) \cdot (an - 1)}{4 \cdot (an + 1)} = 0 \quad fn - \frac{(3 \cdot an + 1) \cdot (an - 1)}{4 \cdot (an + 1)} = 0$

$fg - \frac{(an - 1) \cdot \sqrt{(an + 3) \cdot (3 \cdot an + 1)}}{(4 \cdot an + 4)} = 0 \quad ae - \frac{2 \cdot an}{an + 1} = 0 \quad af - \frac{an^2 + 6 \cdot an + 1}{4 \cdot (an + 1)} = 0$

$ao - \frac{an \cdot \left(an^2 + 6 \cdot an + 1\right)}{(an + 1)^3} = 0 \quad mo - \frac{an \cdot \sqrt{(an + 3) \cdot (3 \cdot an + 1)} \cdot (an - 1)}{(an + 1)^3} = 0$

$\mathbf{bo} := \mathbf{ao} - \mathbf{ab}$

$\mathbf{on} := \mathbf{bn} - \mathbf{bo}$

$$\mathbf{bo} - \frac{\mathbf{bn}\cdot(3\cdot\mathbf{bn}+4)}{(\mathbf{bn}+2)^3} = 0$$

$$\mathbf{on} - \frac{\mathbf{bn}\cdot(\mathbf{bn}+1)^2\cdot(\mathbf{bn}+4)}{(\mathbf{bn}+2)^3} = 0$$

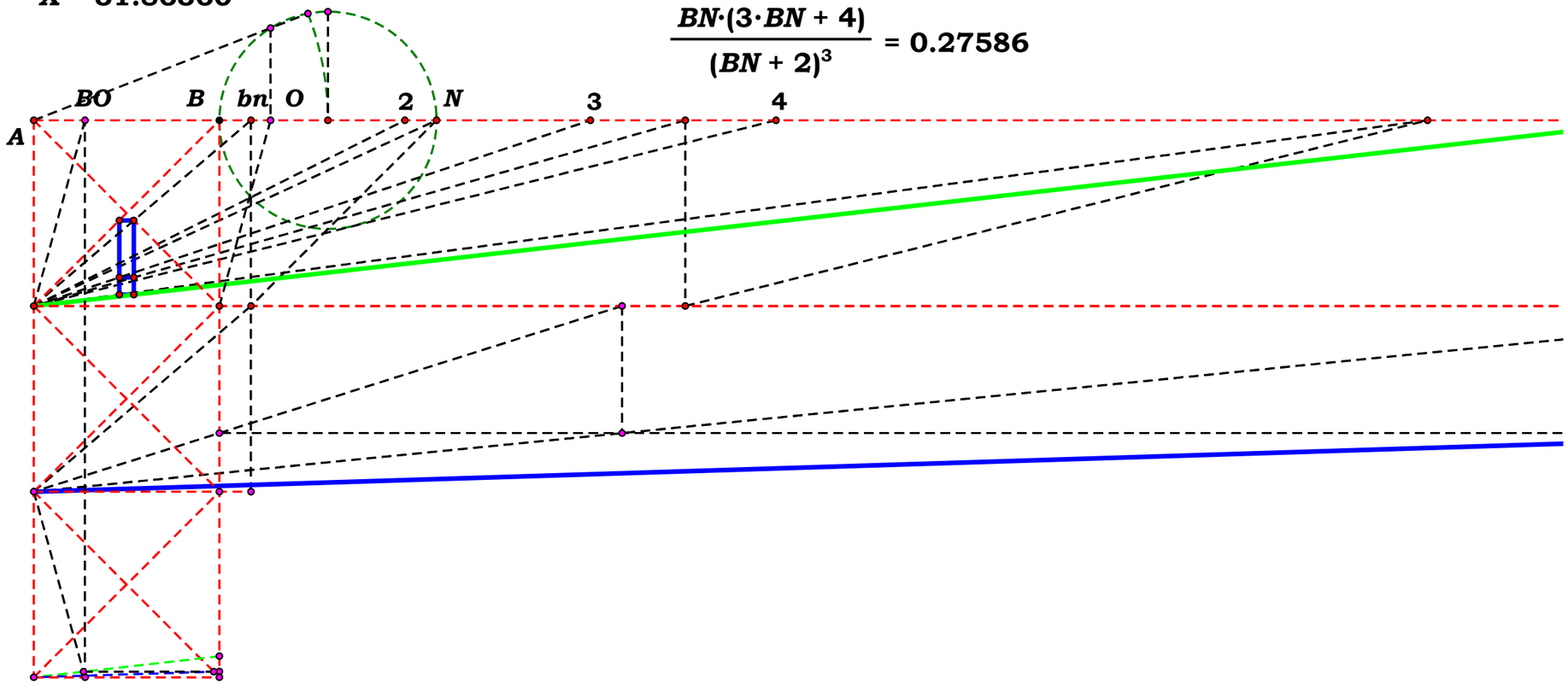
$$\mathbf{bo} - \frac{(\mathbf{an}-1)\cdot(3\cdot\mathbf{an}+1)}{(\mathbf{an}+1)^3} = 0$$

$$\mathbf{on} - \frac{\mathbf{an}^2\cdot(\mathbf{an}-1)\cdot(\mathbf{an}+3)}{(\mathbf{an}+1)^3} = 0$$

$N = 2.17028$
$B = 1.00000$
$BN = 1.17028$
$X = 31.86360$

$Y = 8.78984$
$O = 1.27586$
$BO = 0.27586$

$BN\cdot(3\cdot BN + 4) = 8.78984$
$(BN + 2)^3 = 31.86360$
$\frac{BN\cdot(3\cdot BN + 4)}{(BN + 2)^3} + B = 1.27586$
$\frac{BN\cdot(3\cdot BN + 4)}{(BN + 2)^3} = 0.27586$



Taking it easy

Descriptions.

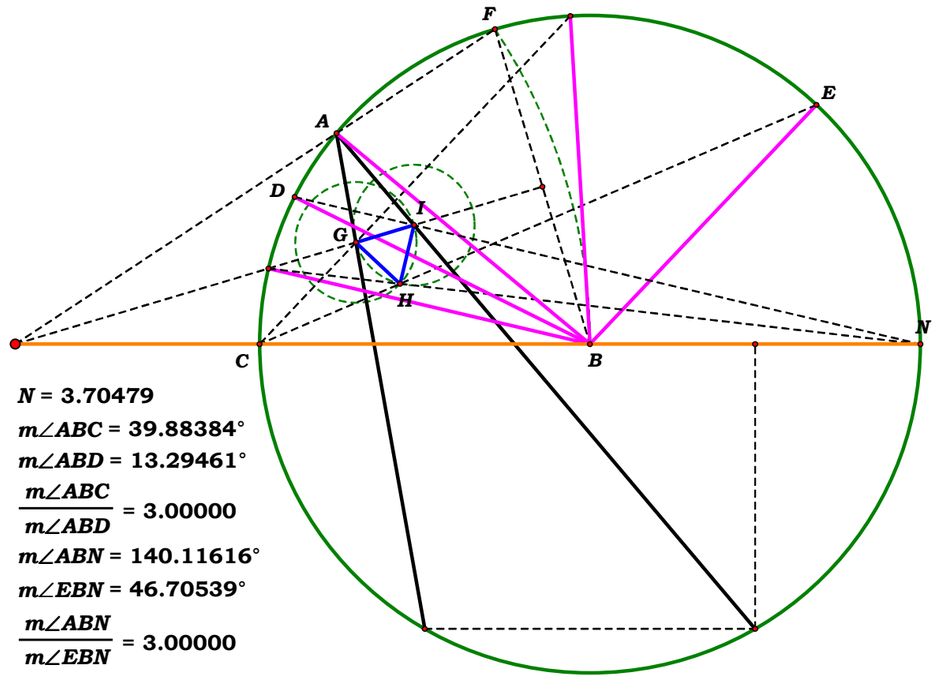
Now, when the brain wants to take a vacation, then any hard work is not on the menu.

Divide CN, BN, BF, by 2 and fill the rest in because basically, it is simpler then the Archimedean Paper Trisector and you always end up with the equilateral triangle in the center.

Not hard to find, not hard to remember.

Unit.

Given.



$N = 3.70479$

$m\angle ABC = 39.88384°$

$m\angle ABD = 13.29461°$

$\frac{m\angle ABC}{m\angle ABD} = 3.00000$

$m\angle ABN = 140.11616°$

$m\angle EBN = 46.70539°$

$\frac{m\angle ABN}{m\angle EBN} = 3.00000$

$m\angle GHI = 60.00000°$

Definitions.

Coffee and sci-fi.

Unit.

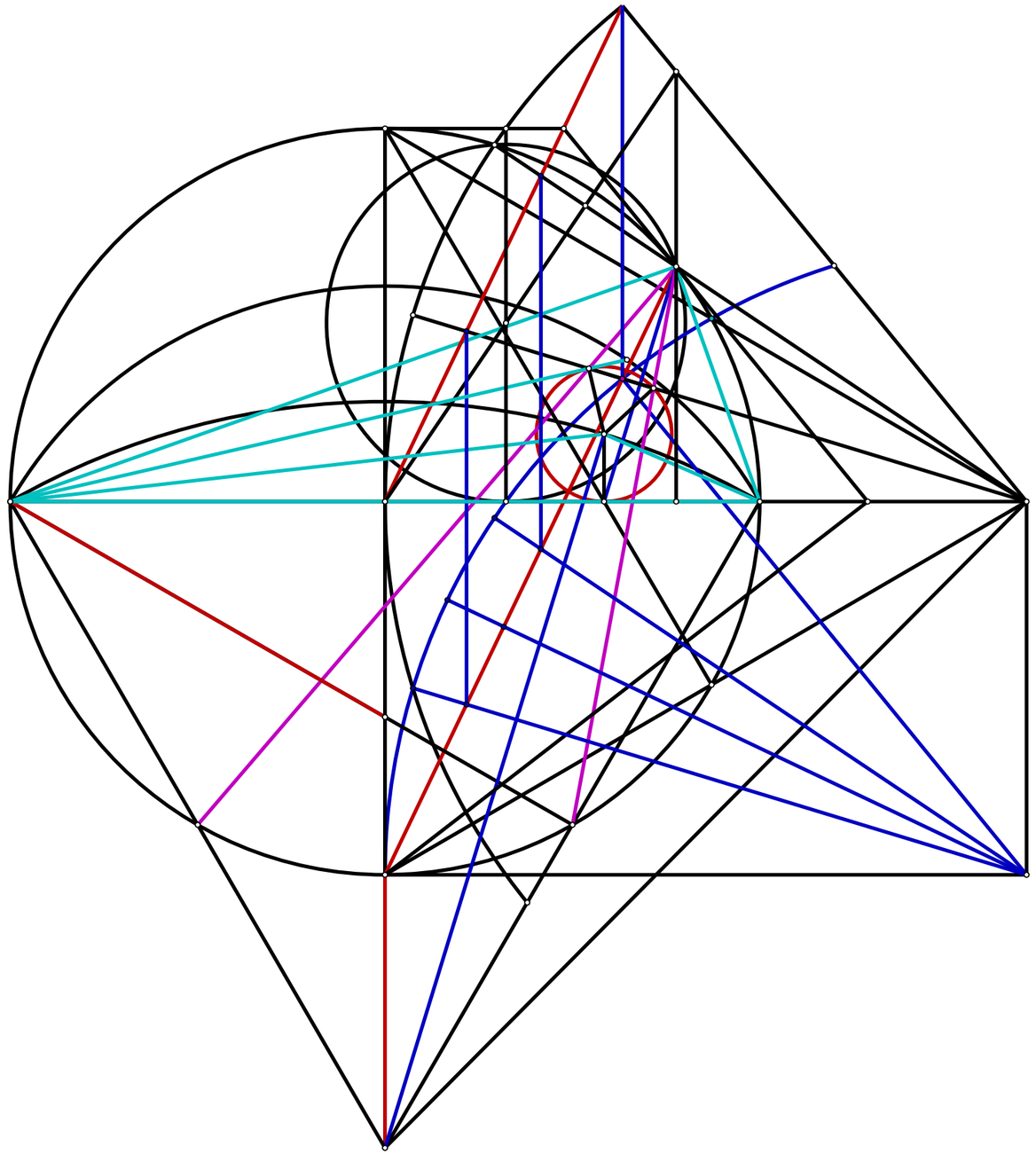
Given.

Wall Paper

Descriptions.

Picking mushrooms.

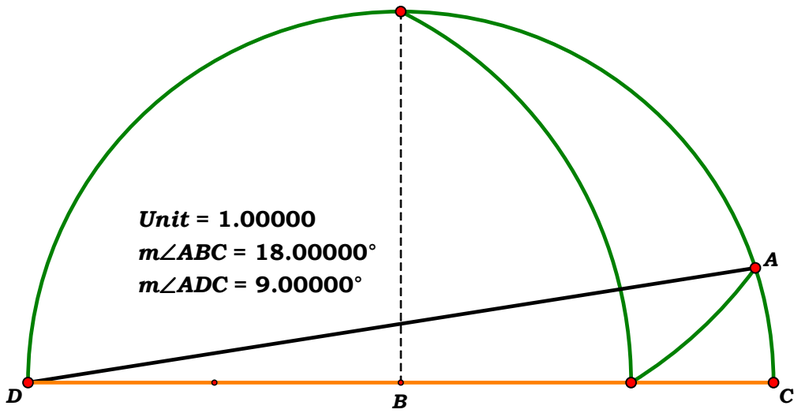
Definitions.



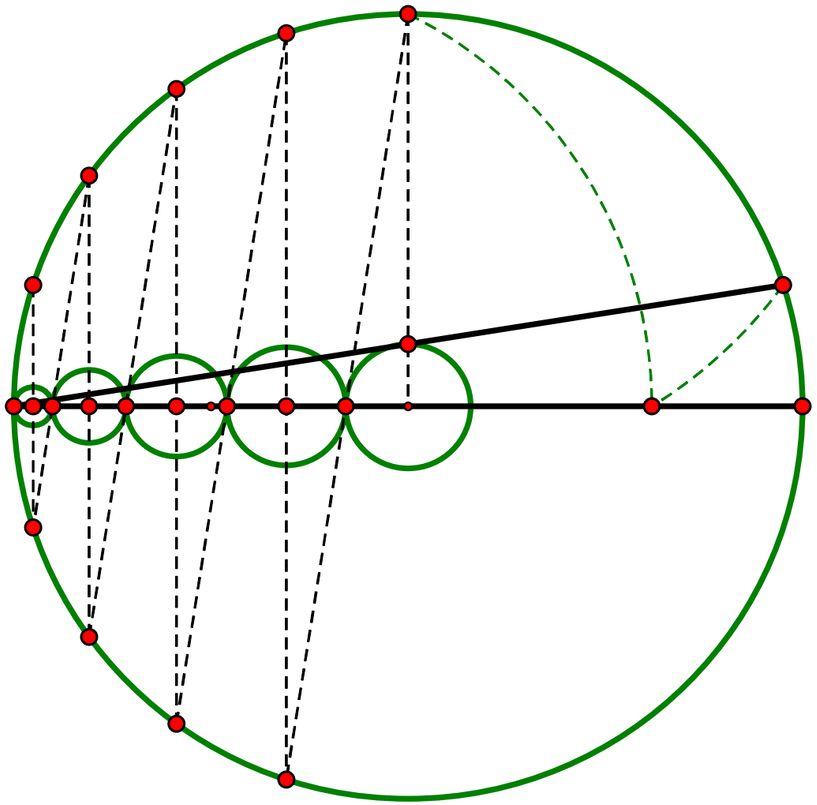
Eliptic Progression or
Baby it is cold outside.

Descriptions.

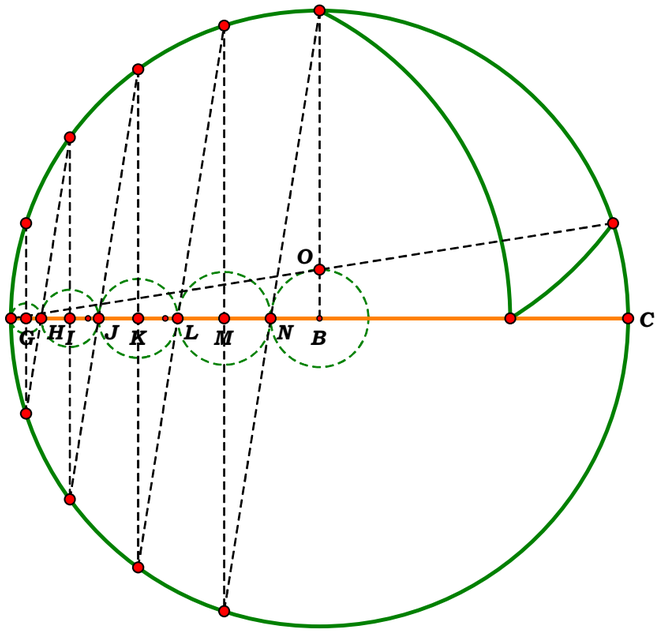
A little review on how to draw a pentagon which will be needed for our weather report.



I can straight forward use the figure to divide the whole circle into twenty parts. This is the proportional method, instead of drawing twenty individual circles aithmetically.



The area of each circle has a specific mathematical relationship.



Area ⊙*GH* = 0.29403 cm²
Area ⊙*IJ* = 1.06380 cm²
Area ⊙*KL* = 2.01529 cm²
Area ⊙*MN* = 2.78506 cm²
Area ⊙*BO* = 3.07909 cm²

(Area ⊙*GH*) + (Area ⊙*MN*) − (Area ⊙*BO*) = 0.00000 cm²
(Area ⊙*IJ*) + (Area ⊙*KL*) − (Area ⊙*BO*) = 0.00000 cm²

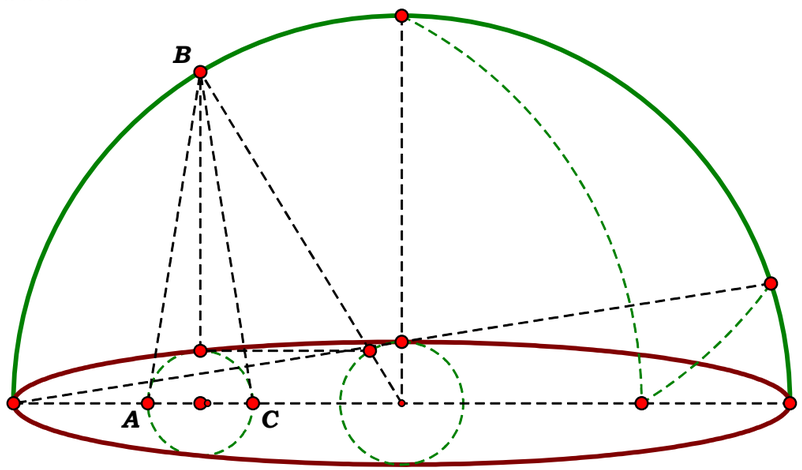
$$\frac{\text{Area } \odot MN}{\text{Area } \odot GH} = 9.47214$$

$$\frac{\text{Area } \odot KL}{\text{Area } \odot IJ} = 1.89443$$

$$\frac{\dfrac{\text{Area } \odot MN}{\text{Area } \odot GH}}{\dfrac{\text{Area } \odot KL}{\text{Area } \odot IJ}} = 5.00000$$

That relationsip is elliptical so that no matter where B is and AC has enough line to construct AC, it will always be the same angle as the circle on the minor axis.

$m\angle ABC$ = 18.00000°

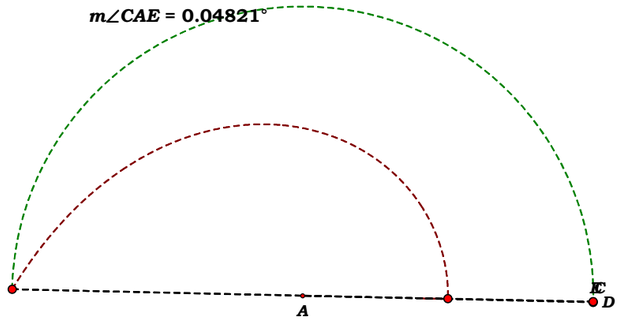


The totle number of circles will always be the number of parts that the quarter circle will be divided.

$\frac{m\angle CAD}{m\angle CAE}$ = 3.00000

$m\angle CAD$ = 0.14462°

$m\angle CAE$ = 0.04821°



Unit = 1.00000

Area ⊙*AB* = 0.07516 cm²

Area ⊙*CD* = 0.28920 cm²

Area ⊙*EF* = 0.60953 cm²

Area ⊙*GH* = 0.98738 cm²

Area ⊙*IJ* = 1.36524 cm²

Area ⊙*KL* = 1.68557 cm²

Area ⊙*MN* = 1.89961 cm²

Area ⊙*OP* = 1.97477 cm²

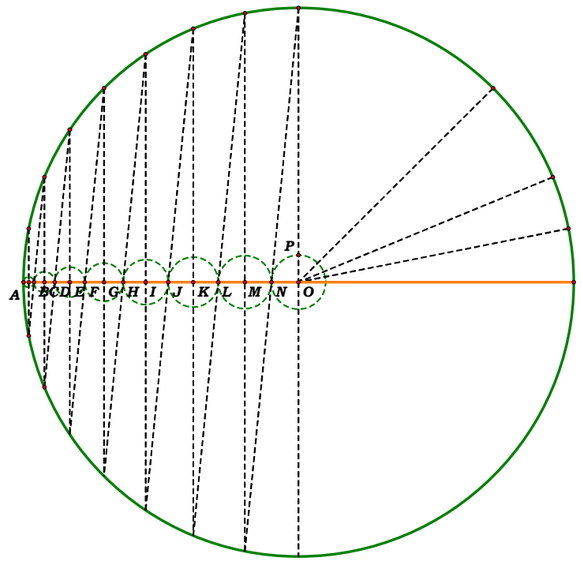
(Area ⊙*AB*) + (Area ⊙*MN*) = 1.97477 cm²

(Area ⊙*CD*) + (Area ⊙*KL*) = 1.97477 cm²

(Area ⊙*EF*) + (Area ⊙*IJ*) = 1.97477 cm²

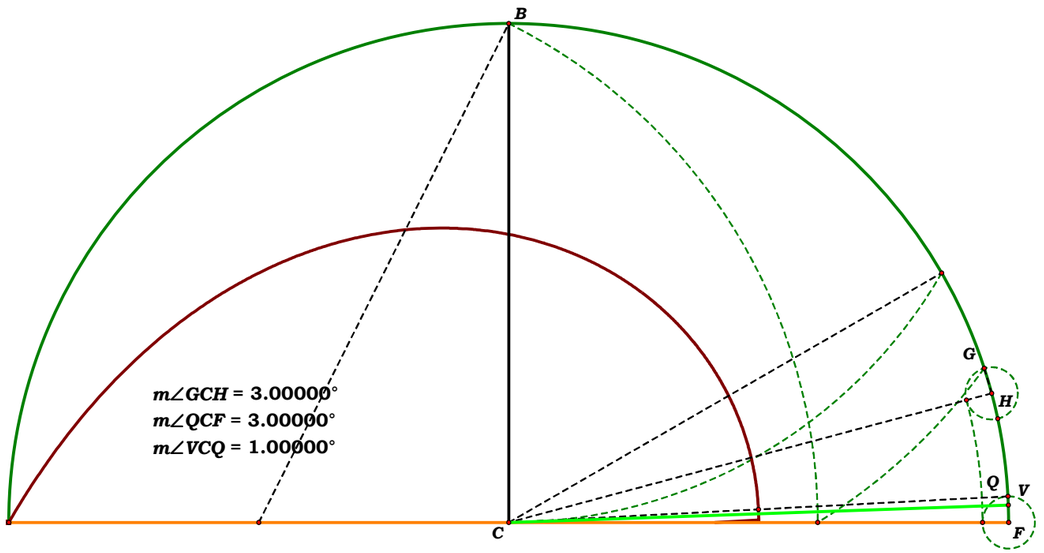
2·(Area ⊙*GH*) = 1.97477 cm²

$\frac{2\cdot(\text{Area } \odot GH)}{\text{Area } \odot GH}$ = 2.00000



So that the difference between 15 and 18 gives 3, which when trisected gives us 1 degree by which one can, if they have nothing better to do, draw away at it to keep them warm and fuzzy.

I made me a little Total Recall tool so I can sleep in. I wonder how big a screen I would need to see my tools angle?



Blast from the Past

Descriptions.

$AB := 1 \quad N := 5 \quad AF := AB \cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EK := BE \quad AE := AB + BE \quad DE := \frac{EK^2}{AE}$

$EF := BE \quad FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ \quad EQ = 0.307135$

Definitions.

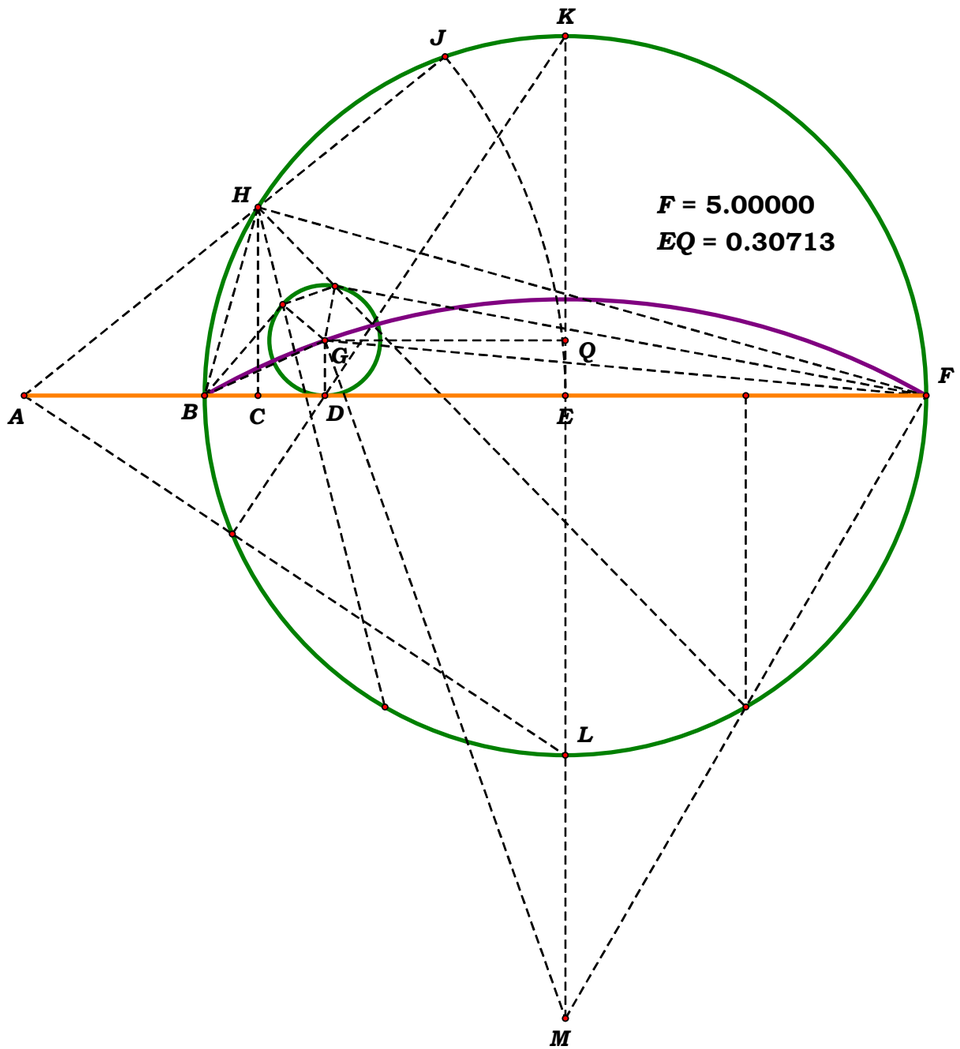
$AB - 1 = 0 \quad N - N = 0 \quad AF - N = 0 \quad BF - (N - 1) = 0$

$BE - \frac{N - 1}{2} = 0 \quad EK - BE = 0 \quad AE - \left(\frac{N + 1}{2}\right) = 0$

$DE - \frac{(N - 1)^2}{2 \cdot (N + 1)} = 0 \quad EF - BE = 0 \quad FM - BF = 0$

$EM - \frac{\sqrt{3} \cdot (N - 1)}{2} = 0 \quad GM - FM = 0 \quad GQ - DE = 0 \quad MQ - \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)}{2 \cdot (N + 1)} = 0$

$EQ - \frac{(N - 1) \cdot \left[\sqrt{[(N + 3) \cdot (3 \cdot N + 1)]} - \sqrt{3} \cdot N - \sqrt{3}\right]}{2 \cdot (N + 1)} = 0 \quad DG - EQ = 0 \quad EQ = 0.307135$



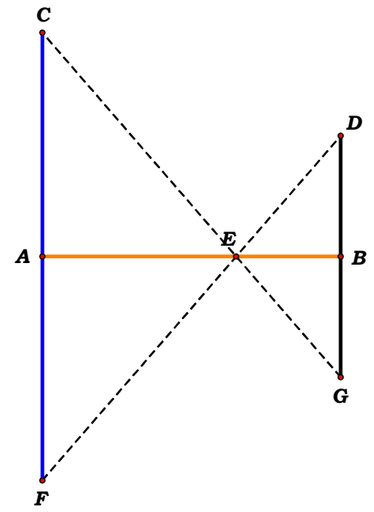
Treasure Island

Cartography Review

The missing treasure is at X, find its coordinates

One of the things writing up dynamic figures does, is make one sloppy when it comes to static figures, one habitually tries to start off with a variable, which is not actually permitted unless it is a study plate to help learn where to find the static for input.
So, in order to help me get out of a bad habit, I decided to take a vacation to Treasure Island. Once arriving I found out that in order to leave the Island, I had to find the treasure maked on the map. It held a first class ticket home.

A = 0.00000
B = 1.00000
C = 0.75346
D = 0.40636
E = 0.64964
H = 1.53933



AB = 6.25062 cm
AC = 4.70958 cm
BD = 2.54000 cm
AE = 4.06063 cm
AH = 9.62175 cm

Descriptions.

$AB := 1 \quad AC := .75346 \quad BD := .40636$

$BC := \sqrt{AB^2 + AC^2} \qquad BH := AB \cdot \frac{BD}{AC} \qquad AH := AB + BH$

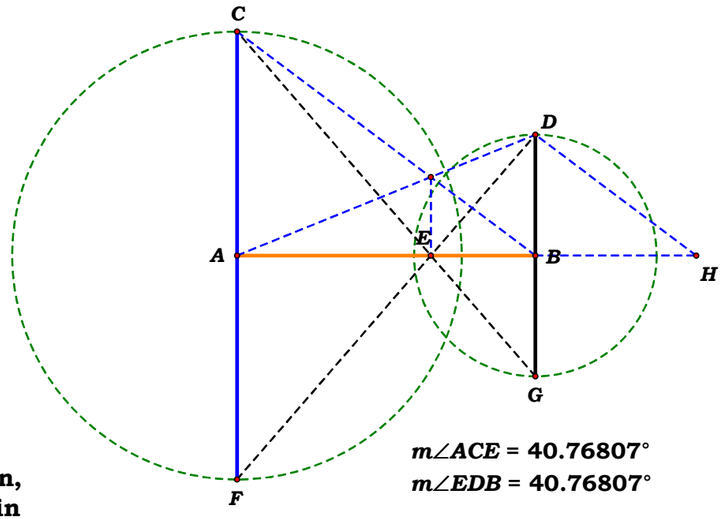
$BE := \frac{BH \cdot AB}{AH} \qquad AE := AB - BE$

Definitions.

$AB := AB \qquad AC := AC \qquad BD := BD \qquad BC - \sqrt{AB^2 + AC^2} = 0$

$BH - AB \cdot \frac{BD}{AC} = 0 \qquad AH - \frac{AB \cdot (AC + BD)}{AC} = 0$

$BE - \frac{AB \cdot BD}{AC + BD} = 0 \qquad AE - \frac{AB \cdot AC}{AC + BD} = 0$



m∠ACE = 40.76807°
m∠EDB = 40.76807°

This may be helpful in angle writeups as X always sets the angle, or the proportion. As written by Euclid, angle is an inclination, or again, propotion. Therefore, contrary to current dogma, proportion starts in Book 1. Angle and ratio are synonyms.

The Geometric Unit

The geometric circle affords us two distinct triangles to work with, one using the diameter, and the other the radius and therefore, allows us to convert the Arithmetic circle, into the Geometric one which is determined by the radius.

Descriptions.

$AC := 1 \quad BC := \frac{AC}{2} \quad CD := \sqrt{\frac{2}{3}} \qquad AD := \sqrt{\frac{1}{3}}$

$S_1 := AC \qquad S_2 := AD \qquad S_3 := CD$

$AE := \frac{AC^2 + AD^2 - CD^2}{2 \cdot AC} \qquad AE = 0.333333 \qquad AD^2 = 0.333333$

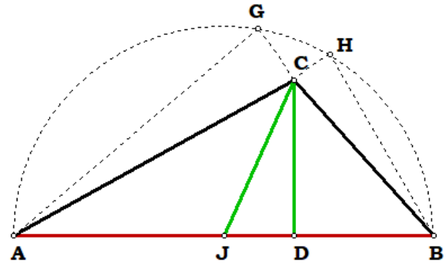
$CE := AC - AE \qquad DE := \sqrt{AE \cdot (1 - AE)}$

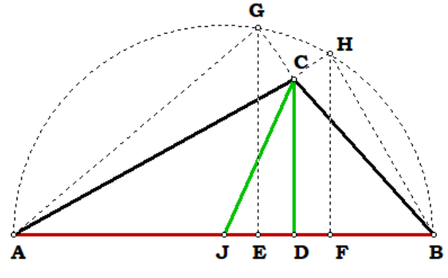
Definitions.

$AE - \left(1 - CD^2\right) = 0 \qquad CE - CD^2 = 0$

$DE - \sqrt{\left(CD^2 - CD^4\right)} = 0$

$AD - \sqrt{1 - CD^2} = 0 \qquad \frac{1}{3} + \frac{2}{3} = 1$





Pythagoras Revisited for the Circle

$S_1 := AB$

$S_2 := AC$

$S_3 := BC$

$EF - \frac{S_1^2 - S_2^2 - S_3^2}{S_1} = 0 \qquad DE - \frac{S_1^2 - S_2^2 - S_3^2}{2 \cdot S_1} = 0$

$AD - \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^2 - S_2^2 + S_3^2}{2 \cdot S_1} = 0$

$DJ - \frac{\sqrt{\left(S_2^2 - S_3^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^2 - S_1^2 + 2 \cdot S_3^2}}{2} = 0$

$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0$

$A = 0.00000$

$AE = 0.33333$

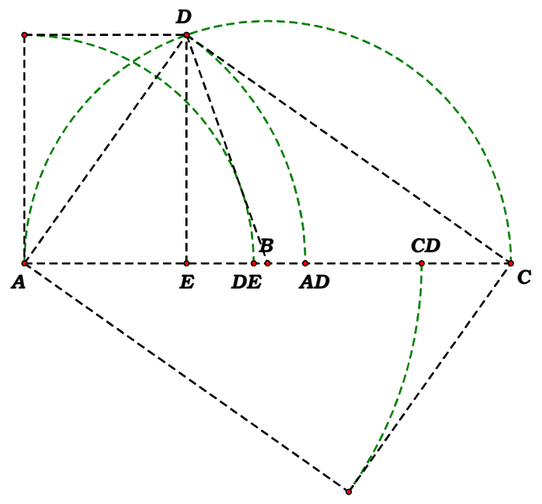
$DE = 0.47140$

$B = 0.50000$

$AD = 0.57735$

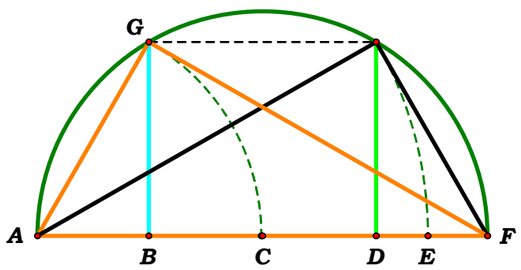
$CD = 0.81650$

$C = 1.00000$



Where did the pie go?

$A = 0.00000$
$B = 0.24744$
$C = 0.49743$
$D = 0.75256$
$E = 0.86750$
$F = 1.00000$
$G = 0.43152$

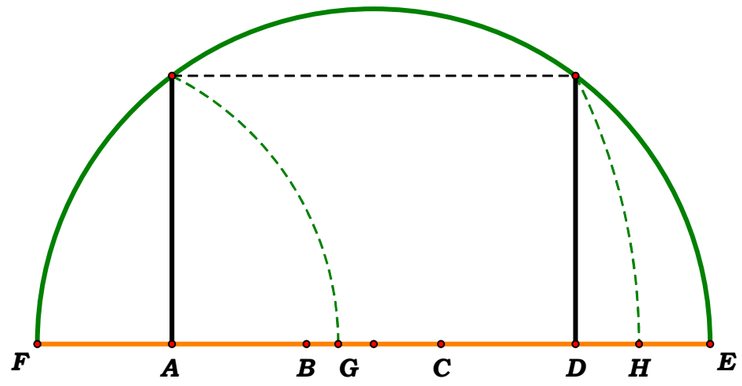


$AF = 8.34083$ cm
$AB = 2.06386$ cm
$AD = 6.27697$ cm
$AC = 4.14901$ cm
$AE = 7.23569$ cm

$B + D = 1.00000$ $\quad (AB + AD) - AF = 0.00000$ cm

$C^2 + E^2 = 1.00000$ $\quad \dfrac{AC^2 + AE^2}{AF^2} = 1.00000$

Which is according to the sum of the squares etc., of Pythagoras translated into simple Arithmetic. And as Pythagoras Revisited would show, all of triangular equatios resolve to arithmetic expressions of proportion, exactly as the angle is defined in Book 1 of the elements as an inclination, i.e., proportion. One will notice, π has noting whatsoever to do with it.



$A = 0.20000$ $\quad \dfrac{1}{A} = 5.00000$
$B = 0.40000$
$C = 0.60000$ $\quad \dfrac{2}{B} = 5.00000$
$D = 0.80000$
$E = 1.00000$ $\quad \dfrac{3}{C} = 5.00000$
$\dfrac{4}{D} = 5.00000$
$\dfrac{5}{E} = 5.00000$

$G = 0.44721$ $\quad G^2 = 0.20000$
$H = 0.89443$ $\quad H^2 = 0.80000$

$A + D = 1.00000$
$G^2 + H^2 = 1.00000$

Ellipse 2

Descriptions.

$AC := 1 \qquad AB := \frac{AC}{2} \qquad AL := \frac{1}{\sqrt{2}}$

$BL := AL - AB \qquad AE := AB - BL \qquad AD := \frac{AE}{2}$

$CD := AC - AD \qquad DF := \sqrt{AD \cdot CD} \qquad FH := 2 \cdot DF$

$EH := \sqrt{DF^2 + AD^2} \qquad EG := \sqrt{AB^2 + BL^2}$

$GH := EH + EG$

$2 \cdot FH^2 = 1$

Definitions.

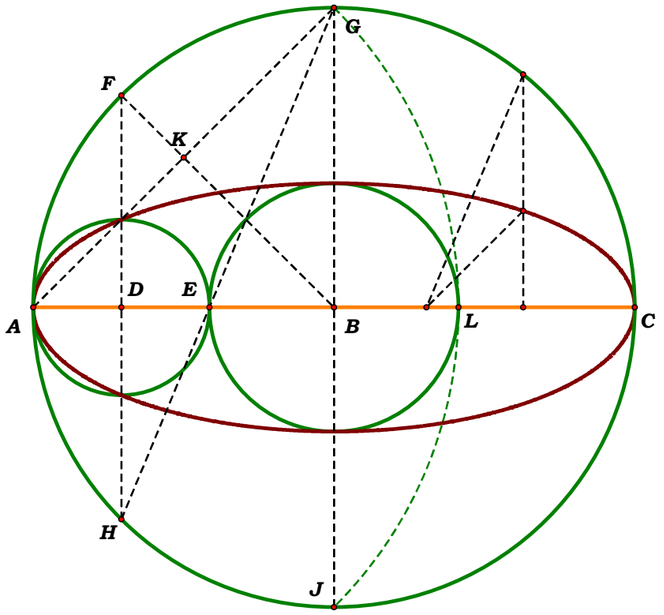
$AC - 1 = 0 \qquad AB - \frac{1}{2} = 0 \qquad AL - \frac{1}{\sqrt{2}} = 0 \qquad BL - \frac{\sqrt{2} - 1}{2} = 0$

$AE - \frac{2 - \sqrt{2}}{2} = 0 \qquad AD - \frac{2 - \sqrt{2}}{4} = 0 \qquad CD - \frac{\sqrt{2} + 2}{4} = 0$

$DF - \frac{\sqrt{2}}{4} = 0 \qquad FH - \frac{\sqrt{2}}{2} = 0 \qquad EH - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0 \qquad EG - \left(\frac{\sqrt{2 - \sqrt{2}}}{\sqrt{2}} \right) = 0$

$GH - \frac{\left(\sqrt{2} + 1\right) \cdot \sqrt{2 - \sqrt{2}}}{2} = 0$

A = 0.00000
D = 0.14645
E = 0.29289
B = 0.50000
L = 0.70711
C = 1.00000



Ellipse 3

Descriptions.

$AC := 1 \quad AB := \frac{AC}{2} \quad AF := \frac{AB}{2} \quad CF := AC - AF$

$BF := AB - AF \quad DK := BF \quad FL := \sqrt{AF \cdot CF} \quad AD := AB - FL$

$AE := 2 \cdot AD \quad DK := BF \quad EF := AF - AE \quad AG := AF + EF$

$BG := AB - AG \quad AK := \sqrt{DK^2 + AD^2} \quad KN := 2 \cdot DK$

$LO := 2 \cdot FL$

$LO^2 + KN^2 = 1$

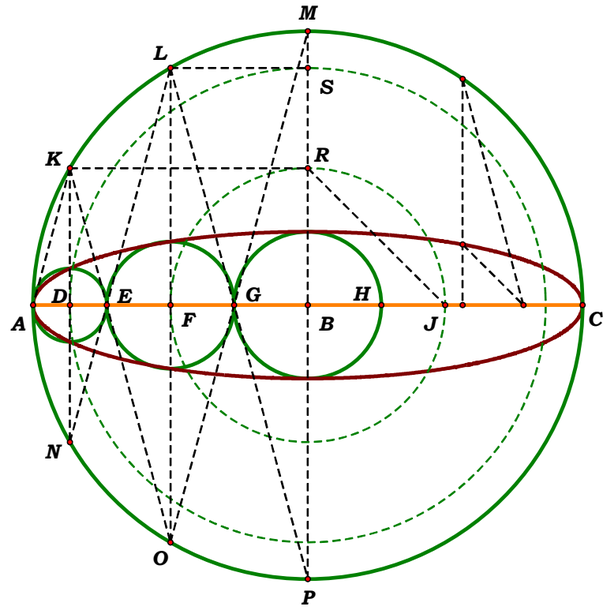
Definitions.

$AC - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AF - \frac{1}{4} = 0 \quad CF - \frac{3}{4} = 0 \quad BF - \frac{1}{4} = 0 \quad DK - BF = 0$

$FL - \frac{\sqrt{3}}{2^2} = 0 \quad AD - \frac{2 - \sqrt{3}}{4} = 0 \quad AE - \frac{2 - \sqrt{3}}{2} = 0 \quad DK - BF = 0 \quad EF - \frac{2 \cdot \sqrt{3} - 3}{4} = 0$

$AG - \frac{2 \cdot \sqrt{3} - 2}{4} = 0 \quad BG - \frac{2 - \sqrt{3}}{2} = 0 \quad AK - \frac{\sqrt{2 - \sqrt{3}}}{2} = 0 \quad KN - \frac{1}{2} = 0 \quad LO - \frac{\sqrt{3}}{2} = 0$

A = 0.00000
D = 0.06699
E = 0.13397
F = 0.25000
G = 0.36603
B = 0.50000
H = 0.63397
J = 0.75000
H = 0.63397
J = 0.75000
C = 1.00000
D = 0.25000
L = 0.43301



Ellipse 4

Descriptions.

$AC := 1 \quad AB := \frac{AC}{2} \quad AR := \sqrt{2 \cdot AB^2} \quad AY := \frac{AR}{2} \quad BY := AY$

$OY := AB - BY \quad AF := OY \quad FO := BY \quad AO := \sqrt{FO^2 + AF^2}$

$AW := \frac{AO}{2} \quad BW := \sqrt{AB^2 - AW^2} \quad NW := AB - BW \quad AD := NW$

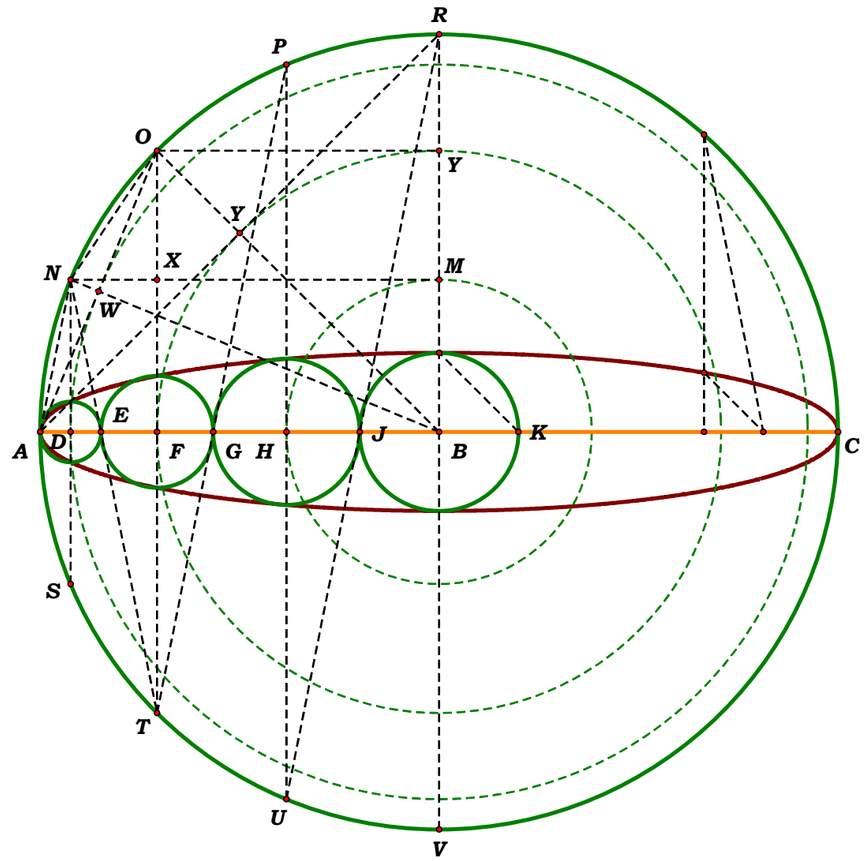
$CD := AC - AD \quad DN := \sqrt{AD \cdot CD} \quad AE := 2 \cdot AD \quad EF := AF - AE$

$AG := AF + EF \quad AH := AB - DN \quad GH := AH - AG \quad AJ := AH + GH$

$BJ := AB - AJ \quad BD := AB - AD \quad HP := BD \quad NS := 2 \cdot DN \quad OT := 2 \cdot FO \quad PU := 2 \cdot HP$

$PU^2 + NS^2 = 1 \quad 2 \cdot OT^2 = 1$

$A = 0.00000$
$D = 0.03806$
$E = 0.07612$
$F = 0.14645$
$G = 0.21677$
$H = 0.30866$
$J = 0.40054$
$B = 0.50000$
$N = 0.19134$
$O = 0.35355$
$P = 0.46194$



Definitions.

$AC - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AR - \frac{\sqrt{2}}{2} = 0 \quad AY - \frac{\sqrt{2}}{4} = 0 \quad BY - AY = 0 \quad OY - \frac{2 - \sqrt{2}}{4} = 0 \quad AF - OY = 0 \quad FO - BY = 0$

$AO - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0 \quad AW - \frac{\sqrt{2 - \sqrt{2}}}{4} = 0 \quad BW - \frac{\sqrt{2 + \sqrt{2}}}{2^2} = 0 \quad NW - \frac{2 - \sqrt{\sqrt{2} + 2}}{4} = 0 \quad AD - NW = 0 \quad CD - \frac{2 + \sqrt{\sqrt{2} + 2}}{4} = 0 \quad DN - \frac{\sqrt{2 - \sqrt{2}}}{2^2} = 0 \quad AE - \frac{2 - \sqrt{\sqrt{2} + 2}}{2} = 0$

$EF - \frac{2 \cdot \sqrt{\sqrt{2} + 2} - \sqrt{2} - 2}{4} = 0 \quad AG - \frac{\sqrt{\sqrt{2} + 2} - \sqrt{2}}{2} = 0 \quad AH - \frac{2 - \sqrt{2 - \sqrt{2}}}{4} = 0 \quad GH - \frac{2 \cdot \left(\sqrt{2} - \sqrt{\sqrt{2} + 2} + 1\right) - \sqrt{2 - \sqrt{2}}}{4} = 0 \quad AJ - \frac{\sqrt{2} - \sqrt{\sqrt{2} + 2} - \sqrt{2 - \sqrt{2}} + 2}{2} = 0$

$BJ - \left(\frac{\sqrt{2 - \sqrt{2}} + \sqrt{\sqrt{2} + 2} - \sqrt{2} - 1}{2}\right) = 0 \quad BD - \frac{\sqrt{\sqrt{2} + 2}}{4} = 0 \quad HP - BD = 0 \quad NS - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0 \quad OT - \frac{\sqrt{2}}{2} = 0 \quad PU - \frac{\sqrt{\sqrt{2} + 2}}{2} = 0$

The 54 degree massacre.

Descriptions.

$AG := 1 \quad AC := \frac{AG}{2} \quad AB := \frac{AG}{4} \quad BC := AC - AB$

$BH := \sqrt{BC^2 + AC^2} \quad AE := AB + BH \quad CE := AE - AC$

$HE := \sqrt{CE^2 + AC^2} \quad S_1 := AC \quad S_2 := HE \quad S_3 := AC$

$HM := \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} \quad CM := AC - HM \quad FM := \sqrt{HM \cdot (AG - HM)}$

$AL := AC + FM \quad GL := AG - AL \quad FG := \sqrt{CM^2 + GL^2}$

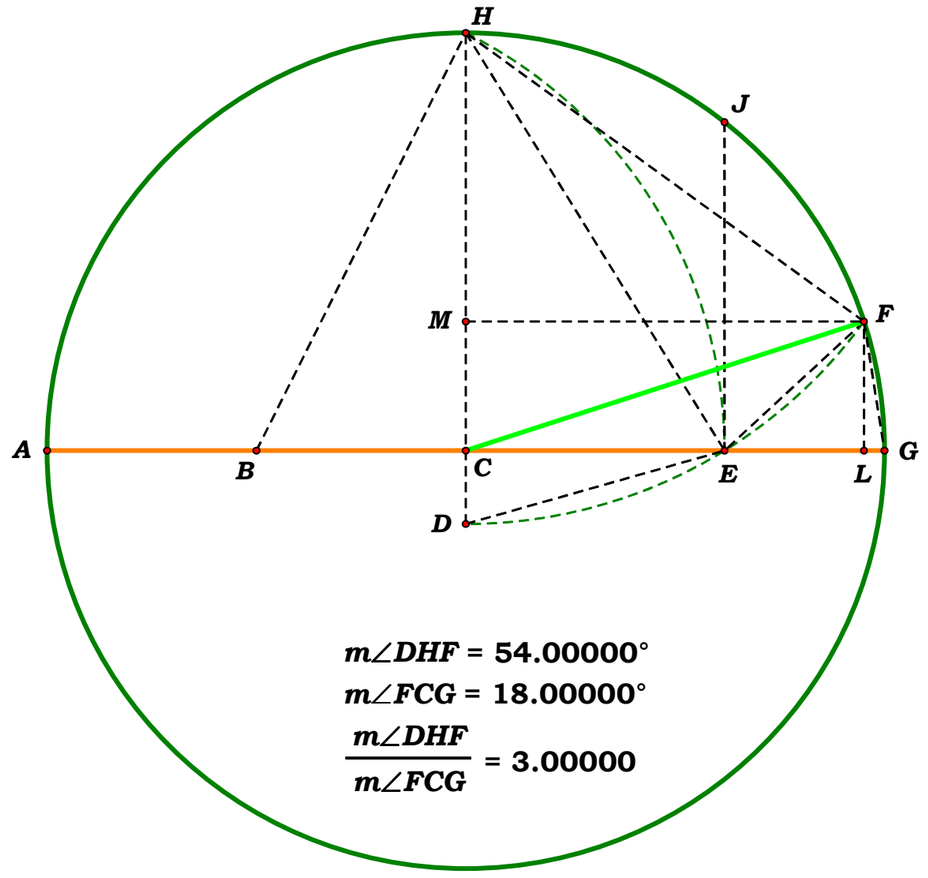
Definitions.

$AG - 1 = 0 \quad AC - \frac{1}{2} = 0 \quad AB - \frac{1}{4} = 0 \quad BC - \frac{1}{4} = 0 \quad BH - \frac{\sqrt{5}}{2^2} = 0$

$AE - \frac{1+\sqrt{5}}{4} = 0 \quad CE - \frac{\sqrt{5}-1}{4} = 0 \quad HE - \frac{\sqrt{5-\sqrt{5}}}{2^{\frac{3}{2}}} = 0 \quad HM - \frac{5-\sqrt{5}}{8} = 0$

$CM - \frac{\sqrt{5}-1}{8} = 0 \quad FM - \frac{\sqrt{5+\sqrt{5}}}{\sqrt{2^5}} = 0 \quad AL - \frac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad GL - \frac{4-\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad FG - \frac{\sqrt{4-\sqrt{2\cdot\sqrt{5}+10}}}{\sqrt{8}} = 0$

$A = 0.00000$
$B = 0.25000$
$C = 0.50000$
$E = 0.80902$
$L = 0.97553$
$G = 1.00000$
$F = 0.15451$
$J = 0.39308$
$M = 0.15451$



$m\angle DHF = 54.00000°$
$m\angle FCG = 18.00000°$
$\frac{m\angle DHF}{m\angle FCG} = 3.00000$

Ellipse 5

Descriptions.

$AC := 1 \quad AB := \frac{AC}{2} \quad AN := \frac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} \quad CN := AC - AN$

$MN := CN \quad NS := \sqrt{AN\cdot CN} \quad BF := NS \quad AF := AB + BF$

$CF := AC - AF \quad FO := \sqrt{AF\cdot CF} \quad BE := \frac{BF\cdot AB}{AB+FO} \quad AE := AB + BE$

$EF := BF - BE \quad AG := AF + EF \quad BN := AN - AB \quad DS := \sqrt{BN^2 + CF^2}$

$SX := \frac{DS}{2} \quad ST := 2\cdot NS \quad PX := \sqrt{ST^2 - SX^2} \quad BX := AB - PX$

$BK := BX \quad AK := AB + BK \quad CK := AC - AK \quad KR := \sqrt{AK\cdot CK}$

$BH := KR \quad CM := 2\cdot CN \quad KM := BK - CM \quad AH := AB + BH$

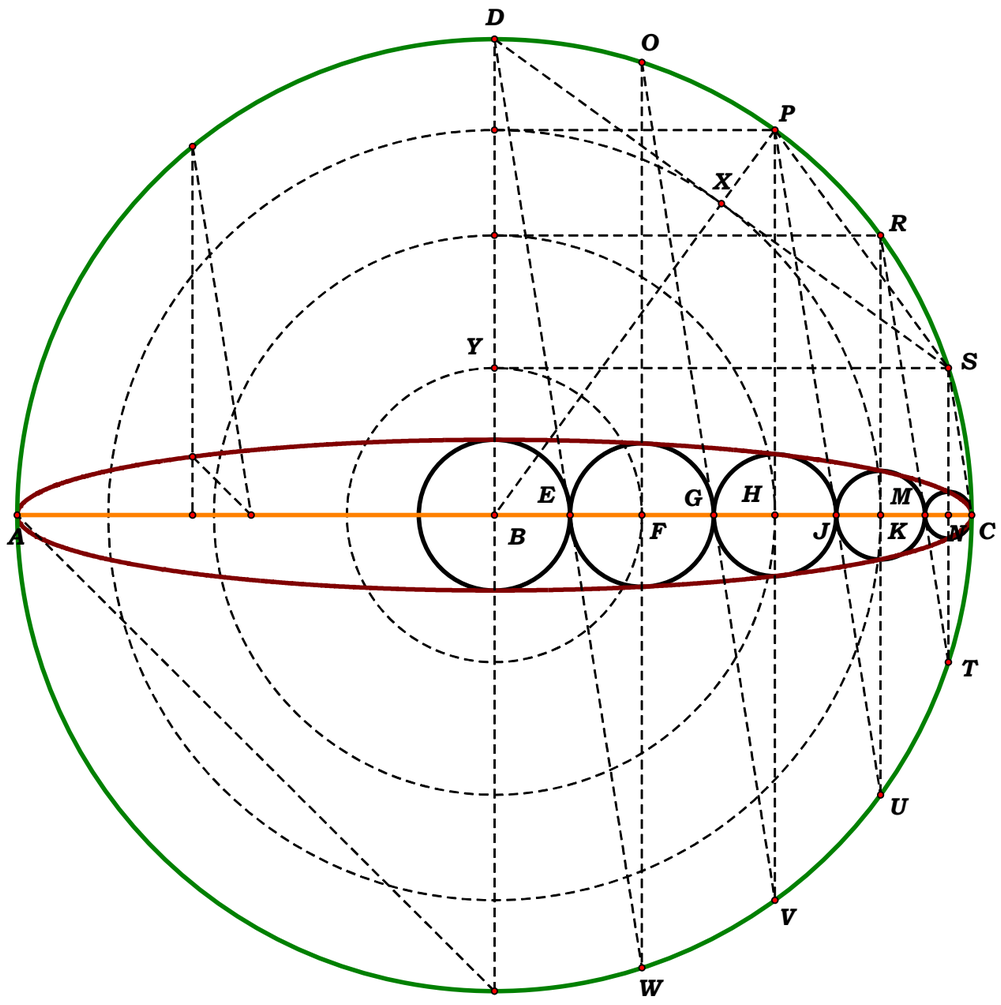
$GH := AH - AG \quad JH := GH \quad AJ := AH + GH \quad JK := AK - AJ$

$AM := AK + JK \quad CH := AC - AH \quad HP := \sqrt{AH\cdot CH}$

$OW := 2\cdot FO \quad PV := 2\cdot HP \quad RU := 2\cdot KR$

$ST^2 + OW^2 = 1 \quad PV^2 + RU^2 = 1$

A = 0.00000
B = 0.50000
E = 0.57919
F = 0.65451
G = 0.72982
H = 0.79389
J = 0.85796
K = 0.90451
M = 0.95106
N = 0.97553
C = 1.00000
O = 0.47553
P = 0.40451
R = 0.29389
S = 0.15451



Definitions.

$AC - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AN - \frac{4+\sqrt{2\cdot\sqrt{5}+10}}{8} = 0 \quad CN - \left(\frac{4-\sqrt{2\cdot\sqrt{5}+10}}{8}\right) = 0 \quad MN - CN = 0 \quad AE - \frac{\sqrt{2\cdot\sqrt{5}+10} - \sqrt{10\cdot\sqrt{5}+50} + 2\cdot\sqrt{5}+2}{\left(12-4\cdot\sqrt{5}\right)} = 0$

$$NS - \frac{\sqrt{3-\sqrt{5}}}{\sqrt{2^5}} = 0 \qquad BF - NS = 0 \qquad AF - \frac{3+\sqrt{5}}{8} = 0$$

$$CF - \frac{5-\sqrt{5}}{8} = 0 \qquad FO - \frac{\sqrt{5+\sqrt{5}}}{\sqrt{2^5}} = 0 \qquad BE - \frac{\left(\sqrt{5}-1\right)\cdot\left(\sqrt{2\cdot\sqrt{5}+10}-4\right)}{2\cdot\left(2\cdot\sqrt{5}-6\right)} = 0$$

Definitions.

$$EF - \frac{\left(\sqrt{5}-1\right)\cdot\left(\sqrt{5}-2\cdot\sqrt{2\cdot\sqrt{5}+10}+5\right)}{8\cdot\left(\sqrt{5}-3\right)} = 0 \qquad AG - \frac{\left(\sqrt{5}+1\right)\cdot\left(\sqrt{2\cdot\sqrt{5}+10}-2\right)}{8} = 0 \qquad BN - \frac{\sqrt{2\cdot\sqrt{5}+10}}{8} = 0$$

$$DS - \frac{\sqrt{5-\sqrt{5}}}{2\cdot\sqrt{2}} = 0 \qquad SX - \frac{\sqrt{5-\sqrt{5}}}{4\cdot\sqrt{2}} = 0 \qquad ST - \frac{2\cdot\sqrt{3-\sqrt{5}}}{\sqrt{2^5}} = 0 \qquad PX - \frac{3-\sqrt{5}}{8} = 0 \qquad BX - \frac{1+\sqrt{5}}{8} = 0 \qquad BK - BX = 0$$

$$AK - \frac{5+\sqrt{5}}{8} = 0 \qquad CK - \frac{3-\sqrt{5}}{8} = 0 \qquad KR - \frac{\sqrt{5-\sqrt{5}}}{\sqrt{2^5}} = 0 \qquad BH - KR = 0 \qquad CM - \frac{4-\sqrt{2\cdot\sqrt{5}+10}}{4} = 0$$

$$KM - \frac{2\cdot\sqrt{2\cdot\sqrt{5}+10}+\sqrt{5}-7}{8} = 0 \qquad AH - \frac{4+\sqrt{2}\cdot\sqrt{5-\sqrt{5}}}{8} = 0 \qquad GH - \frac{\sqrt{2}\cdot\sqrt{5-\sqrt{5}}-\sqrt{2\cdot\sqrt{5}+10}-\sqrt{10\cdot\sqrt{5}+50}+2\cdot\sqrt{5}+6}{8} = 0$$

$$JH - GH = 0 \qquad AJ - \frac{2\cdot\sqrt{2}\cdot\sqrt{5-\sqrt{5}}-\sqrt{2\cdot\sqrt{5}+10}-\sqrt{10\cdot\sqrt{5}+50}+2\cdot\sqrt{5}+10}{8} = 0 \qquad JK - \frac{\sqrt{2\cdot\sqrt{5}+10}-2\cdot\sqrt{10-2\cdot\sqrt{5}}+\sqrt{10\cdot\sqrt{5}+50}-\sqrt{5}-5}{8} = 0$$

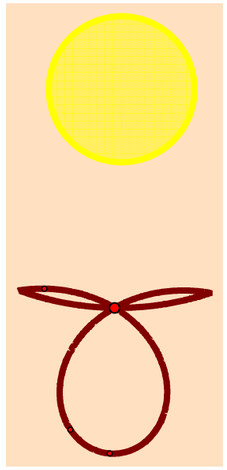
$$AM - \frac{\sqrt{2\cdot\sqrt{5}+10}-2\cdot\sqrt{10-2\cdot\sqrt{5}}+\sqrt{10\cdot\sqrt{5}+50}}{8} = 0 \qquad CH - \frac{4-\sqrt{2}\cdot\sqrt{5-\sqrt{5}}}{8} = 0 \qquad HP - \frac{1+\sqrt{5}}{8} = 0 \qquad OW - \frac{2\cdot\sqrt{5+\sqrt{5}}}{\sqrt{2^5}} = 0$$

$$PV - \frac{2\cdot\left(1+\sqrt{5}\right)}{8} = 0 \qquad RU - \frac{2\cdot\sqrt{5-\sqrt{5}}}{\sqrt{2^5}} = 0$$

The Rabbit Trisector

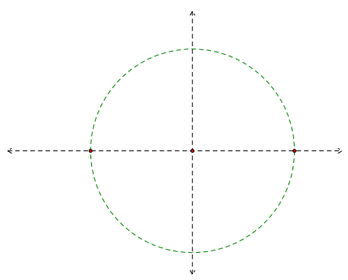
Saturday, June 17, 2023

While cleaning out some old plates for Three Pieces of Paper, which I am now working on, I rant (sic) into a single file in a directory for 2006. I also found, that while sitting there, it was not idle, for I found that it had a kid, or rather a little bunny.

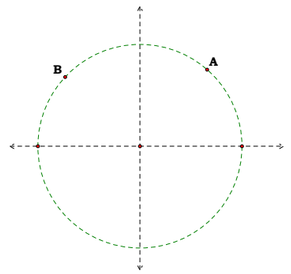


Not being a grown-up bunny, it cannot do anything really, 'cept watch the sun and grow for when it grows up, it will become, by using its ears, a master angle trisector. Hard to believe, I know, but it is a fact which his father will demonstrate for you. His father is rather famous, having made his first movie debut in the film classic, Harvey, which you might want to find a copy of, for in it, you will learn that Harvey is a Pooka, and invisible.

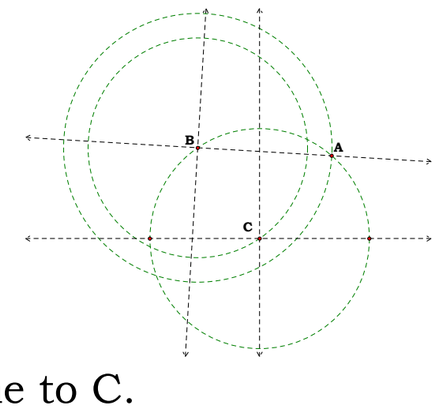
I am, if you have not yet guessed it, an amateur Pooka Hunter. And this is the story of how I found Harvey after he retired from the motion picture industry.



The reason that Harvey was so hard to find is, perhaps you already know, that he is not the product of the Straightedge, or Arithmetic recursion, but of the circle, or Geometric recursion. Therefore, one must learn the magic of the circle in order to peer into the invisible world. I will start there, with the circle and divide it into its quadrants for Harvey can trisect any angle in any quadrant.

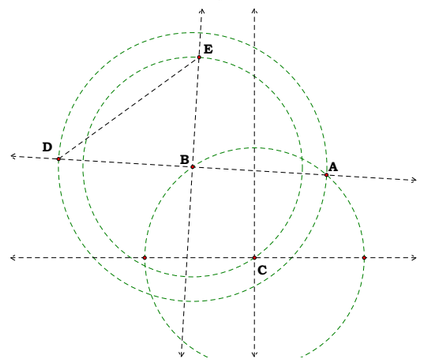


Place upon it, two points, point A for the angle, and B to construct the loci, or Loki, as some have called him, but his name is really Harvey and his origins will be made invisible after we get done outlining him.

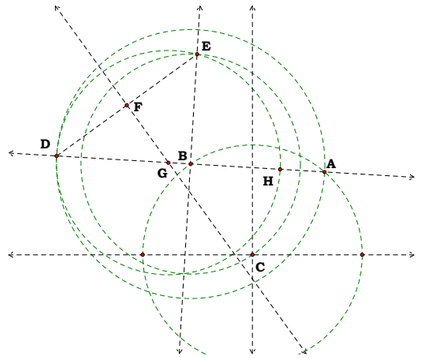


From B, draw two circles, one to A and one to C.

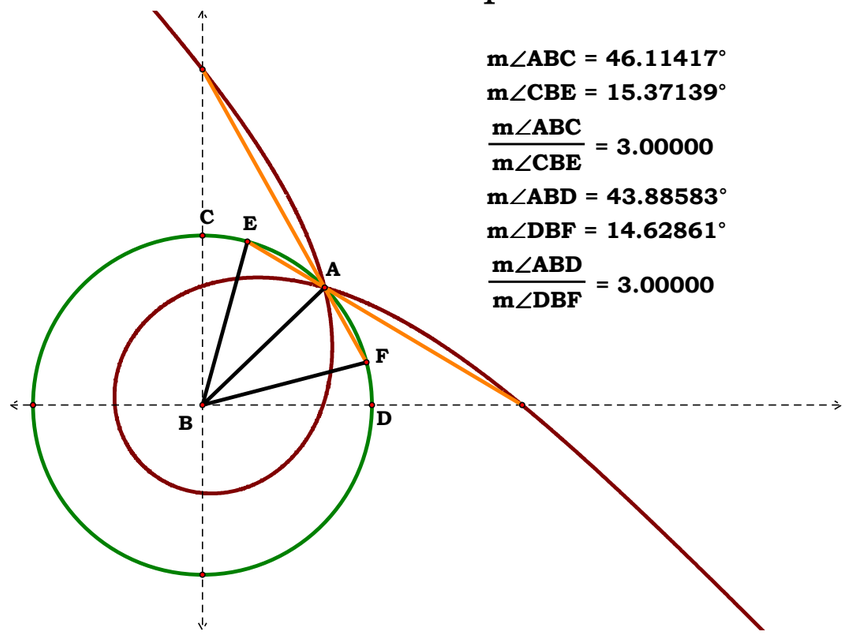
Construct a segment DE, where D is on the circle AB, and E on the circle BC.



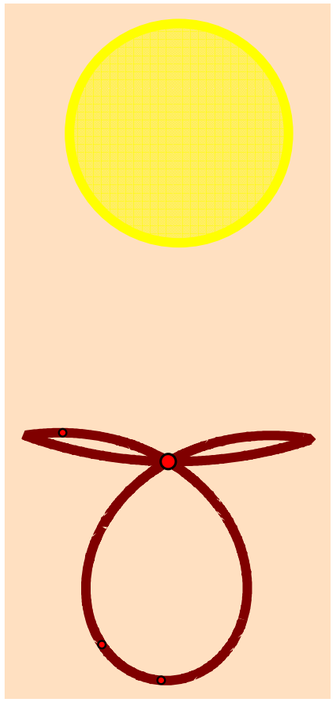
From the mid-point of DE, construct the perpendicular FG. At G through D and E, draw the third circle and denote its intersection with AB as H, and construct the Loci of H relative to B.



The end result can be viewed in Harvey's dressing room, to view Harvey and his magic ears. Keep in mind that one will have to redraw the intersection lines for the quadrant that A resides in.



So, if you make a mistake in the drawing, well, you just may end up with another brat fostered by Harvey, watching the sun.

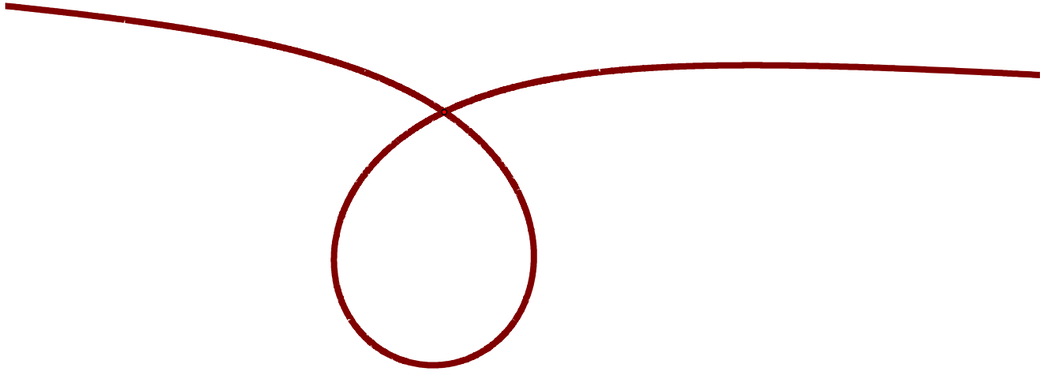


Harvey, The Full Service Rabbit

Sunday, June 18, 2023

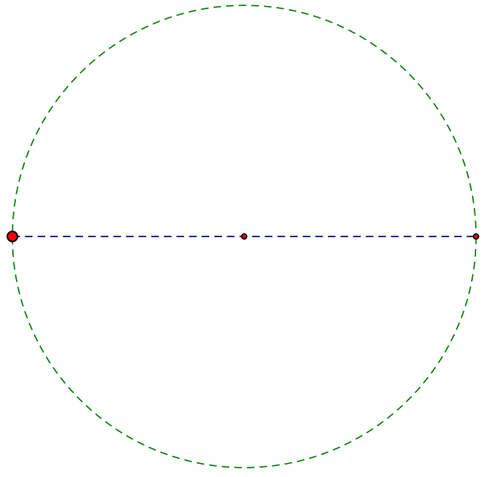
Back in ’06, I found Harvey. You might be familiar with Harvey, he made a feature film once with James Steward, by the same name. After Harvey retired, no one has seen him, or rather, since he was invisible, no one has ever seen him ’cept those who see pooka’s, ’cause Harvey is a pooka. Well, I ran into Harvey, as I said, and he tells me that he can trisect any angle using his head or his ears, whichever was required. I said to him, so what is that to me? If I cannot do it, then it is of no interest to me. I can teach you, said he, and so I learnt.

The following is how to draw Harvey’s head, and I got this straight from the rabbit himself.



That is exactly what the shape of Harvey's head is, and this is how to draw it, because, it can do what no human has been able to do, so we have to leave it up to the magic of a pooka.

One starts drawing Harvey's head with a perfect circle, like so.



So, watch the two construction vids, and be warned, a point in any number of places can make a loci, and the figure can make many, but for now, we need only one method for this tool.

Trisection

Descriptions.

The most common sense method of finding the loci for angular division is very simple, B will always be proportional to A. So, let us write it up in a slightly different form.

$AD := 1 \quad AB := \frac{AD}{2} \quad AC := .5833333$

$AM := 2 \cdot AD \quad CD := AD - AC \quad CJ := \sqrt{AC \cdot CD}$

$AJ := \sqrt{AC^2 + CJ^2} \quad JF := AJ \quad JK := AB \quad JG := \frac{AB^2}{JF}$

$AG := AJ - JG \quad AH := \frac{AC \cdot AG}{AJ} \quad AH = 0.333333 \quad GH := \frac{CJ \cdot AG}{AJ} \quad GH = 0.281718$

Definitions.

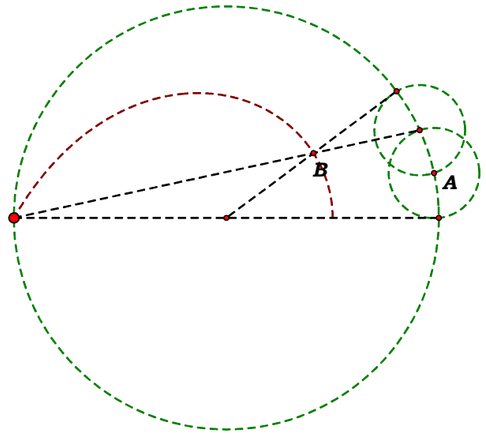
$AD - 1 = 0 \quad AB - \frac{1}{2} = 0 \quad AC - AC = 0 \quad AM - 2 = 0 \quad CD - (1 - AC) = 0$

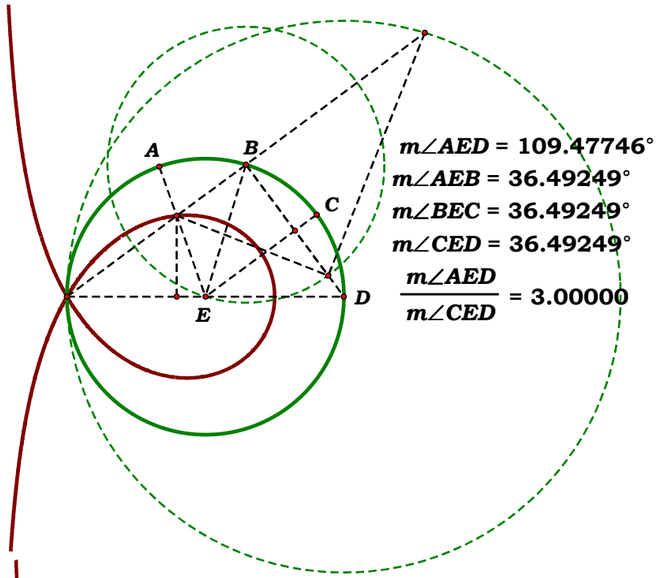
$CJ - \sqrt{(AC - AC^2)} = 0 \quad AJ - \sqrt{AC} = 0 \quad JF - AJ = 0 \quad JK - AB = 0$

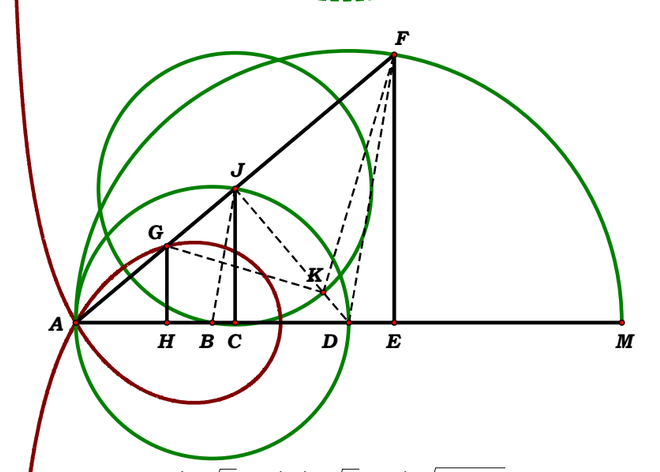
$JG - \frac{1}{4 \cdot \sqrt{AC}} = 0 \quad AG - \frac{(2 \cdot \sqrt{AC} - 1) \cdot (2 \cdot \sqrt{AC} + 1)}{4 \cdot \sqrt{AC}} = 0$

$GH - \frac{(2 \cdot \sqrt{AC} - 1) \cdot (2 \cdot \sqrt{AC} + 1) \cdot \sqrt{AC - AC^2}}{4 \cdot AC} = 0 \quad AH - \frac{(2 \cdot \sqrt{AC} - 1) \cdot (2 \cdot \sqrt{AC} + 1)}{4} = 0$

Note from Euclid to Trigonometry: I'sm sorry, did I spoil your moment?





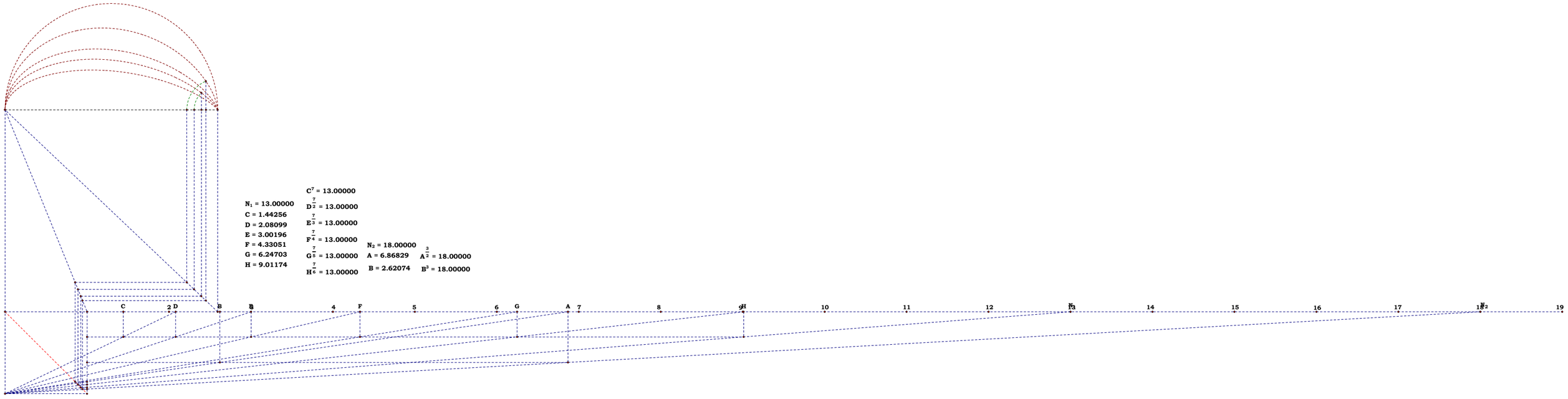


A = 0.00000
H = 0.33333
B = 0.50000
C = 0.58333
D = 1.00000
E = 1.16667
N = 2.00000
G = 0.28172
J = 0.49301
F = 0.98601

$G - \frac{(2 \cdot \sqrt{C} - 1) \cdot (2 \cdot \sqrt{C} + 1) \cdot \sqrt{C - C^2}}{4 \cdot C} = 0.00000$

$H - \frac{(2 \cdot \sqrt{C} - 1) \cdot (2 \cdot \sqrt{C} + 1)}{4} = 0.00000$

Whenever one has a fraction, say 7/1, 7/2 etc, instead of thinking something stupid, such as seven divided by one, say of a set of seven, this is the fist, and of a set of seven, this is the second.



Mansion of the Mind

Saturday, June 10, 2023

Our mind is a container of our experience, that experience, is fleeting and is quickly invisible to all. However, I do construct sign-posts, memories of it through grammar. When the inner sight catches the relationships between things, it becomes quite a lot of furniture in our mind, and there is so much of it for some of us, we never have time to examine and sort it all out. But it cannot be passed on, unless in its last recorded form.

When it is said, that the Temple of God is within, it is also written, that it can be proven by two, or more, reliable witnesses. The only reliable witness I know of is the Rectified Name and that is a biological and provable, fact. It is also a provable fact, that I have lived my life among the poor, but the poor do not beg, they steal your life, because for them, they have no other choice. Eventually, you have nothing left to defend yourself with, which means… what it means. It is a hard thing, when you freely share your supper with the world, and they then rob you of your life. There can only be justice in the world, when that very same world knows judgment. So, the only forgiveness one can ever understand, is when the evil has learnt to be good. And the only justification for what life one has had, is what they have done to make it so.

The Elements

Wednesday, July 5, 2023

We learn by experience, and learning means that we make something intelligible of our experiences. Most people do not even know the difference between the perceptible and intelligible. The intelligible is the ability to do our own work as a mind, i.e., to parse information in accordance with grammatical operations performed with a binary. Euclid's *Elements* is a compilation of interesting particulars about geometric figures, and it starts by constructing a particular figure, an equilateral triangle. It is, however filled with examples of using the only two methods of binary recursion possible, induction which is arithmetic, and deduction which is geometric.

Therefore, the intelligible method of developing geometry is after one has acquired enough information to do so, and organizing it in accordance with the intelligible, when one has acquired the intelligence to do so. Euclid did his part, providing a record of data, now we should do ours.

Given information, all we can do with it is perform operations on it; arithmetic and geometric. Therefore, the finished product of learning the elements will start with a unit, and effect recursion on it arithmetically, and then geometrically. One will find that in arithmetic recursion, we start with the one, and we have called it counting, i.e., assigning arithmetic names to the result, which, when we finally stop counting we end up with a final proportion to our starting unit. We should call counting, multiplication of our unit. One is being added to one, but a specific number of times when we use it. This is also called induction. We are actually adding equal verbs, or relative differences, to equal verbs a given multiple number of times.

In Geometric Recursion, we never change the verb, but apply the correlative or noun, and it is called division. We end up with a series of fractions. Therefore, in geometric recursion, we do not find the unit until after all the operations have been performed. This induction brings in relative differences, and starts with the unit; in deduction we use the relative difference given, and divide it and find the unit after all the operations have been perform with it. The unit comes last. Arithmetic starts with a unit, and goes outward without end, while Geometry starts with a unit thing, and divides it to inward without end. The ordered presentation of the Elements start with what we should have learnt in the way of grammatical operations upon anything.

Back in 1989, I called my work, One Circle, One Square, One line, which factually ended up describing the final figure for doing all the simple and complex grammatical operations and expressed in the grammar of geometry.

Our job, as a mind, is recursion of information in accordance with the expressed function of a life support system starting with a standard of behavior called a unit. This recursion starts with a given unit, and we show the two methods by which to effect recursion with it. From there we learn how to develop

a figure which does all the operations that can be done on everything by pairing both types of recursion into a single figure.

This presentation of the *Elements* is then the second possible method of developing functional geometry, and the results covers the entire range of judgment, of grammatical operations.

Euclid's presentation is a compilation, as an exploratory and quite frankly most of the propositions are presented in the forensic mode, which is as it should be.

There is little or nothing forensic about the finished product of Euclidean Geometry, Euclid taught us how to write the math to a figure and all we have to do is apply it using our single figure. What Euclid taught us, is how to find a single figure for doing all the operations, and for even proofing attachments to it to extend it in particular ways. Therefore, there is no new Euclidean proofs here, one can find those, if they need them, in Euclid's presentation, which is prior to this work. Therefore, if you think you are sufficiently tired and bored with Euclid's presentation, then you should be able to proof everything required of our starting figure for doing all the operations on information with.

If you believe that you can skip Euclid, then you do not know what you are about at all.

Preserved in Euclid is the data, now how do we reformat Euclid's Elements so as to better prepare a mind to do its own work? A mind must learn to ask questions and learn how to lead one's self to answer them.

We are all smart in some things and dumb in others, that is why I question that I can do this part alone. What makes sense to me, are things which most find very hard to make sense of. A mind which has not learnt to ask and answer questions is not functional.

I can start it, but probably not finish it. Keep moving, just keep moving.

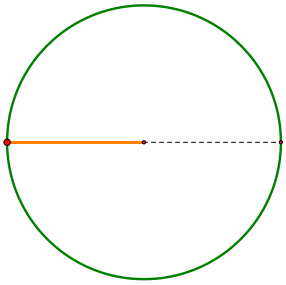
However, there is a need to take some effort into putting Euclid's presentation aright by way of the use of names. One of the reason's this has to be done is that a single naming convention is not used in what we have today of that work, and that is probably because what we have today is not only part or most of Euclid's original work and only repeats demonstration by using other terms, but a work which has been edited by various hands throughout history. So, a rectification of names is required to prepare an acceptable copy of Euclid's Elements which will actually help people understand that a change in a name does not always mean a change in topic.

Therefore, I have done what Plato recommended, what our own biologically defined job demands; I have taken all the particular examples of the Elements of Euclid, and reduced them to a single figure for showing the results of any figure.

Introduction to Book 1

In geometry, we denote a unit, a thing, as such:

And any possible position of that thing, in a binary relationship as such:

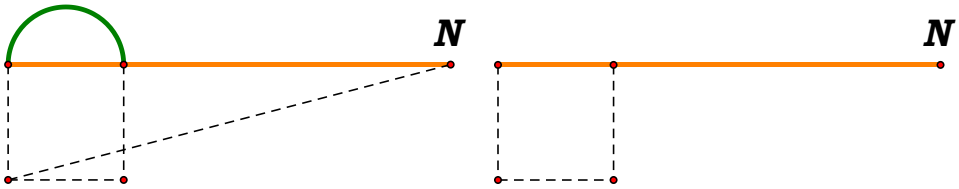


And we have four physical tools by which to record them in our grammar called pen, paper, straightedge, and compass, however, if we are working in a virtual environment, then these are also virtualized, so when I say pen, paper, straightedge and compass, I mean that if you only have these in physical form, you use those, but if you have them in virtual form, you use those.

The straightedge and compass represent two distinct methods of recursion, arithmetic, and geometric, the one, and the universe of discourse for the one and the ability to effect total recall with them. You might know them as whole numbers and fractions. The circle is the product of that fraction called a reciprocal.

Although every system of grammar is effected by the recursion of the unit, of one, this means that constructing a circle using a unit, is actually easy, but it is also cumbersome, so we take it as a given, in short, it is a given that a unit involves a relative which can be both one and the many. The circle is the reciprocal of the unit. We have two tools for convenience as the compass actually represents also, a product of physical memory, to be recursively used as a tool, therefore in regard to our givens in geometry we are given, paper, pen, straightedge, and our first construct using total recall produced by the simple reciprocal, or geometric unit. Again, the unit and its reciprocal, and the ability to record our results for Total Recall. If you are so simple minded as to not comprehend these simple and obvious facts, you might eventually learn them.

Now, we can combine these in a special way to make a Grammar Matrix Processor; a cooking pot as if you will.



A great many operations do not require the lid on our pot, but for more intricate meals, it is required, so we more often than not leave it as our first

attachment to our pot. We can, however, if we want, demonstrate its construction as a reciprocal before showing it as an attachment. We are going to learn how to construct our processing pot, and use it, to perform all of our grammatical operations with, multiplication, division, addition, subtraction, both arithmetic and geometric. And we are going to learn that we can increase its ability by attachments which will be introduced as we go.

Euclid has taught the world how to proof a figure, now one has modern math programs to proof any figure with using simple algebra.

Exercises 1 to 5

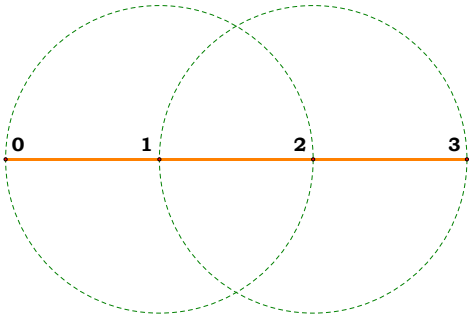
Wednesday, July 5, 2023

Book 1 Exercise 1

With your pen, paper, and straightedge, construct the universal binary one.

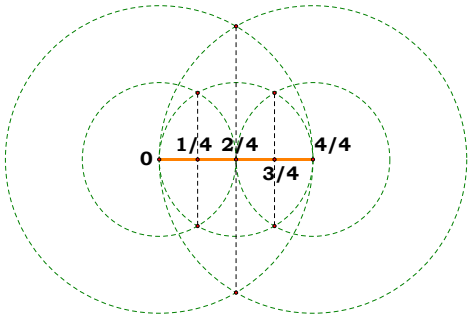
Book 1 Exercise 2

With your pen, paper, straightedge, and compass, multiply the one by three.



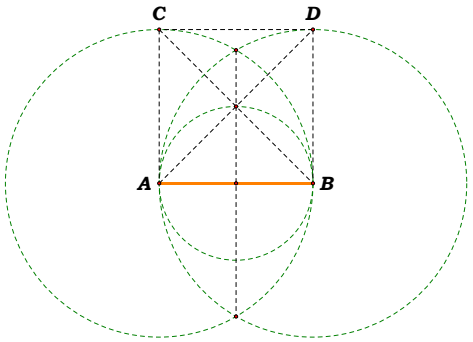
Book 1 Exercise 3

With your pen, paper, straightedge, and compass, divide the one into four.



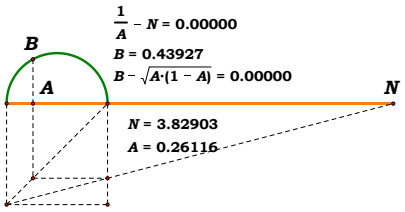
Book 1 Exercise 4

With your pen, paper, straight, compass, and unit, describe what is commonly called a square.



Book 1 Exercise 5

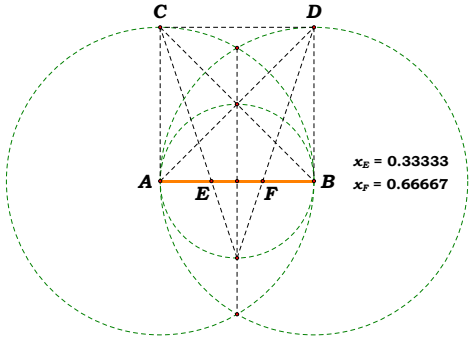
Now, adding a handle, we have our pot by which to cook with. Finish the construction of our pot, and use it to show the reciprocal and how it constructs the circle from which we derive the compass.



We have learnt how to multiply and divide, and to produce a working pot along with proving that our compass provides us with the reciprocal of the unit. We can test our construction, learning to validate simple mathematics, but setting A to whole numbers, and seeing the result of the resulting reciprocal. Take some time and play with it and learn.

Those propositions of Euclid's which teach the simple things about lines and circles, we will let him familiarize us with those.

Wait, oh dear. Someone in the back is yelping about dividing our unit into three before we add the handle,



Perhaps it is best, for this is also an introduction to cube roots. We can, however, divide it into anything we wish using the handle.

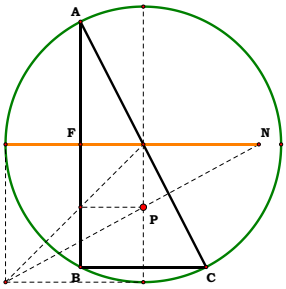
If we pay attention to the Elements of Euclid, it is composed of two parts. The simple relationships between figures, and learning the math before we have found the right way to do so, in fact, the majority of the Elements factually have no real graphics by which to follow the demonstrations.

Exercises 6 to 7

Thursday, July 6, 2023

Exercise 6

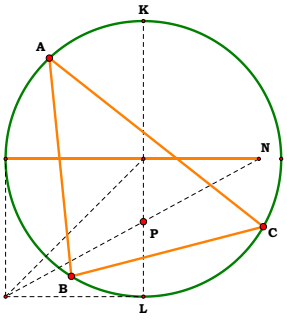
Pair the Circle that is divided into equal units, angles, to Arithmetic.



$N = 1.83836$
$F = 0.54396$
$\frac{1}{F} - N = 0.00000$
$A = 0.88996$
$C = 1.45604$
$AB = 1.77992$
$BC = 0.91208$
$AC = 2.00000$
$m\angle ABC = 90.00000°$
$m\angle BAC = 27.13174°$
$m\angle BCA = 62.86826°$
$\sin(m\angle BCA) - \frac{AB}{AC} = 0.00000$
$\sin(m\angle BAC) - \frac{BC}{AC} = 0.00000$
$\sin(m\angle ABC) - \frac{AC}{AC} = 0.00000$

In the Elements of Euclid, an angle is defined as the inclination and meeting of two lines, i.e., a proportion. Study how to proportion the angle to arithmetic by the following figure.

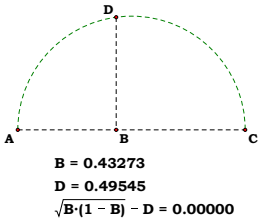
$KL = 2.87073$ in.
$AB = 2.28472$ in.
$BC = 2.06527$ in.
$AC = 2.83737$ in.
$\frac{AB}{KL} = 0.79586$
$\frac{BC}{KL} = 0.71942$
$\frac{AC}{KL} = 0.98838$
$m\angle ABC = 81.25629°$
$m\angle BCA = 52.73701°$
$m\angle CAB = 46.00670°$
$\sin(m\angle ABC) - \frac{AC}{KL} = 0.00000$
$\sin(m\angle BCA) - \frac{AB}{KL} = 0.00000$
$\sin(m\angle CAB) - \frac{BC}{KL} = 0.00000$
$\sin^{-1}\left(\frac{AB}{KL}\right) - m\angle BCA = 0.00000°$
$\sin^{-1}\left(\frac{BC}{KL}\right) - m\angle CAB = 0.00000°$
$\sin^{-1}\left(\frac{AC}{KL}\right) - m\angle ABC = 0.00000°$



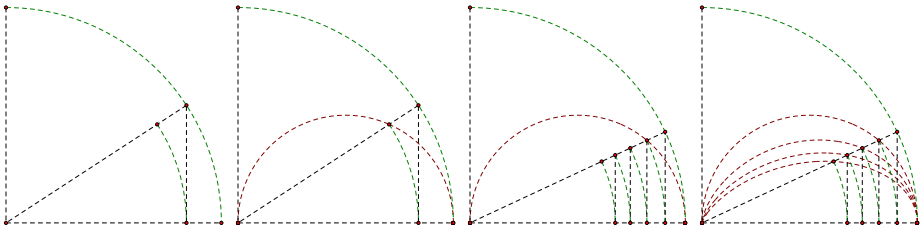
If one has studied Trigonometry, the figure proves that almost all of it is mythology. One can use just one function, and not a basket full of them, the sine function for any triangle whatsoever and there is absolutely no need for any of them to be a right-angled triangle.

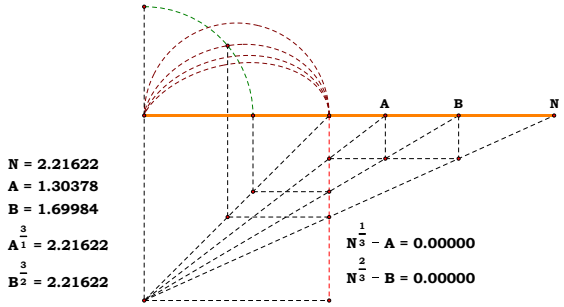
With the arcsine, or if you prefer, inverse sine, you can use the ratio of a segment to find any angle of any triangle given only its sides. Or one can put each segment back into the standard figure and find out the number from which it derived as a reciprocal.

Lesson, the reciprocal.



You should consider this figure from Euclid's Elements. What is it telling you? That a circle is the product of the root of two values. This means that the same figure can be used to find the other value if one is given. In short, the circle is the reciprocal of multiplication and division. So, we are going to produce not only the circle for the square and the root, but of any exponent of a value multiplied by itself, so that we can do complete exponential results. This called a continued proportion of a line multiplied by itself. Oddly enough, there have been recent so-called math author's claiming this does not exist in geometry while also claiming that the notation is as a rabbit pulled out of a hat.





We are not doing anything different from constructing the first of a series, we are only recursively applying the same process. And, the figure can be constructed from either direction, but the direction you construct it from is the direction you have to use it for, just like in the original figure found in Euclid. The sequence is square roots and powers, cube roots and powers, etc. One can construct any exponential series with a given unit. The figure given in Euclid, actually tells us that a circle is the product of a reciprocal, and thus, exponential series is the continuance of constructing the reciprocal. One can then either call it the circle, or the power of two, then the power of three, four and I stopped at five. So that exponential notation is with the starting unit on top if it is the reciprocal, and on the bottom if it is the number, the sequence of that recursion on the remaining place holder. Of three, this is the first, or second, etc. This is the same way one expresses ordinary fractions, just in a different place.

Lesson 7.

Construct and play with the figure.

We have seen, that as the triangle, every segment is expressible as the product of a reciprocal, so too, all of exponential manipulation. These relationships are simply part of reality, and we only discover them, they are not the product of magic, but of binary recursion.

The Exercise

Friday, July 7, 2023

The Exercise is the reformatting of Euclid's Elements. It is undeniably the data, not in the best format, not presented in the best way, etc., but undeniably required. It needs to be repaired and the probability of me being able to do it alone is very slim, but if that is the way it has to be, then that is the way.

First of all, all data we learn by is examined forensically. We have to not only learn how to ask and answer questions, but also about something which leads to specific results, a mind as functional as it can be.

How do we pass on to another binary information processing such that the Art eventually flourishes?

How do you pass on the revelation that the mind, doing its own work, is the most important thing there ever can be? I don't know.

The Exercises

Monday, July 3, 2023

Try to recall the various methods one may have been instructed on which have been aimed at the organization of information. Have any of them been aimed at the universal, or have they all been aimed at the particular?

What do I mean by this?

If all we can do is parse information, i.e., organize it, and as there are two different identities we are using, Arithmetic and Geometric, and our common notion's provide us with only four basic operations of grammar, then would you claim, that each proposition is dependent upon particular definitions, and proven by those figures which can before it, or that all information is processed in exactly the same way, by our available operations upon any thing?

In short, one may say that one can say such and such is true because you have learnt to do it at a particular time in the past, but that is false reasoning, i.e., the proofs in the Elements, are not rendered down to intelligible fact at all, and are all, very bad for teaching.

Book 1. Propositions.

Book 1 Proposition 1.

On a given finite straight line to construct an equilateral triangle.

Book 1 Proposition 2.

To place, at a given point (*as an extremity*), *a straight line equal, to a given straight line.*

Book 1 Proposition 3.

Given two unequal straight lines, to cut off from the greater, a straight line equal, to the less.

Book 1 Proposition 4.

If two triangles have the two sides equal, to two sides respectively, and have the angles contained by the equal straight lines equal, they will, also, have the base equal, to the base, the triangle will be equal, to the triangle, and the remaining angles will be equal, to the remaining angles respectively, namely those which the equal sides subtend.

Book 1 Proposition 5.

In isosceles triangles the angles at the base are equal, to one another, and, if the equal straight lines be produced further, the angles under the base will be equal, to one another.

Book 1 Proposition 6.

If in a triangle two angles be equal, to one another, the sides which subtend the equal angles will, also, *be equal, to one another.*

Book 1 Proposition 7.

Given two straight lines constructed on a straight line (*from its extremities*) *and meeting in a point, there cannot be constructed on the same straight line* (*from its extremities*), *and on the same side of it, two other straight lines meeting in another point and equal, to the former two respectively, namely each to that which has the same extremity with it.*

Book 1 Proposition 8.

If two triangles have the two sides equal, to two sides respectively, and have, also, *the base equal, to the base, they will,* also, *have the angles equal which are contained by the equal straight lines.*

Book 1 Proposition 9.

To bisect a given rectilineal angle.

Book 1 Proposition 10.

To bisect a given finite straight line.

Book 1 Proposition 11.

To draw a straight line at right angles to a given straight line from a given point on it.

Book 1 Proposition 12.

To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.

Book 1 Proposition 13.

If a straight line set up on a straight line make angles, it will make either two right angles or angles equal, to two right angles.

Book 1 Proposition 14.

If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal, to two right angles, the two straight lines will be in a straight line with one another.

Book 1 Proposition 15.

If two straight lines cut one another, they make the vertical angles equal, to one another.

Book 1 Proposition 16.

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.

Book 1 Proposition 17.

In any triangle, two angles taken together in any manner are less than two right angles.

Book 1 Proposition 18.

In any triangle the greater side subtends the greater angle.

Book 1 Proposition 19.

In any triangle the greater angle is subtended by the greater side.

Book 1 Proposition 20.

In any triangle two sides taken together in any manner are greater than the remaining one.

Book 1 Proposition 21.

If on one of the sides of a triangle, from its extremities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater angle.

Book 1 Proposition 22.

Out of three straight lines, which are equal, to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one. [Book 1 Book 1 Proposition 20]

Book 1 Proposition 23.

On a given *straight line and at a point on it to construct a rectilineal angle equal, to* a given *rectilineal angle.*

Book 1 Proposition 24.

If two triangles have the two sides equal, to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will, also, *have the base greater than the base.*

Book 1 Proposition 25.

If two triangles have the two sides equal, to two sides respectively, but have the base greater than the base, they will, also, *have the one of the angles contained by the equal straight lines greater than the other.*

Book 1 Proposition 26.

If two triangles have the two angles equal, to two angles respectively, and one side equal, to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will, also, *have the remaining sides equal, to the remaining sides and the remaining angle to the remaining angle.*

Book 1 Proposition 27.

If a straight line falling on two straight lines make the alternate angles equal, to one another, the straight lines will be parallel to one another.

Book 1 Proposition 28.

If a straight line falling on two straight lines make the exterior angle equal, to the interior and opposite angle on the same side, or the interior angles on the same side equal, to two right angles, the straight lines will be parallel to one another.

Book 1 Proposition 29.

A straight line falling on parallel straight lines makes the alternate angles equal, to one another, the exterior angle equal, to the interior and opposite angle, and the interior angles on the same side equal, to two right angles.

Book 1 Proposition 30.

Straight lines parallel to the same straight line are, also, *parallel to one another.*

Book 1 Proposition 31.

Through a given *point to draw a straight line parallel to* a given *straight line.*

Book 1 Proposition 32.

*In any triangle, if one of the sides be produced, the exterior angle is equal, to the two interior and opposite angles, and the three interior angles of the triangle are equal, to two right angle*s.

Book 1 Proposition 33.

The straight lines joining equal and parallel straight lines (*at the extremities which are*) *in the same directions* (*respectively*) *are themselves*, also, *equal and parallel.*

Book 1 Proposition 34.

In parallelogrammic areas the opposite sides and angles are equal, to one another, and the diameter bisects the areas.

Book 1 Proposition 35.

Parallelograms which are on the same base and in the same parallels are equal, to one another.

Book 1 Proposition 36.

Parallelograms which are on equal bases and in the same parallels are equal, to one another.

Book 1 Proposition 37.

Triangles which are on the same base and in the same parallels are equal, to one another.

Book 1 Proposition 38.

T*riangles which are on equal bases and in the same parallels are equal to one another.*

Book 1 Proposition 39.

Equal triangles which are on the same base and on the same side are, also, *in the same parallels.*

Book 1 Proposition 40.

Equal triangles which are on equal bases and on the same side are, also, *in the same parallels.*

Book 1 Proposition 41.

If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle.

Book 1 Proposition 42.

To construct in a given rectilineal angle a parallelogram equal, to a given triangle.

Book 1 Proposition 43.

In any parallelogram the complements of the parallelograms about the diameter are equal, to one another.

Book 1 Proposition 44.

To a given straight line to apply, in a given rectilineal angle, a parallelogram equal, to a given triangle.

Book 1 Proposition 45.

To construct, in a given rectilineal angle, a parallelogram equal, to a given rectilineal figure.

Book 1 Proposition 46.

On a given straight line to describe a square.

Book 1 Proposition 47.

In right-angled triangles the square, on the side subtending the right angle is equal, to the squares on the sides containing the right angle.

Book 1 Proposition 48.

If in a triangle the square, on one of the sides be equal, to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.

Book 2. Propositions.

Book 2 Proposition 1.

If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal, to the rectangles contained by the uncut straight line and each, of the segments.

Book 2 Proposition 2.

If a straight line be cut at random, the rectangle contained by the whole and both of the segments is equal, to the square, on the whole.

Book 2 Proposition 3.

If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal, to the rectangle contained by the segments and the square, on the aforesaid segment.

Book 2 Proposition 4.

If a straight line be cut at random, the square, on the whole is equal, to the squares on the segments and twice the rectangle contained by the segments.

Book 2 Proposition 5.

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square, on the straight line between the points of section is equal, to the square, on the half.

Book 2 Proposition 6.

If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square, on the half is equal, to the square, on the straight line made up of the half and the added straight line.

Book 2 Proposition 7.

If a straight line be cut at random, the square, on the whole and that on one of the segments both together are equal, to twice the rectangle contained by the whole and the said segment and the square, on the remaining segment.

Book 2 Proposition 8.

If a straight line be cut at random, four times the rectangle contained by the whole and one of the segments together with the square, on the remaining segment is equal, to the square described on the whole and the aforesaid segment as on one straight line.

Book 2 Proposition 9.

If a straight line be cut into equal and unequal segments, the squares on the unequal segments of the whole are double of the square, on the half and of the square, on the straight line between the points of section.

Book 2 Proposition 10.

If a straight line be bisected, and a straight line be added to it in a straight line, the square, on the whole with the added straight line and the square, on the added straight line both together are double of the square, on the half and of the square described on the straight line made up of the half and the added straight line as on one straight line.

Book 2 Proposition 11.

To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal, to the square, on the remaining segment.

Book 2 Proposition 12.

In obtuse-angled triangles, the square, on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.

Book 2 Proposition 13.

In acute-angled triangles the square, on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the straight line cut off within by the perpendicular towards the acute angle.

Book 2 Proposition 14.

To construct a square equal, to a given *rectilineal figure.*

Book 3. Propositions.

Book 3 Proposition 1.

To find the center of a given circle.

Book 3 Proposition 2.

If on the circumference of a circle two points be taken at random, the straight line joining the points will fall within the circle.

Book 3 Proposition 3.

If in a circle, a straight line through the center bisect a straight line not through the center, it, also, *cuts it at right angles; and if it cut it at right angles, it,* also, *bisects it.*

Book 3 Proposition 4.

If in a circle two straight lines cut one another which are not through the center, they do not bisect one another.

Book 3 Proposition 5.

If two circles cut one another, they will not have the same center.

Book 3 Proposition 6.

If two circles touch one another, they will not have the same center.

Book 3 Proposition 7.

If, on the diameter of a circle, a point be taken which is not the center of the circle, and from the point straight lines fall upon the circle, that will be greatest on which the center is, the remainder of the same diameter will be least, and of the rest the nearer to the straight line through the center is always greater than the more remote, and only two equal straight lines will fall from the point on the circle, one on each side of the least straight line.

Book 3 Proposition 8.

If a point be taken outside a circle and from the point straight lines be drawn through to the circle, one of which is through the center and the others are drawn at random, then, of the straight lines which fall on the concave circumference, that through the center is greatest, while of the rest the nearer to that through the center is always greater than the more remote, but, of the straight lines falling on the convex circumference, that between the point and the diameter is least, while of the rest the nearer to the least is always less than the more remote, and only

two equal straight lines will fall on the circle from the point, one on each side of the least.

Book 3 Proposition 9.

If a point be taken within a circle, and more than two equal straight lines fall from the point on the circle, the point taken is the center of the circle.

Book 3 Proposition 10.

A circle does not cut a circle at more points than two.

Book 3 Proposition 11.

If two circles touch one another internally, and their centers be taken, the straight line joining their centers, if it be, also, *produced, will fall on the point of contact of the circles.*

[Book 3 Proposition 12.

If two circles touch one another externally, the straight line joining their centers will pass through the point of contact.

Book 3 Proposition 13.

A circle does not touch a circle at more points than one, whether it touch it internally or externally.

Book 3 Proposition 14.

In a circle, equal straight lines are equally distant from the center, and those which are equally distant from the center are equal, to one another.

Book 3 Proposition 15.

Of straight lines in a circle the diameter is greatest, and of the rest the nearer to the center is always greater than the more remote.

Book 3 Proposition 16.

The straight line drawn at right angles to the diameter of a circle *from its extremity will fall outside the circle, and into the space between the straight line and the circumference another straight line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute rectilineal angle.*

Book 3 Proposition 17.

From a given point to draw a straight line touching a given circle.

Book 3 Proposition 18.

If a straight line touch a circle, and a straight line be joined from the center to the point of contact, the straight line so joined will be perpendicular to the tangent.

Book 3 Proposition 19.

If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the tangent, the center of the circle will be on the straight line so drawn.

Book 3 Proposition 20.

In a circle the angle at the center is double of the angle at the circumference, when the angles have the same circumference as base.

Book 3 Proposition 21.

In a circle, the angles in the same segment are equal, to one another.

Book 3 Proposition 22.

The opposite angles of quadrilaterals in circles are equal, to two right angles.

Book 3 Proposition 23.

On the same straight line there cannot be constructed two similar and unequal segments of circles on the same side.

Book 3 Proposition 24.

Similar segments of circles on equal straight lines are equal, to one another.

Book 3 Proposition 25.

Given a segment of a circle, to describe the complete circle of which it is a segment.

Book 3 Proposition 26.

In equal circles equal angles stand on equal circumferences, whether they stand at the centers or at the circumferences.

Book 3 Proposition 27.

In equal circles, angles standing on equal circumferences are equal, to one another, whether they stand at the centers or at the circumferences.

Book 3 Proposition 28.

In equal circles equal straight lines cut off equal circumferences, the greater equal, to the greater and the less to the less.

Book 3 Proposition 29.

In equal circles equal circumferences are subtended by equal straight lines.

Book 3 Proposition 30.

To bisect a given circumference.

Book 3 Proposition 31.

In a circle, the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle; and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.

Book 3 Proposition 32.

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal, to the angles in the alternate segments of the circle.

Book 3 Proposition 33.

On a given straight line to describe a segment of a circle admitting an angle equal, to a given rectilineal angle.

Book 3 Proposition 34.

From a given circle to cut off a segment admitting an angle equal, to a given rectilineal angle.

Book 3 Proposition 35.

If in a circle two straight lines cut one another, the rectangle contained by the segments of the one is equal, to the rectangle contained by the segments of the other.

Book 3 Proposition 36.

If a point be taken outside a circle and from it there fall on the circle two straight lines, and if one of them cut the circle and the other touch it, the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference will be equal, to the square, on the tangent.

Book 3 Proposition 37.

If a point be taken outside a circle and from the point there fall on the circle two straight lines, if one of them cut the circle, and the other fall on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference be equal, to the square, on the straight line which falls on the circle, the straight line which falls on it will touch the circle.

Book 4. Propositions

Book 4 Proposition 1.

Into a given circle to fit a straight line equal, to a given straight line which is not greater than the diameter of the circle.

Book 4 Proposition 2.

In a given circle to inscribe a triangle equiangular with a given triangle.

Book 4 Proposition 3.

About a given circle to circumscribe a triangle equiangular with a given triangle.

Book 4 Proposition 4.

In a given triangle to inscribe a circle.

Book 4 Proposition 5.

About a given triangle to circumscribe a circle.

Book 4 Proposition 6.

In a given circle to inscribe a square.

Book 4 Proposition 7.

About a given circle to circumscribe a square.

Book 4 Proposition 8.

In a given square to inscribe a circle.

Book 4 Proposition 9.

About a given square to circumscribe a circle.

Book 4 Proposition 10.

To construct an isosceles triangle having each, of the angles at the base double of the remaining one.

Book 4 Proposition 11.

In a given circle to inscribe an equilateral and equiangular pentagon.

Book 4 Proposition 12.

About a given circle to circumscribe an equilateral and equiangular pentagon.

Book 4 Proposition 13.

In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

Book 4 Proposition 14.

About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.

Book 4 Proposition 15.

In a given circle to inscribe an equilateral and equiangular hexagon.

Book 4 Proposition 16.

In a given circle, to inscribe a fifteen-angled figure which shall be both equilateral and equiangular.

Book 5. Propositions.

Book 5 Proposition 1.

If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one of the magnitudes is of one, that multiple, also, *will all be of all.*

Book 5 Proposition 2.

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth, also, *be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will*, also, *be the same multiple of the second that the sum of the third and sixth is of the fourth.*

Book 5 Proposition 3.

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then, also, equally *the magnitudes taken will be equimultiples respectively, the one of the second and the other of the fourth.*

Book 5 Proposition 4.

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will, also, *have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.*

Book 5 Proposition 5.

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will, also, *be the same multiple of the remainder that the whole is of the whole.*

Book 5 Proposition 6.

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders, also, *are either equal, to the same or equimultiples of them.*

Book 5 Proposition 7.

Equal magnitudes have to the same the same ratio, as, also, *has the same to equal magnitudes.*

Book 5 Proposition 8.

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater.

Book 5 Proposition 9.

Magnitudes which have the same ratio to the same are equal, to one another; and magnitudes to which the same has the same ratio are equal.

Book 5 Proposition 10.

Of magnitudes which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

Book 5 Proposition 11.

Ratios which are the same with the same ratio are, also, *the same with one another.*

Book 5 Proposition 12.

If any number of magnitudes be proportional as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents.

Book 5 Proposition 13.

If a first magnitude have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will, also, *have to the second a greater ratio than the fifth to the sixth.*

Book 5 Proposition 14.

If a first magnitude have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will, also, *be greater than the fourth; if equal, equal; and if less, less.*

Book 5 Proposition 15.

Parts have the same ratio as the same multiples of them taken in corresponding order.

Book 5 Proposition 16.

If four magnitudes be proportional, they will, also, *be proportional alternately.*

Book 5 Proposition 17.

If magnitudes be proportional taken jointly, *they will*, also, *be proportional* taken separately.

Book 5 Proposition 18.

If magnitudes be proportional taken separately, *they will*, also, *be proportional* taken jointly.

Book 5 Proposition 19.

If, as a whole is to a whole, so is a part subtracted to a part subtracted, the remainder will, also, *be to the remainder as whole to whole.*

Book 5 Proposition 20.

If there be three magnitudes, and others equal, to them in multitude, which taken two and two are in the same ratio, and if equally *the first be greater than the third, the fourth will*, also, *be greater than the sixth*; *if equal, equal*; *and, if less, less.*

Book 5 Proposition 21.

If there be three magnitudes, and others equal, to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, then, if equally *the first magnitude is greater than the third, the fourth will*, also, *be greater than the sixth*; *if equal, equal*; *and if less, less.*

Book 5 Proposition 22.

If there be any number of magnitudes whatever, and others equal, to them in multitude, which taken two and two together are in the same ratio, they will, also, *be in the same ratio* equally.

Book 5 Proposition 23.

If there be three magnitudes, and others equal, to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will, also, *be in the same ratio* equally.

Book 5 Proposition 24:

If a first magnitude have to a second the same ratio as a third has to a fourth, and, also, *a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.*

Book 5 Proposition 25:

If four magnitudes be proportional, the greatest and the least are greater than the remaining two.

Book 6. Propositions.

Book 6 Proposition 1.

Triangles and parallelograms which are under the same height are to one another as their bases.

Book 6 Proposition 2.

If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally; and, if the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.

Book 6 Proposition 3.

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.

Book 6 Proposition 4.

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

Book 6 Proposition 5.

If two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Book 6 Proposition 6.

If two triangles have one angle equal, to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Book 6 Proposition 7.

If two triangles have one angle equal, to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.

Book 6 Proposition 8.

If, in a right-angled triangle, a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.

Book 6 Proposition 9.

From a given straight line to cut off a prescribed part.

Book 6 Proposition 10.

To cut a given uncut straight line similarly to a given cut straight line.

Book 6 Proposition 11.

To two given straight lines to find a third proportional.

Book 6 Proposition 12.

To three given straight lines to find a fourth proportional.

Book 6 Proposition 13.

To two given straight lines to find a mean proportional.

Book 6 Proposition 14.

In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Book 6 Proposition 15.

In equal triangles which have one angle equal, to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal, to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

Book 6 Proposition 16.

If four straight lines be proportional, the rectangle contained by the extremes is equal, to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal, to the rectangle contained by the means, the four straight lines will be proportional.

Book 6 Proposition 17.

If three straight lines be proportional, the rectangle contained by the extremes is equal, to the square, on the mean; and, if the rectangle contained by the extremes be equal, to the square, on the mean, the three straight lines will be proportional.

Book 6 Proposition 18.

On a given straight line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.

Book 6 Proposition 19.

Similar triangles are to one another in the duplicate ratio of the corresponding sides.

Book 6 Proposition 20.

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

Book 6 Proposition 21.

Figures which are similar to the same rectilineal figure are, also, *similar to one another.*

Book 6 Proposition 22.

If four straight lines be proportional, the rectilineal figures similar and similarly described upon them will, also, *be proportional*; *and, if the rectilineal figures similar and similarly described upon them be proportional, the straight lines will themselves*, also, *be proportional.*

Book 6 Proposition 23.

Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides.

Book 6 Proposition 24.

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

Book 6 Proposition 25.

To construct one and the same figure similar to a given rectilineal figure and equal, to another given rectilineal figure.

Book 6 Proposition 26.

If from a parallelogram there be taken away a parallelogram similar and similarly situated to the whole and having a common angle with it, it is about the same diameter with the whole.

Book 6 Proposition 27.

Of all the parallelograms applied to the same straight line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the defect.

Book 6 Proposition 28.

To a given straight line to apply a parallelogram equal, to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: thus the given rectilineal figure must not be greater than the parallelogram described on the half of the straight line and similar to the defect.

Book 6 Proposition 29.

To a given straight line to apply a parallelogram equal, to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one.

Book 6 Proposition 30.

To cut a given finite straight line in extreme and mean ratio.

Book 6 Proposition 31.

In right-angled triangles the figure on the side subtending the right angle is equal, to the similar and similarly described figures on the sides containing the right angle.

Book 6 Proposition 32.

If two triangles having two sides proportional to two sides be placed together at one angle so that their corresponding sides are, also, *parallel, the remaining sides of the triangles will be in a straight line.*

Book 6 Proposition 33.

In equal circles angles have the same ratio as the circumferences on which they stand, whether they stand at the centers or at the circumferences.

Book 7. Propositions.

Book 7 Proposition 1.

Two unequal numbers being set out, and the less being continually subtracted in turn from the greater, if the number which is left never measures the one before it until a unit is left, the original numbers will be prime to one another.

Book 7 Proposition 2.

Given two numbers not prime to one another, to find their greatest common measure.

Book 7 Proposition 3.

Given three numbers not prime to one another, to find their greatest common measure.

Book 7 Proposition 4.

Any number is either a part or parts of any number, the less of the greater.

Book 7 Proposition **5.**

If a number be a part of a number, and another be the same part of another, the sum will, also, *be the same part of the sum that the one is of the one.*

Book 7 Proposition 6.

If a number be parts of a number, and another be the same parts of another, the sum will, also, *be the same parts of the sum that the one is of the one.*

Book 7 Proposition 7.

If a number be that part of a number, which a number subtracted is of a number subtracted, the remainder will, also, *be the same part of the remainder that the whole is of the whole.*

Book 7 Proposition 8.

If a number be the same parts of a number that a number subtracted is of a number subtracted, the remainder will, also, *be the same parts of the remainder that the whole is of the whole.*

Book 7 Proposition 9.

If a number be a part of a number, and another be the same part of another, alternately also, whatever part or parts the first is of the third, the same part, or the same parts, will the second, also, *be of the fourth.*

Book 7 Proposition 10.

If a number be parts of a number, and another be the same parts of another, alternately also, whatever parts or part the first is of the third, the same parts or the same part will the second, also, *be of the fourth.*

Book 7 Proposition 11.

If, as whole is to whole, so is a number subtracted to a number subtracted, the remainder will, also, *be to the remainder as whole to whole.*

Book 7 Proposition 12.

If there be as many numbers as we please in proportion, then, as one of the antecedents is to one of the consequents, so are all the antecedents to all the consequents.

Book 7 Proposition 13.

If four numbers be proportional, they will, also, *be proportional alternately.*

Book 7 Proposition 14.

If there be as many numbers as we please, and others equal, to them in multitude, which taken two and two are in the same ratio, they will, also, *be in the same ratio* equally.

Book 7 Proposition 15.

If an unit measure any number, and another number measure any other number the same number of times, alternately also, the unit will measure the third number the same number of times that the second measures the fourth.

Book 7 Proposition 16.

If two numbers, by multiplying one another, make certain numbers, the numbers so produced will be equal, to one another.

Book 7 Proposition 17.

If a number, by multiplying two numbers, make certain numbers, the numbers so produced will have the same ratio as the numbers multiplied.

Book 7 Proposition 18.

If two numbers, by multiplying any number, make certain numbers, the numbers so produced will have the same ratio as the multipliers.

Book 7 Proposition 19.

If four numbers be proportional the number produced from the first and fourth will be equal, to the number produced from the second and third; *and*, *if the number produced from the first and fourth be equal, to that produced from the second and third*, *the four numbers will be proportional*.

Book 7 Proposition 20.

The least numbers of those which have the same ratio with them measure those which have the same ratio the same number of times, *the greater the greater and the less the less*.

Book 7 Proposition 21.

Numbers prime to one another are the least of those which have the same ratio with them.

Book 7 Proposition 22.

The least numbers of those which have the same ratio with them are prime to one another.

Book 7 Proposition 23.

If two numbers be prime to one another, *the number which measures the one of them will be prime to the remaining number*.

Book 7 Proposition 24.

If two numbers be prime to any number, *their product*, also, *will be prime to the same*.

Book 7 Proposition **25.**

If two numbers be prime to one another, *the product of one of them into itself will be prime to the remaining one*.

Book 7 Proposition 26.

If two numbers be prime to two numbers, *both to each*, *their products*, also, *will be prime to one another*.

Book 7 Proposition 27.

If two numbers be prime to one another, *and each*, *by multiplying itself*, *make a certain number*, *the products will be prime to one another*; *and*, *if the original numbers*, *by multiplying the products*, *make certain numbers*, *the latter will*, also, *be prime to one another* [*and this is always the case with the extremes*].

Book 7 Proposition 28.

If two numbers be prime to one another, the sum will, also, *be prime to each, of them; and, if the sum to two numbers be prime to any one of them, the original numbers will,* also, *be prime to one another.*

Book 7 Proposition 29.

Any prime number is prime to any number which it does not measure.

Book 7 Proposition 30.

If two numbers, by multiplying one another, make some number, and any prime number measure the product, it will, also, *measure one of the original numbers.*

Book 7 Proposition 31.

Any composite number is measured by some prime number.

Book 7 Proposition 32.

Any number either is prime or is measured by some prime number.

Book 7 Proposition 33.

Given as many numbers as we please, to find the least of those which have the same ratio with them.

Book 7 Proposition 34.

Given two numbers, to find the least number which they measure.

Book 7 Proposition 35.

If two numbers measure any number, the least number measured by them will, also, *measure the same.*

Book 7 Proposition 36.

Given three numbers, to find the least number which they measure.

Book 7 Proposition 37.

If a number be measured by any number, the number which is measured will have a part called by the same name as the measuring number.

Book 7 Proposition 38.

If a number have any part whatever, it will be measured by a number called by the same name as the part.

Book 7 Proposition 39.

To find the number which is the least that will have given parts.

Book 8. Propositions.

Book 8 Proposition 1.

If there be as many numbers as we please in continued proportion, and the extremes of them be prime to one another, the numbers are the least of those which have the same ratio with them.

Book 8 Proposition 2.

To find numbers in continued proportion, as many as may be prescribed, and the least that are in a given ratio.

Book 8 Proposition 3.

If as many numbers as we please in continued proportion be the least of those which have the same ratio with them, the extremes of them are prime to one another.

Book 8 Proposition 4.

Given as many ratios as we please in least numbers, to find numbers in continued proportion which are the least in the given ratios.

Book 8 Proposition 5.

Plane numbers have to one another the ratio compounded of the ratios of their sides.

Book 8 Proposition 6.

If there be as many numbers as we please in continued proportion, and the first do not measure the second, neither will any other measure any other.

Book 8 Proposition 7.

If there be as many numbers as we please in continued proportion and the first measure the last, it will measure the second also.

Book 8 Proposition 8.

If between two numbers there fall numbers in continued proportion with them, then, however many numbers fall between them in continued proportion, so many will, also, *fall in continued proportion between the numbers which have the same ratio with the original numbers.*

Book 8 Proposition 9.

If two numbers be prime to one another, and numbers fall between them in continued proportion then however many numbers fall between them in continued proportion, so many will, also, *fall between each, of them and an unit in continued proportion.*

Book 8 Proposition 10.

If numbers fall between each, of two numbers and an unit in continued proportion, however many numbers fall between each, of them and an unit in continued proportion, so many, also, *will fall between the numbers themselves in continued proportion.*

Book 8 Proposition 11.

Between two square numbers there is one mean proportional number, and the square has to the square the ratio duplicate of that which the side has to the side.

Book 8 Proposition 12.

Between two cube numbers there are two mean proportional numbers, and the cube has to the cube the ratio triplicate of that which the side has to the side.

Book 8 Proposition 13.

If there be as many numbers as we please in continued proportion, and each, by multiplying itself, make some number, the products will be proportional; and, If the original numbers, by multiplying the products, make certain numbers, the latter will, also, *be proportional.*

Book 8 Proposition 14.

If a square measure a square, the side will, also, *measure the side; and, if the side measure the side, the square will,* also, *measure the square.*

Book 8 Proposition 15.

If a cube number measure a cube number, the side will, also, *measure the side; and, if the side measure the side, the cube will,* also, *measure the cube.*

Book 8 Proposition 16.

If a square number do not measure a square number, neither will the side measure the side; and, if the side do not measure the side, neither will the square measure the square.

Book 8 Proposition 17.

If a cube number do not measure a cube number, neither will the side measure the side; and, if the side do not measure the side, neither will the cube measure the cube.

Book 8 Proposition 18.

Between two similar plane numbers there is one mean proportional number; and the plane number has to the plane number the ratio duplicate of that which the corresponding side has to the corresponding side.

Book 8 Proposition 19.

Between two similar solid numbers there fall two mean proportional numbers; and the solid number has to the similar solid number the ratio triplicate of that which the corresponding side has to the corresponding side.

Book 8 Proposition 20.

If one mean proportional number fall between two numbers, the numbers will be similar plane numbers.

Book 8 Proposition 21.

If two mean proportional numbers fall between two numbers, the numbers are similar solid numbers.

Book 8 Proposition 22.

If three numbers be in continued proportion, and the first be square, the third will, also, *be square.*

Book 8 Proposition 23.

If four numbers be in continued proportion, and the first be cube, the fourth will, also, *be cube.*

Book 8 Proposition 24.

If two numbers have to one another the ratio which a square number has to a square number, and the first be square, the second will, also, *be square.*

Book 8 Proposition 25.

If two numbers have to one another the ratio which a cube number has to a cube number, and the first be cube, the second will, also, *be cube.*

Book 8 Proposition 26.

Similar plane numbers have to one another the ratio which a square number has to a square number.

Book 8 Proposition 27.

Similar solid numbers have to one another the ratio which a cube number has to a cube number.

Book 9. Propositions.

Book 9 Proposition 1.

If two similar plane numbers, by multiplying one another, make some number, the product will be square.

Book 9 Proposition 2.

If two numbers, by multiplying one another, make a square number, they are similar plane numbers.

Book 9 Proposition 3.

If a cube number, by multiplying itself, make some number, the product will be cube.

Book 9 Proposition 4.

If a cube number, by multiplying a cube number, make some number, the product will be cube.

Book 9 Proposition 5.

If a cube number, by multiplying any number, make a cube number, the multiplied number will, also, *be cube.*

Book 9 Proposition 6.

If a number, by multiplying itself, make a cube number, it will itself, also, *be cube.*

Book 9 Proposition 7.

If a composite number, by multiplying any number, make some number, the product will be solid.

Book 9 Proposition 8.

If as many numbers as we please beginning from an unit be in continued proportion, the third from the unit will be square, as will, also, *those which successively leave out one; the fourth will be cube, as will,* also, *all those which leave out two; and the seventh will be at once cube and square, as will,* also, *those which leave out five.*

Book 9 Proposition 9.

If as many numbers as we please, beginning from an unit, be in continued proportion, and the number after the unit be square, all the rest will, also, *be square. And, if the number after the unit be cube, all the rest will,* also, *be cube.*

Book 9 Proposition 10.

If as many numbers as we please beginning from an unit be in continued proportion, and the number after the unit be not square, neither will any other be square except the third from the unit and all those which leave out one. And if the number after the unit be not cube, neither will any other be cube except the fourth from the unit and all those which leave out two.

Book 9 Proposition 11.

If as many numbers as we please beginning from an unit be in continued proportion, the less measures the greater according to some one of the numbers which have place among the proportional numbers.

Book 9 Proposition 12.

If as many numbers as we please beginning from an unit be in continued proportion, by however many prime numbers the last is measured, the next to the unit will, also, *be measured by the same.*

Book 9 Proposition 13.

If as many numbers as we please beginning from an unit be in continued proportion, and the number after the unit be prime, the greatest will not be measured by any except those which, have a place among the proportional numbers.

Book 9 Proposition 14.

If a number be the least that is measured by prime numbers, it will not be measured by any other prime number except those originally measuring it.

Book 9 Proposition 15.

If three numbers in continued proportion be the least of those which have the same ratio with them, any two whatever added together will be prime to the remaining number.

Book 9 Proposition 16.

If two numbers be prime to one another, the second will not be to any other number as the first is to the second.

Book 9 Proposition 17.

If there be as many numbers as we please in continued proportion, and the extremes of them be prime to one another, the last will not be to any other number as the first to the second.

Book 9 Proposition 18.

Given two numbers, to investigate whether it is possible to find a third proportional to them.

Book 9 Proposition 19.

Given three numbers, to investigate when it is possible to find a fourth proportional to them.

Book 9 Proposition 20.

Prime numbers are more than any assigned multitude of prime numbers.

Book 9 Proposition 21.

If as many even numbers as we please be added together, the whole is even.

Book 9 Proposition 22.

If as many odd numbers as we please be added together, and their multitude be even, the whole will be even.

Book 9 Proposition 23.

If as many odd numbers as we please be added together, and their multitude be odd, the whole will, also, *be odd.*

Book 9 Proposition 24.

If from an even number an even number be subtracted, the remainder will be even.

Book 9 Proposition 25.

If from an even number an odd number be subtracted, the remainder will be odd.

Book 9 Proposition 26.

If from an odd number an odd number be subtracted, the remainder will be even.

Book 9 Proposition 27.

If from an odd number an even number be subtracted, the remainder will be odd.

Book 9 Proposition 28.

If an odd number, by multiplying an even number, make some number, the product will be even.

Book 9 Proposition 29.

If an odd number, by multiplying an odd number, make some number, the product will be odd.

Book 9 Proposition 30.

If an odd number measure an even number, it will, also, *measure the half of it.*

Book 9 Proposition 31.

If an odd number be prime to any number, it will, also, *be prime to the double of it.*

Book 9 Proposition 32.

Each, of the numbers which are continually doubled beginning from a dyad is even-times even only.

Book 9 Proposition 33.

If a number have its half odd, it is even-times odd only.

Book 9 Proposition 34.

If a number neither be one of those which are continually doubled from a dyad, nor have its half odd, it is both even-times even and even-times odd.

Book 9 Proposition 35.

If as many numbers as we please be in continued proportion, and there be subtracted from the second and the last numbers equal, to the first, then, as the excess of the second is to the first, so will the excess of the last be to all those before it.

Book 9 Proposition 36.

If as many numbers as we please beginning from an unit be set out continuously in double proportion, until the sum of all becomes prime, and if the sum multiplied into the last make some number, the product will be perfect.

Book 10. Propositions.

Book 10 Proposition 1.

Two unequal magnitudes being set out, if from the greater there be subtracted a magnitude greater than its half and from that which is left a magnitude greater than its half and if this process be repeated continually there will be left some magnitude which will be less than the lesser magnitude set out.

Book 10 Proposition 2.

If when the less of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it, the magnitudes will be incommensurable.

Book 10 Proposition 3.

Given two commensurable magnitudes, to find their greatest common measure.

Book 10 Proposition 4.

Given three commensurable magnitudes, to find their greatest common measure.

Book 10 Proposition 5.

Commensurable magnitudes have to one another the ratio which a number has to a number.

Book 10 Proposition 6.

If two magnitudes have to one another the ratio which a number has to a number, the magnitudes will be commensurable.

Book 10 Proposition 7.

Incommensurable magnitudes have not to one another the ratio which a number has to a number.

Book 10 Proposition 8.

If two magnitudes have not to one another the ratio which a number has to a number, the magnitudes will be incommensurable.

Book 10 Proposition 9.

The squares on straight lines commensurable in length have to one another the ratio which a square number has to a square number; and squares which

have to one another the ratio which a square number has to a square number will, also, *have their sides commensurable in length. But the squares on straight lines incommensurable in length have not to one another the ratio which a square number has to a square number*; *and squares which have not to one another the ratio which a square number has to a square number will not have their sides commensurable in length either.*

[Book 10 Proposition 10.

To find two straight lines incommensurable, *the one in length only*, *and the other in square also, with an assigned straight line.*

Book 10 Proposition 11.

If four magnitudes be proportional, *and the first be commensurable with the second*, *the third will*, also, *be commensurable with the fourth*; *and*, *if the first be incommensurable with the second*, *the third will*, also, *be incommensurable with the fourth.*

Book 10 Proposition 12.

Magnitudes commensurable with the same magnitude are commensurable with one another also.

Book 10 Proposition 13.

If two magnitudes be commensurable, *and the one of them be incommensurable with any magnitude*, *the remaining one will*, also, *be incommensurable with the same.*

Book 10 Proposition 14.

If four straight lines be proportional, *and the square, on the first be greater than the square, on the second by the square, on a straight line commensurable with the first*, *the square, on the third will*, also, *be greater than the square, on the fourth by the square, on a straight line commensurable with the third.*

And, *if the square, on the first be greater than the square, on the second by the square, on a straight line incommensurable with the first*, *the square, on the third will*, also, *be greater than the square, on the fourth by the square, on a straight line incommensurable with the third.*

Book 10 Proposition 15.

If two commensurable magnitudes be added together, *the whole will*, also, *be commensurable with each, of them*; *and*, *if the whole be commensurable with one of them*, *the original magnitudes will*, also, *be commensurable.*

Book 10 Proposition 16.

If two incommensurable magnitudes be added together, the whole will, also, *be incommensurable with each, of them; and, if the whole be incommensurable with one of them, the original magnitudes will,* also, *be incommensurable.*

Book 10 Proposition 17.

If there be two unequal straight lines, and to the greater there be applied a parallelogram equal, to the fourth part of the square, on the less and deficient by a square figure, and if it divide it into parts which are commensurable in length, then the square, on the greater will be greater than the square, on the less by the square, on a straight line commensurable with the greater.

And, if the square, on the greater be greater than the square, on the less by the square, on a straight line commensurable with the greater, and if there be applied to the greater a parallelogram equal, to the fourth part of the square, on the less and deficient by a square figure, it will divide it into parts which are commensurable in length.

Book 10 Proposition 18.

If there be two unequal straight lines, and to the greater there be applied a parallelogram equal, to the fourth part of the square, on the less and deficient by a square figure, and if it divide it into parts which are incommensurable, the square, on the greater will be greater than the square, on the less by the square, on a straight line incommensurable with the greater.

And, if the square, on the greater be greater than the square, on the less by the square, on a straight line incommensurable with the greater, and if there be applied to the greater a parallelogram equal, to the fourth part of the square, on the less and deficient by a square figure, it divides it into parts which are incommensurable.

Book 10 Proposition 19.

The rectangle, contained by rational straight lines commensurable in length, is rational.

Book 10 Proposition 20.

If a rational area be applied to a rational straight line, it produces as breadth a straight line rational and commensurable, in length, with the straight line to which it is applied.

Book 10 Proposition 21.

The rectangle contained by rational straight lines commensurable in square, only is irrational, and the side of the square equal, to it is irrational. Let the latter be called **medial**.

Book 10 Proposition 22.

The square, on a medial straight line, if applied to a rational straight line, produces as breadth a straight line rational and incommensurable, in length, with that to which it is applied.

Book 10 Proposition 23.

A straight line commensurable with a medial straight line is medial.

Book 10 Proposition 24.

The rectangle contained by medial straight lines commensurable in length is medial.

Book 10 Proposition 25.

The rectangle contained by medial straight lines commensurable in square, only is either rational or medial.

Book 10 Proposition 26.

A medial area does not exceed a medial area by a rational area.

Book 10 Proposition 27.

To find medial straight lines commensurable in square, only which contain a rational rectangle.

Book 10 Proposition 28.

To find medial straight lines commensurable, in square, only which contain a medial rectangle.

Book 10 Proposition 29.

To find two rational straight lines commensurable, in square, only and such that the square, on the greater is greater than the square, on the less by the square, on a straight line commensurable in length with the greater.

Book 10 Proposition 30.

To find two rational straight lines commensurable in square, only and such that the square, on the greater is greater than the square, on the less by the square, on a straight line incommensurable in length with the greater.

Book 10 Proposition 31.

To find two medial straight lines commensurable in square, only, containing a rational rectangle, and such that the square, on the greater is greater than the square, on the less by the square, on a straight line commensurable in length with the greater.

Book 10 Proposition 32.

To find two medial straight lines commensurable, in square, only, containing a medial rectangle, and such that the square, on the greater is greater than the square, on the less by the square, on a straight line commensurable with the greater.

Book 10 Proposition 33.

To find two straight lines incommensurable, in square, which make the sum of the squares on them rational but the rectangle contained by them medial.

Book 10 Proposition 34.

To find two straight lines incommensurable, in square, which make the sum of the squares on them medial but the rectangle contained by them rational.

Book 10 Proposition 35.

To find two straight lines incommensurable, in square, which make the sum of the squares on them medial and the rectangle contained by them medial and moreover incommensurable with the sum of the squares on them.

Book 10 Proposition 36.

If two rational straight lines commensurable, in square, only be added together, the whole is irrational; and let it be called **binomial**.

Book 10 Proposition 37.

If two medial straight lines commensurable, in square, only and containing a rational rectangle be added together, the whole is irrational; and let it be called a **first bimedial** *straight line.*

Book 10 Proposition 38.

If two medial straight lines commensurable, in square, only and containing a medial rectangle be added together, the whole is irrational; and let it be called a **second bimedial** *straight line.*

Book 10 Proposition 39.

If two straight lines incommensurable, in square, which make the sum of the squares on them rational, but the rectangle contained by them medial, be added together, the whole straight line is irrational; and let it be called **major**.

Book 10 Proposition 40.

If two straight lines incommensurable, in square, which make the sum of the squares on them medial, but the rectangle contained by them rational, be added together, the whole straight line is irrational; and let it be called the **side of a rational plus a medial area**.

Book 10 Proposition 41.

If two straight lines incommensurable, in square, which make the sum of the squares on them medial, and the rectangle contained by them medial and, also, *incommensurable with the sum of the squares on them, be added together, the whole straight line is irrational; and let it be called the* **side of the sum of two medial areas**.

Book 10 Proposition 42.

A binomial straight line is divided into its terms at one point only.

Book 10 Proposition 43.

A first bimedial straight line is divided at one point only.

Book 10 Proposition 44.

A second bimedial straight line is divided at one point only.

Book 10 Proposition 45.

A major straight line is divided at one and the same point only.

Book 10 Proposition 46.

The side of a rational plus a medial area is divided at one point only.

Book 10 Proposition 47.

The side of the sum of two medial areas is divided at one point only.

Book 10 Proposition 48.

To find the first binomial straight line.

Book 10 Proposition 49.

To find the second binomial straight line.

Book 10 Proposition 50.

To find the third binomial straight line.

Book 10 Proposition 51.

To find the fourth binomial straight line.

Book 10 Proposition 52.

To find the fifth binomial straight line.

Book 10 Proposition 53.

To find the sixth binomial straight line.

Book 10 Proposition 54.

If an area be contained by a rational straight line and the first binomial, the "side", of the area is the irrational straight line which is called binomial.

Book 10 Proposition 55.

If an area be contained by a rational straight line and the second binomial the "side", of the area is the irrational straight line which is called a first bimedial.

Book 10 Proposition 56.

If an area be contained by a rational straight line and the third binomial, the "side", of the area, is the irrational straight line called a second bimedial.

Book 10 Proposition 57.

If an area be contained by a rational straight line and the fourth binomial, the "side", of the area, is the irrational straight line called major.

Book 10 Proposition 58.

If an area be contained by a rational straight line and the fifth binomial, the "side", of the area, is the irrational straight line called the side of a rational plus a medial area.

Book 10 Proposition 59.

If an area be contained by a rational straight line and the sixth binomial, the "side", of the area, is the irrational straight line called the side of the sum of two medial areas.

Book 10 Proposition 60.

The square, on the binomial straight line applied to a rational straight line produces as breadth the first binomial.

Book 10 Proposition 61.

The square, on the first bimedial straight line applied to a rational straight line produces as breadth the second binomial.

Book 10 Proposition 62.

The square, on the second bimedial straight line applied to a rational straight line produces as breadth the third binomial.

Book 10 Proposition 63.

The square, on the major straight line applied to a rational straight line produces as breadth the fourth binomial.

Book 10 Proposition 64.

The square, on the side of a rational plus a medial area applied to a rational straight line produces as breadth the fifth binomial.

Book 10 Proposition 65.

The square, on the side of the sum of two medial areas applied to a rational straight line produces as breadth the sixth binomial.

Book 10 Proposition 66.

A straight line commensurable in length with a binomial straight line is itself, also, *binomial and the same, in order.*

Book 10 Proposition 67.

A straight line commensurable in length with a bimedial straight line is itself, also, *bimedial and the same, in order.*

Book 10 Proposition 68.

A straight line commensurable with a major straight line is itself, also, *major.*

Book 10 Proposition 69.

A straight line commensurable with the side of a rational plus a medial area is itself, also, *the side of a rational plus a medial area.*

Book 10 Proposition 70.

A straight line commensurable with the side of the sum of two medial areas is the side of the sum of two medial areas.

Book 10 Proposition 71.

If a rational and a medial area be added together, four irrational straight lines arise, namely a binomial or a first bimedial or a major or a side of a rational plus a medial area.

Book 10 Proposition 72.

If two medial areas incommensurable with one another be added together the remaining two irrational straight lines arise, namely either a second bimedial or a side of the sum of two medial areas.

Book 10 Proposition 73.

If from a rational straight line there be subtracted a rational straight line commensurable with the whole, in square, only, the remainder is irrational; and let it be called an **apotome**.

Book 10 Proposition 74.

If from a medial straight line there be subtracted a medial straight line which is commensurable with the whole, in square, only, and which contains with the whole a rational rectangle, the remainder is irrational. And let it be called a **first apotome of a medial** *straight line.*

Book 10 Proposition 75.

If from a medial straight line there be subtracted a medial straight line which is commensurable with the whole, in square, only, and which contains with the whole a medial rectangle, the remainder is irrational; and let it be called a **second apotome of a medial** *straight line.*

Book 10 Proposition 76.

If from a straight line there be subtracted a straight line which is incommensurable, in square, with the whole and which with the whole makes the squares on them added together rational, but the rectangle contained by them medial, the remainder is irrational; and let it be called **minor**.

Book 10 Proposition 77.

If from a straight line there be subtracted a straight line which is incommensurable, in square, with the whole, and which with the whole makes the sum of the squares on them medial, but twice the rectangle contained by them rational, the remainder is irrational: and let it be called **that which produces with a rational area a medial whole**.

Book 10 Proposition 78.

If from a straight line there be subtracted a straight line which is incommensurable, in square, with the whole and which with the whole makes the sum of the squares on them medial, twice the rectangle contained by them medial, and further the squares on them incommensurable with twice the rectangle contained by them, the remainder is irrational; and let it be called **that which produces**, **with a medial area**, **a medial whole**.

Book 10 Proposition 79.

To an apotome only one rational straight line can be annexed which is commensurable with the whole, in square, only.

Book 10 Proposition 80.

To a first apotome of a medial straight line only one medial straight line can be annexed which is commensurable with the whole, in square, only and which contains with the whole a rational rectangle.

Book 10 Proposition 81.

To a second apotome of a medial straight line only one medial straight line can be annexed which is commensurable with the whole, in square, only, and which contains with the whole a medial rectangle.

Book 10 Proposition 82.

To a minor straight line only one straight line can be annexed which is incommensurable, in square, with the whole and which makes, with the whole, the sum of the squares on them rational, but twice the rectangle contained by them, medial.

Book 10 Proposition 83.

To a straight line which produces with a rational area a medial whole only one straight line can be annexed which is incommensurable, in square, with the whole straight line and which with the whole straight line makes the sum of the squares on them medial, but twice the rectangle contained by them rational.

Book 10 Proposition 84.

To a straight line which produces with a medial area a medial whole only one straight line can be annexed which is incommensurable, in square, with the whole straight line and which with the whole straight line makes the sum of the squares on them medial and twice the rectangle contained by them both medial and, also, *incommensurable with the sum of the squares on them.*

Book 10 Proposition 85.

To find the first apotome.

Book 10 Proposition 86.

To find the second apotome.

Book 10 Proposition 87.

To find the third apotome.

Book 10 Proposition 88.

To find the fourth apotome.

Book 10 Proposition 89.

To find the fifth apotome.

Book 10 Proposition 90.

To find the sixth apotome.

Book 10 Proposition 91.

If an area be contained by a rational straight line and a first apotome, the "side", of the area, is an apotome.

Book 10 Proposition 92.

If an area be contained by a rational straight line and a second apotome, the "side", of the area, is a first apotome of a medial straight line.

Book 10 Proposition 93.

If an area be contained by a rational straight line and a third apotome, the "side", of the area, is a second apotome of a medial straight line.

Book 10 Proposition 94.

If an area be contained by a rational straight line and a fourth apotome, the "side", of the area, is minor.

Book 10 Proposition 95.

If an area be contained by a rational straight line and a fifth apotome, the "side", of the area, is a straight line which produces with a rational area a medial whole.

Book 10 Proposition 96.

If an area be contained by a rational straight line and a sixth apotome, the "side", of the area, is a straight line which produces with a medial area a medial whole.

Book 10 Proposition 97.

The square, on an apotome applied to a rational straight line produces as breadth a first apotome.

Book 10 Proposition 98.

The square, on a first apotome of a medial straight line applied to a rational straight line produces as breadth a second apotome.

Book 10 Proposition 99.

The square, on a second apotome of a medial straight line applied to a rational straight line produces as breadth a third apotome.

Book 10 Proposition 100.

The square, on a minor straight line applied to a rational straight line produces as breadth a fourth apotome.

Book 10 Proposition 101.

The square, on the straight line which produces with a rational area a medial whole, if applied to a rational straight line, produces as breadth a fifth apotome.

Book 10 Proposition 102.

The square, on the straight line which produces with a medial area a medial whole, if applied to a rational straight line, produces as breadth a sixth apotome.

Book 10 Proposition 103.

A straight line commensurable, in length, with an apotome is an apotome and the same, in order.

Book 10 Proposition 104.

A straight line commensurable with an apotome of a medial straight line is an apotome of a medial straight line and the same, in order.

Book 10 Proposition 105.

A straight line commensurable with a minor straight line is minor.

Book 10 Proposition 106.

A straight line commensurable with that which produces with a rational area a medial whole is a straight line which produces with a rational area a medial whole.

Book 10 Proposition 107.

A straight line commensurable with that which produces, with a medial area, a medial whole, is itself, also, *a straight line which produces, with a medial area, a medial whole.*

Book 10 Proposition 108.

If from a rational area a medial area be subtracted, the "side", of the remaining area, becomes one of two irrational straight lines, either an apotome or a minor straight line.

Book 10 Proposition 109.

If, from a medial area, a rational area be subtracted, there arise two other irrational straight lines, either a first apotome of a medial straight line or a straight line which produces with a rational area a medial whole.

Book 10 Proposition 110.

If from a medial area there be subtracted a medial area incommensurable with the whole, the two remaining irrational straight lines arise, either a second apotome of a medial straight line or a straight line which produces with a medial area a medial whole.

Book 10 Proposition 111.

The apotome is not the same with the binomial straight line.

Book 10 Proposition 112.

The square, on a rational straight line applied to the binomial straight line produces as breadth an apotome the terms of which are commensurable with the terms of the binomial and moreover in the same ratio; and further the apotome so arising will have the same order as the binomial straight line.

Book 10 Proposition 113.

The square, on a rational straight line, if applied to an apotome, produces as breadth the binomial straight line the terms of which are commensurable with the terms of the apotome and in the same ratio; and further the binomial so arising has the same order as the apotome.

Book 10 Proposition 114.

If an area be contained by an apotome and the binomial straight line the terms of which are commensurable with the terms of the apotome and in the same ratio, the "side", of the area, is rational.

Book 10 Proposition 115.

From a medial straight line there arise irrational straight lines infinite in number, and none of them is the same as any of the preceding.

Book 11. Propositions.

Book 11 Proposition 1.

A part of a straight line cannot be in the plane of reference and a part in a plane more elevated.

Book 11 Proposition 2.

If two straight lines cut one another, they are in one plane, and every triangle is in one plane.

Book 11 Proposition 3.

If two planes cut one another, their common section is a straight line.

Book 11 Proposition 4.

If a straight line be set up at right angles to two straight lines which cut one another, at their common point of section, it will, also, *be at right angles to the plane through them.*

Book 11 Proposition 5.

If a straight line be set up at right angles to three straight lines which meet one another, at their common point of section, the three straight lines are in one plane.

Book 11 Proposition 6.

If two straight lines be at right angles to the same plane, the straight lines will be parallel.

Book 11 Proposition 7.

If two straight lines be parallel and points be taken at random on each, of them, the straight line joining the points is in the same plane with the parallel straight lines.

Book 11 Proposition 8.

If two straight lines be parallel, and one of them be at right angles to any plane, the remaining one will, also, *be at right angles to the same plane.*

Book 11 Proposition 9.

Straight lines which are parallel to the same straight line and are not in the same plane with it are, also, *parallel to one another.*

Book 11 Proposition 10.

If two straight lines meeting one another be parallel to two straight lines meeting one another not in the same plane, they will contain equal angles.

Book 11 Proposition 11.

From a given elevated point to draw a straight line perpendicular to a given plane.

Book 11 Proposition 12.

To set up a straight line at right angles to a given plane from a given point in it.

Book 11 Proposition 13.

From the same point two straight lines cannot be set up at right angles to the same plane on the same side.

Book 11 Proposition 14.

Planes to which the same straight line is at right angles will be parallel.

Book 11 Proposition 15.

If two straight lines meeting one another be parallel to two straight lines meeting one another, not being in the same plane, the planes through them are parallel.

Book 11 Proposition 16.

If two parallel planes be cut by any plane, their common sections are parallel.

Book 11 Proposition 17.

If two straight lines be cut by parallel planes, they will be cut in the same ratios.

Book 11 Proposition 18.

If a straight line be at right angles to any plane, all the planes through it will, also, *be at right angles to the same plane.*

Book 11 Proposition 19.

If two planes which cut one another be at right angles to any plane, their common section will, also, *be at right angles to the same plane.*

Book 11 Proposition 20.

If a solid angle be contained by three plane angles, any two, taken together in any manner, are greater than the remaining one.

Book 11 Proposition 21.

Any solid angle is contained by plane angles less than four right angles.

Book 11 Proposition 22.

If there be three plane angles of which two, taken together in any manner, are greater than the remaining one, and they are contained by equal straight lines, it is possible to construct a triangle out of the straight lines joining the extremities of the equal straight lines.

Book 11 Proposition 23.

To construct a solid angle out of three plane angles two of which, taken together in any manner, are greater than the remaining one: thus the three angles must be less than four right angles.

Book 11 Proposition 24.

If a solid be contained by parallel planes, the opposite planes in it are equal and parallelogrammic.

Book 11 Proposition 25.

If a parallelepipedal solid be cut by a plane which is parallel to the opposite planes, then, as the base is to the base, so will the solid be to the solid.

Book 11 Proposition 26.

On a given straight line, and at a given point on it, to construct a solid angle equal, to a given solid angle.

Book 11 Proposition 27.

On a given straight line to describe a parallelepipedal solid similar and similarly situated to a given parallelepipedal solid.

Book 11 Proposition 28.

If a parallelepipedal solid be cut by a plane through the diagonals of the opposite planes, the solid will be bisected by the plane.

Book 11 Proposition 30.

Parallelepipedal solids which are on the same base and of the same height, and in which the extremities of the sides which stand up are not on the same straight lines, are equal, to one another.

Book 11 Proposition 31.

Parallelepipedal solids which are on equal bases and of the same height are equal, to one another.

Book 11 Proposition 32.

Parallelepipedal solids which are of the same height are to one another as their bases.

Book 11 Proposition 33.

Similar parallelepipedal solids are to one another in the triplicate ratio of their corresponding sides.

Book 11 Proposition 34.

In equal parallelepipedal solids the bases are reciprocally proportional to the heights; and those parallelepipedal solids in which the bases are reciprocally proportional to the heights are equal.

Book 11 Proposition 35.

If there be two equal plane angles, and on their vertices there be set up elevated straight lines containing equal angles with the original straight lines respectively, if on the elevated straight lines points be taken at random and perpendiculars be drawn from them to the planes in which the original angles are, and if from the points so arising in the planes straight lines be joined to the vertices of the original angles, they will contain, with the elevated straight lines, equal angles.

Book 11 Proposition 36.

If three straight lines be proportional, the parallelepipedal solid formed out of the three is equal, to the parallelepipedal solid on the mean which is equilateral, but equiangular with the aforesaid solid.

Book 11 Proposition 37.

If four straight lines be proportional, the parallelepipedal solids on them which are similar and similarly described will, also, *be proportional; and, if the parallelepipedal solids on them which are similar and similarly described be proportional, the straight lines will themselves,* also, *be proportiona*l.

Book 11 Proposition 38.

If the sides of the opposite planes of a cube be bisected, and planes be carried through the points of section, the common section of the planes and the diameter of the cube bisect one another.

Book 11 Proposition 39.

If there be two prisms of equal height, and one have a parallelogram as base and the other a triangle, and if the parallelogram be double of the triangle, the prisms will be equal.

Book 12. Propositions.

Book 12 Proposition 1.

Similar polygons inscribed in circles are to one another as the squares on the diameters.

Book 12 Proposition 2.

Circles are to one another as the squares on the diameters.

Book 12 Proposition 3.

Any pyramid which has a triangular base is divided into two pyramids equal and similar to one another, similar to the whole and having triangular bases, and into two equal prisms; *and the two* prisms *are greater than the half of the whole pyramid.*

Book 12 Proposition 4.

If there be two pyramids of the same height which have triangular bases, and each, of them be divided into two pyramids equal, to one another and similar to the whole, and into two equal prisms, then, as the base of the one pyramid is to the base of the other pyramid, so will all the prisms in the one pyramid be to all the prisms, being equal in multitude, in the other pyramid.

Book 12 Proposition 5.

Pyramids which are of the same height and have triangular bases are to one another as the bases.

Book 12 Proposition 6.

Pyramids which are of the same height and have polygonal bases are to one another as the bases.

Book 12 Proposition 7.

Any prism which has a triangular base is divided into three pyramids equal, to one another which have triangular bases.

Book 12 Proposition 8.

Similar pyramids which have triangular bases are in the triplicate ratio of their corresponding sides.

Book 12 Proposition 9.

In equal pyramids which have triangular bases the bases are reciprocally proportional to the heights; and those pyramids in which the bases are reciprocally proportional to the heights are equal.

Book 12 Proposition 10.

Any cone is a third part of the cylinder which has the same base with it and equal height.

Book 12 Proposition 11.

Cones and cylinders which are of the same height are to one another as their bases.

Book 12 Proposition 12.

Similar cones and cylinders are to one another in the triplicate ratio of the diameters in their bases.

Book 12 Proposition 13.

If a cylinder be cut by a plane which is parallel to its opposite planes, then, as the cylinder is to the cylinder, so will the axis be to the axis.

Book 12 Proposition 14.

Cones and cylinders which are on equal bases are to one another as their heights.

Book 12 Proposition 15.

In equal cones and cylinders the bases are reciprocally proportional to the heights; and those cones and cylinders in which the bases are reciprocally proportional to the heights are equal.

Book 12 Proposition 16.

Given two circles about the same center, to inscribe in the greater circle an equilateral polygon with an even number of sides which does not touch the lesser circle.

Book 12 Proposition 17.

Given two spheres about the same center, to inscribe in the greater sphere a polyhedral solid which does not touch the lesser sphere at its surface.

Book 12 Proposition 18.

Spheres are to one another in the triplicate ratio of their respective diameters.

Book 13. Propositions.

Book 13 Proposition 1.

If a straight line be cut in extreme and mean ratio, the square, on the greater segment added to the half of the whole is five times the square, on the half.

Book 13 Proposition 2.

If the square, on a straight line be five times the square, on a segment of it, then, when the double of the said segment is cut in extreme and mean ratio, the greater segment is the remaining part of the original straight line.

Book 13 Proposition 3.

If a straight line be cut in extreme and mean ratio, the square, on the lesser segment added to the half of the greater segment is five times the square, on the half of the greater segment.

Book 13 Proposition 4.

If a straight line be cut in extreme and mean ratio, the square, on the whole and the square, on the lesser segment together are triple of the square, on the greater segment.

Book 13 Proposition 5.

If a straight line be cut in extreme and mean ratio, and there be added to it a straight line equal, to the greater segment, the whole straight line has been cut in extreme and mean ratio, and the original straight line is the greater segment.

Book 13 Proposition 6.

If a rational straight line be cut in extreme and mean ratio, each, of the segments is the irrational straight line called apotome.

Book 13 Proposition 7.

If three angles of an equilateral pentagon, taken either in order or not in order, be equal, the pentagon will be equiangular.

Book 13 Proposition 8.

If in an equilateral and equiangular pentagon straight lines subtend two angles taken in order, they cut one another in extreme and mean ratio, and their greater segments are equal, to the side of the pentagon.

Book 13 Proposition 9.

If the side of the hexagon and that of the decagon inscribed in the same circle be added together, the whole straight line has been cut in extreme and mean ratio, and its greater segment is the side of the hexagon.

Book 13 Proposition 10.

If an equilateral pentagon be inscribed in a circle, the square, on the side of the pentagon is equal, to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.

Book 13 Proposition 11.

If in a circle which has its diameter rational an equilateral pentagon be inscribed, the side of the pentagon is the irrational straight line called minor.

Book 13 Proposition 12.

If an equilateral triangle be inscribed in a circle, the square, on the side of the triangle is triple of the square, on the radius of the circle.

Book 13 Proposition 13.

To construct a pyramid, to comprehend it in a given sphere, and to prove that the square, on the diameter of the sphere is one and a half times the square, on the side of the pyramid.

Book 13 Proposition 14.

To construct an octahedron and comprehend it in a sphere, as in the preceding case; and to prove that the square, on the diameter of the sphere is double of the square, on the side of the octahedron.

Book 13 Proposition 15.

To construct a cube and comprehend it in a sphere, like the pyramid; and to prove that the square, on the diameter of the sphere is triple of the square, on the side of the cube.

Book 13 Proposition 16.

To construct an icosahedron and comprehend it in a sphere, like the aforesaid figures; and to prove that the side of the icosahedron is the irrational straight line called minor.

Book 13 Proposition 17.

To construct a dodecahedron and comprehend it in a sphere, like the aforesaid figures, and to prove that the side of the dodecahedron is the irrational straight line called apotome.

Book 13 Proposition 18.

To set out the sides of the five figures and to compare them with one another.

BOOK I. Definitions.

1. A **point** is that which has no part.

2. A **line** is breadthless length.

3. The extremities of a line are **points**.

4. A **straight line** is a line which lies evenly with the points on itself.

5. A **surface** is that which has length and breadth only.

6. The extremities of a surface are **lines**.

7. A **plane surface** is a surface which lies evenly with the straight lines on itself.

8. A **plane angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

9. And when the lines containing the angle are straight, the angle is called **rectilineal**.

10. When a straight line set up on a straight line makes the adjacent angles equal, to one another, each, of the equal angles is **right**, and the straight line standing on the other is called a **perpendicular** to that on which it stands.

11. An **obtuse angle** is an angle greater than a right angle.

12. An **acute angle** is an angle less than a right angle.

13. A **boundary** is that which is an extremity of anything.

14. A **figure** is that which is contained by any boundary or boundaries.

15. A **circle** is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal, to one another;

16. And the point is called the **centre** of the circle.

17. A **diameter** of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line, also, bisects the circle.

18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.

19. **Rectilineal figures** are those which are contained by straight lines, **trilateral** figures being those contained by three, **quadrilateral** those contained by four, and **multilateral** those contained by more than four straight lines.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.

21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.

22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal, to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.

23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

BOOK II. Definitions.

1. Any rectangular parallelogram is said to be **contained** by the two straight lines containing the right angle.

2. And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a **gnomon**.

BOOK III. Definitions.

1. **Equal circles** are those the diameters of which are equal, or the radii of which are equal.

2. A straight line is said to **touch a circle** which, meeting the circle and being produced, does not cut the circle.

3. **Circles** are said to **touch one another** which, meeting one another, do not cut one another.

4. In a circle straight lines are said **to be equally distant from the centre** when the perpendiculars drawn to them from the centre are equal.

5. And that straight line is said to be **at a greater distance** on which the greater perpendicular falls.

6. A **segment of a circle** is the figure contained by a straight line and a circumference of a circle.

7. An **angle of a segment** is that contained by a straight line and a circumference of a circle.

8. An **angle in a segment** is the angle which, when a point is taken on the circumference of the segment and straight lines are joined from it to the extremities of the straight line which is the **base of the segment**, is contained by the straight lines so joined.

9. And, when the straight lines containing the angle cut off a circumference, the angle is said to **stand upon** that circumference.

10. A **sector of a circle** is the figure which, when an angle is constructed at the centre of the circle, is contained by the straight lines containing the angle and the circumference cut off by them.

11. **Similar segments of circles** are those which admit equal angles, or in which the angles are equal, to one another.

BOOK IV. Definitions.

1. A rectilineal figure is said to be **inscribed in a rectilineal figure** when the respective angles of the inscribed figure lie on the respective sides of that in which it is inscribed.

2. Similarly, a figure is said to be **circumscribed about a figure** when the respective sides of the circumscribed figure pass through the respective angles of that about which it is circumscribed.

3. A rectilineal figure is said to be **inscribed in a circle** when each angle of the inscribed figure lies on the circumference of the circle.

4. A rectilineal figure is said to be **circumscribed about a circle**, when each side of the circumscribed figure touches the circumference of the circle.

5. Similarly a circle is said to be **inscribed in a figure** when the circumference of the circle touches each side of the figure in which it is inscribed.

6. A circle is said to be **circumscribed about a figure** when the circumference of the circle passes through each angle of the figure about which it is circumscribed.

7. A straight line is said to be **fitted into a circle** when its extremities are on the circumference of the circle.

BOOK V. Definitions.

1. A magnitude is a **part** of a magnitude, the less of the greater, when it measures the greater.

2. The greater is a **multiple** of the less when it is measured by the less.

3. A **ratio** is a sort of relation in respect of size between two magnitudes of the same kind.

4. Magnitudes are said to **have a ratio** to one another which are capable, when multiplied, of exceeding one another.

5. Magnitudes are said to **be in the same ratio**, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal, to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

6. Let magnitudes which have the same ratio be called **proportional**.

7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to **have a greater ratio** to the second than the third has to the fourth.

8. A proportion in three terms is the least possible.

9. When three magnitudes are proportional, the first is said to have to the third the **duplicate ratio** of that which it has to the second.

10. When four magnitudes are < continuously > proportional, the first is said to have to the fourth the **triplicate ratio** of that which it has to the second, and so on continually, whatever be the proportion.

11. The term **corresponding magnitudes** is used of antecedents in relation to antecedents, and of consequents in relation to consequents.

12. **Alternate ratio** means taking the antecedent in relation to the antecedent and the consequent in relation to the consequent.

13. **Inverse ratio** means taking the consequent as antecedent in relation to the antecedent as consequent.

14. **Composition of a ratio** means taking the antecedent together with the consequent as one in relation to the consequent by itself.

15. **Separation of a ratio** means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself.

16. **Conversion of a ratio** means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

17. A ratio ***ex aequali*** arises when, there being several magnitudes and another set equal, to them in multitude which taken two and two are in the same proportion, as the first is to the last among the first magnitudes, so is the first to the last among the second magnitudes;

Or, in other words, it means taking the extreme terms by virtue of the removal of the intermediate terms.

18. A **perturbed proportion** arises when, there being three magnitudes and another set equal, to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to A third among the first magnitudes, so is a third to the antecedent among the second magnitudes.

BOOK VI. Definitions.

1. **Similar rectilineal figures** are such as have their angles severally equal and the sides about the equal angles proportional.

[2. **Reciprocally related** figures. See note.]

3. A straight line is said to have been **cut in extreme and mean ratio** when, as the whole line is to the greater segment, so is the greater to the less.

4. The **height** of any figure is the perpendicular drawn from the vertex to the base.

BOOK VII. Definitions.

1. An **unit** is that by virtue of which each, of the things that exist is called one.

2. A **number** is a multitude composed of units.

3. A number is **a part** of a number, the less of the greater, when it measures the greater;

4. but **parts** when it does not measure it.

5. The greater number is a **multiple** of the less when it is measured by the less.

6. An **even number** is that which is divisible into two equal parts.

7. An **odd number** is that which is not divisible into two equal parts, or that which differs by an unit from an even number.

8. An **even-times even number** is that which is measured by an even number according to an even number.

9. An **even-times odd number** is that which is measured by an even number according to an odd number.

10. An **odd-times odd number** is that which is measured by an odd number according to an odd number.

11. A **prime number** is that which is measured by an unit alone.

12. Numbers **prime to one another** are those which are measured by an unit alone as a common measure.

13. A **composite number** is that which is measured by some number.

14. Numbers **composite to one another** are those which are measured by some number as a common measure.

15. A number is said to **multiply** a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

16. And, when two numbers having multiplied one another make some number, the number so produced is called **plane**, and its **sides** are the numbers which have multiplied one another.

17. And, when three numbers having multiplied one another make some number, the number so produced is **solid**, and its **sides** are the numbers which have multiplied one another.

18. A **square number** is equal multiplied by equal, or a number which is contained by two equal numbers.

19. And a **cube** is equal multiplied by equal and again by equal, or a number which is contained by three equal numbers.

20. Numbers are **proportional** when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

21. **Similar plane** and **solid** numbers are those which have their sides proportional.

22. A **perfect number** is that which is equal, to its own parts.

BOOK X. Definitions 10-1.

1. Those magnitudes are said to be **commensurable** which are measured by the same measure, and those **incommensurable** which cannot have any common measure.

2. Straight lines are **commensurable in square** when the squares on them are measured by the same area, and **incommensurable in square** when the squares on them cannot possibly have any area as a common measure.

3. With these hypotheses, it is proved that there exist straight lines infinite in multitude which are commensurable and incommensurable respectively, some in length only, and others in square also, with an assigned straight line. Let then the assigned straight line be called **rational**, and those straight lines which are commensurable with it, whether in length and in square or in square, only, **rational**, but those which are incommensurable with it **irrational**.

4. And let the square, on the assigned straight line be called **rational** and those areas which are commensurable with it **rational**, but those which are incommensurable with it **irrational**, and the straight lines which produce them irrational, that is, in case the areas are squares, the sides themselves, but in case they are any other rectilineal figures, the straight lines on which are described squares equal, to them.

Definitions 10-2.

1. Given a rational straight line and a binomial, divided into its terms, such that the square, on the greater term is greater than the square, on the lesser by the square, on a straight line commensurable in length with the greater, then, if the greater term be commensurable in length with the rational straight line set out, let the whole be called a **first binomial** straight line;

2. but if the lesser term be commensurable in length with the rational straight line set out, let the whole be called a **second binomial**;

3. and if neither of the terms be commensurable in length with the rational straight line set out, let the whole be called a **third binomial**.

4. Again, if the square, on the greater term be greater than the square, on the lesser by the square, on a straight line incommensurable in length with the greater, then, if the greater term be commensurable in length with the rational straight line set out, let the whole be called a **fourth binomial**;

5. if the lesser, a **fifth binomial**;

6. and if neither, a **sixth binomial**.

Definitions 10-3.

1. Given a rational straight line and an apotome, if the square, on the whole be greater than the square, on the annex by the square, on a straight line commensurable in length with the whole, and the whole be commensurable in

length with the rational straight line set out, let the apotome be called a **first apotome**.

2. But if the annex be commensurable in length with the rational straight line set out, and the square, on the whole be greater than that on the annex by the square, on a straight line commensurable with the whole, let the apotome be called a **second apotome**.

3. But if neither be commensurable in length with the rational straight line set out, and the square, on the whole be greater than the square, on the annex by the square, on a straight line commensurable with the whole, let the apotome be called a **third apotome**.

4. Again, if the square, on the whole be greater than the square, on the annex by the square, on a straight line incommensurable with the whole, then, if the whole be commensurable in length with the rational straight line set out, let the apotome be called a **fourth apotome**;

5. if the annex be so commensurable, a **fifth**;

6. and, if neither, a **sixth**.

BOOK XI. Definitions.

1. A **solid** is that which has length, breadth, and depth.

2. An **extremity of a solid** is a **surface**.

3. A **straight line** is **at right angles to a plane**, when it makes right angles with all the straight lines which meet it and are in the plane.

4. A **plane** is **at right angles to a plane** when the straight lines drawn, in one of the planes, at right angles to the common section of the planes are at right angles to the remaining plane.

5. The **inclination** of **a straight line to a plane** is, assuming a perpendicular drawn from the extremity of the straight line which is elevated above the plane to the plane, and a straight line joined from the point thus arising to the extremity of the straight line which is in the plane, the angle contained by the straight line so drawn and the straight line standing up.

6. The **inclination** of **a plane to a plane** is the acute angle contained by the straight lines drawn at right angles to the common section at the same point, one in each, of the planes.

7. A plane is said to be **similarly inclined** to a plane as another is to another when the said angles of the inclinations are equal, to one another.

8. **Parallel planes** are those which do not meet.

9. **Similar solid figures** are those contained by similar planes equal in multitude.

10. **Equal and similar solid figures** are those contained by similar planes equal in multitude and in magnitude.

11. A **solid angle** is the inclination constituted by more than two lines which meet one another and are not in the same surface, towards all the lines.

Otherwise: a **solid angle** is that which is contained by more than two plane angles which are not in the same plane and are constructed to one point.

12. A **pyramid** is a solid figure, contained by planes, which is constructed from one plane to one point.

13. A **prism** is a solid figure contained by planes two of which, namely those which are opposite, are equal, similar and parallel, while the rest are parallelograms.

14. When, the diameter of a semicircle remaining fixed, the semicircle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **sphere**.

15. The **axis of the sphere** is the straight line which remains fixed and about which the semicircle is turned.

16. The **centre of the sphere** is the same as that of the semicircle.

17. A **diameter of the** sphere is any straight line drawn through the centre and terminated in both directions by the surface of the sphere.

18. When, one side of those about the right angle in a right-angled triangle remaining fixed, the triangle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **cone**.

And, if the straight line which remains fixed be equal, to the remaining side about the right angle which is carried round, the cone will be **right-angled**; if less, **obtuse-angled**; and if greater, **acute-angled**.

19. The **axis of the cone** is the straight line which remains fixed and about which the triangle is turned.

20. And the **base** is the circle described by the straight line which is carried round.

21. When, one side of those about the right angle in a rectangular parallelogram remaining fixed, the parallelogram is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **cylinder**.

22. The **axis of the cylinder** is the straight line which remains fixed and about which the parallelogram is turned.

23. And the **bases** are the circles described by the two sides opposite to one another which are carried round.

24. **Similar cones and cylinders** are those in which the axes and the diameters of the bases are proportional.

25. A **cube** is a solid figure contained by six equal squares.

26. An **octahedron** is a solid figure contained by eight equal and equilateral triangles.

27. An **icosahedron** is a solid figure contained by twenty equal and equilateral triangles.

28. A **dodecahedron** is a solid figure contained by twelve equal, equilateral, and equiangular pentagons.

BOOK I. Definitions.

This is going to be difficult, trying to word it so as to instill the fact that this grammar is wholly metaphorical. Not something one does every day. What has to be added is the knowledge that this is a written grammar which requires something to record the motion of the hand, such as pen and paper, and the tools to write it with.

A thing is defined as a relative within correlatives and is written by the aid of a straightedge.

1. A **point** is a thing's correlative.

2. A **line** is a thing's relative difference.

3. **Points**, or correlatives, or noun are the container of the relative, or verb.

Here is the problem, one goes from a general notion of a thing, all things, then one wants to specifically denote things whose relative difference is no longer singular, but plural, i.e., from thinking linearly to proportionally. One relative, to two relatives and more, it is an exponential leap in processing.

5. A **two-dimensional** figure has two relative differences.

6. The correlatives of a two-dimensional object are one-dimensional lines, or things.

8. A **two-dimensional ratio** can be expressed as the inclination to one another of two lines in a figure which meet one another and do not lie in a straight line.

10. When a straight line set up on a straight line makes the adjacent angles equal, to one another, each, of the equal angles is **right**, and the straight line standing on the other is called a **perpendicular** to that on which it stands.

11. An **obtuse angle** is an angle greater than a right angle.

12. An **acute angle** is an angle less than a right angle.

13. A **boundary** is that which is an extremity, limit, correlative, noun of anything.

14. A **figure** is the relative difference, noun contained by any boundary or boundaries, correlatives, noun.

15. A **circle** is the reciprocal of one line such that all the results produced by multiplication and division of its verb, or relative, that have one point in common; the equation for the circle is given a unit, divided into x and y, the circle is the loci of the square root of x and y for any point on the line or any other operation of using lines which produces the same loci which is expressible as this one. In short, the compass affords the complete induction and deduction of a given line.

16. And the point is called the **center** of the circle.

17. A **diameter** of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line, also, bisects the circle.

18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the circle.

19. **Rectilineal figures** are those which are contained by straight lines, **trilateral** figures being those contained by three, **quadrilateral** those contained by four, and **multilateral** those contained by more than four straight lines.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.

21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.

22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal, to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.

23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

BOOK II. Definitions.

1. Any rectangular parallelogram is said to be **contained** by the two straight lines containing the right angle.

2. And in any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a **gnomon**.

BOOK III. Definitions.

1. **Equal circles** are those the diameters of which are equal, or the radii of which are equal.

2. A straight line is said to **touch a circle** which, meeting the circle and being produced, does not cut the circle.

3. **Circles** are said to **touch one another** which, meeting one another, do not cut one another.

4. In a circle straight lines are said **to be equally distant from the center** when the perpendiculars drawn to them from the center are equal.

5. And that straight line is said to be **at a greater distance** on which the greater perpendicular falls.

6. A **segment of a circle** is the figure contained by a straight line and a circumference of a circle.

7. An **angle of a segment** is that contained by a straight line and a circumference of a circle.

8. An **angle in a segment** is the angle which, when a point is taken on the circumference of the segment and straight lines are joined from it to the extremities of the straight line which is the **base of the segment**, is contained by the straight lines so joined.

9. And, when the straight lines containing the angle cut off a circumference, the angle is said to **stand upon** that circumference.

10. A **sector of a circle** is the figure which, when an angle is constructed at the center of the circle, is contained by the straight lines containing the angle and the circumference cut off by them.

11. **Similar segments of circles** are those which admit equal angles, or in which the angles are equal, to one another.

BOOK IV. Definitions.

1. A rectilineal figure is said to be **inscribed in a rectilineal figure** when the respective angles of the inscribed figure lie on the respective sides of that in which it is inscribed.

2. Similarly, a figure is said to be **circumscribed about a figure** when the respective sides of the circumscribed figure pass through the respective angles of that about which it is circumscribed.

3. A rectilineal figure is said to be **inscribed in a circle** when each angle of the inscribed figure lies on the circumference of the circle.

4. A rectilineal figure is said to be **circumscribed about a circle**, when each side of the circumscribed figure touches the circumference of the circle.

5. Similarly a circle is said to be **inscribed in a figure** when the circumference of the circle touches each side of the figure in which it is inscribed.

6. A circle is said to be **circumscribed about a figure** when the circumference of the circle passes through each angle of the figure about which it is circumscribed.

7. A straight line is said to be **fitted into a circle** when its extremities are on the circumference of the circle.

BOOK V. Definitions.

1. A magnitude is a **part** of a magnitude, the less of the greater, when it measures the greater.

2. The greater is a **multiple** of the less when it is measured by the less.

3. A **ratio** is a sort of relation in respect of size between two magnitudes of the same kind.

4. Magnitudes are said to **have a ratio** to one another which are capable, when multiplied, of exceeding one another.

5. Magnitudes are said to **be in the same ratio**, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal, to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

6. Let magnitudes which have the same ratio be called **proportional**.

7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to **have a greater ratio** to the second than the third has to the fourth.

8. A proportion in three terms is the least possible.

9. When three magnitudes are proportional, the first is said to have to the third the **duplicate ratio** of that which it has to the second.

10. When four magnitudes are < continuously > proportional, the first is said to have to the fourth the **triplicate ratio** of that which it has to the second, and so on continually, whatever be the proportion.

11. The term **corresponding magnitudes** is used of antecedents in relation to antecedents, and of consequents in relation to consequents.

12. **Alternate ratio** means taking the antecedent in relation to the antecedent and the consequent in relation to the consequent.

13. **Inverse ratio** means taking the consequent as antecedent in relation to the antecedent as consequent.

14. **Composition of a ratio** means taking the antecedent together with the consequent as one in relation to the consequent by itself.

15. **Separation of a ratio** means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself.

16. **Conversion of a ratio** means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

17. A ratio ***ex aequali*** arises when, there being several magnitudes and another set equal, to them in multitude which taken two and two are in the same proportion, as the first is to the last among the first magnitudes, so is the first to the last among the second magnitudes;

Or, in other words, it means taking the extreme terms by virtue of the removal of the intermediate terms.

18. A **perturbed proportion** arises when, there being three magnitudes and another set equal, to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to A third among the first magnitudes, so is a third to the antecedent among the second magnitudes.

BOOK VI. Definitions.

1. **Similar rectilineal figures** are such as have their angles severally equal and the sides about the equal angles proportional.

[2. **Reciprocally related** figures. See note.]

3. A straight line is said to have been **cut in extreme and mean ratio** when, as the whole line is to the greater segment, so is the greater to the less.

4. The **height** of any figure is the perpendicular drawn from the vertex to the base.

BOOK VII. Definitions.

1. An **unit** is that by virtue of which each, of the things that exist is called one.

2. A **number** is a multitude composed of units.

3. A number is **a part** of a number, the less of the greater, when it measures the greater;

4. but **parts** when it does not measure it.

5. The greater number is a **multiple** of the less when it is measured by the less.

6. An **even number** is that which is divisible into two equal parts.

7. An **odd number** is that which is not divisible into two equal parts, or that which differs by an unit from an even number.

8. An **even-times even number** is that which is measured by an even number according to an even number.

9. An **even-times odd number** is that which is measured by an even number according to an odd number.

10. An **odd-times odd number** is that which is measured by an odd number according to an odd number.

11. A **prime number** is that which is measured by an unit alone.

12. Numbers **prime to one another** are those which are measured by an unit alone as a common measure.

13. A **composite number** is that which is measured by some number.

14. Numbers **composite to one another** are those which are measured by some number as a common measure.

15. A number is said to **multiply** a number when that which is multiplied is added to itself as many times as there are units in the other, and thus some number is produced.

16. And, when two numbers having multiplied one another make some number, the number so produced is called **plane**, and its **sides** are the numbers which have multiplied one another.

17. And, when three numbers having multiplied one another make some number, the number so produced is **solid**, and its **sides** are the numbers which have multiplied one another.

18. A **square number** is equal multiplied by equal, or a number which is contained by two equal numbers.

19. And a **cube** is equal multiplied by equal and again by equal, or a number which is contained by three equal numbers.

20. Numbers are **proportional** when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

21. **Similar plane** and **solid** numbers are those which have their sides proportional.

22. A **perfect number** is that which is equal, to its own parts.

BOOK X. Definitions 10-1.

1. Those magnitudes are said to be **commensurable** which are measured by the same measure, and those **incommensurable** which cannot have any common measure.

2. Straight lines are **commensurable in square** when the squares on them are measured by the same area, and **incommensurable in square** when the squares on them cannot possibly have any area as a common measure.

3. With these hypotheses, it is proved that there exist straight lines infinite in multitude which are commensurable and incommensurable respectively, some in length only, and others in square also, with an assigned straight line. Let then the assigned straight line be called **rational**, and those straight lines which are commensurable with it, whether in length and in square or in square, only, **rational**, but those which are incommensurable with it **irrational**.

4. And let the square, on the assigned straight line be called **rational** and those areas which are commensurable with it **rational**, but those which are incommensurable with it **irrational**, and the straight lines which produce them irrational, that is, in case the areas are squares, the sides themselves, but in case they are any other rectilineal figures, the straight lines on which are described squares equal, to them.

Definitions 10-2.

1. Given a rational straight line and a binomial, divided into its terms, such that the square, on the greater term is greater than the square, on the lesser by the square, on a straight line commensurable in length with the greater, then, if the greater term be commensurable in length with the rational straight line set out, let the whole be called a **first binomial** straight line;

2. but if the lesser term be commensurable in length with the rational straight line set out, let the whole be called a **second binomial**;

3. and if neither of the terms be commensurable in length with the rational straight line set out, let the whole be called a **third binomial**.

4. Again, if the square, on the greater term be greater than the square, on the lesser by the square, on a straight line incommensurable in length with the greater, then, if the greater term be commensurable in length with the rational straight line set out, let the whole be called a **fourth binomial**;

5. if the lesser, a **fifth binomial**;

6. and if neither, a **sixth binomial**.

Definitions 10-3.

1. Given a rational straight line and an apotome, if the square, on the whole be greater than the square, on the annex by the square, on a straight line commensurable in length with the whole, and the whole be commensurable in

length with the rational straight line set out, let the apotome be called a **first apotome**.

2. But if the annex be commensurable in length with the rational straight line set out, and the square, on the whole be greater than that on the annex by the square, on a straight line commensurable with the whole, let the apotome be called a **second apotome**.

3. But if neither be commensurable in length with the rational straight line set out, and the square, on the whole be greater than the square, on the annex by the square, on a straight line commensurable with the whole, let the apotome be called a **third apotome**.

4. Again, if the square, on the whole be greater than the square, on the annex by the square, on a straight line incommensurable with the whole, then, if the whole be commensurable in length with the rational straight line set out, let the apotome be called a **fourth apotome**;

5. if the annex be so commensurable, a **fifth**;

6. and, if neither, a **sixth**.

BOOK XI. Definitions.

1. A **solid** is that which has length, breadth, and depth.

2. An **extremity of a solid** is a **surface**.

3. A **straight line** is **at right angles to a plane**, when it makes right angles with all the straight lines which meet it and are in the plane.

4. A **plane** is **at right angles to a plane** when the straight lines drawn, in one of the planes, at right angles to the common section of the planes are at right angles to the remaining plane.

5. The **inclination** of **a straight line to a plane** is, assuming a perpendicular drawn from the extremity of the straight line which is elevated above the plane to the plane, and a straight line joined from the point thus arising to the extremity of the straight line which is in the plane, the angle contained by the straight line so drawn and the straight line standing up.

6. The **inclination** of **a plane to a plane** is the acute angle contained by the straight lines drawn at right angles to the common section at the same point, one in each, of the planes.

7. A plane is said to be **similarly inclined** to a plane as another is to another when the said angles of the inclinations are equal, to one another.

8. **Parallel planes** are those which do not meet.

9. **Similar solid figures** are those contained by similar planes equal in multitude.

10. **Equal and similar solid figures** are those contained by similar planes equal in multitude and in magnitude.

11. A **solid angle** is the inclination constituted by more than two lines which meet one another and are not in the same surface, towards all the lines.

Otherwise: a **solid angle** is that which is contained by more than two plane angles which are not in the same plane and are constructed to one point.

12. A **pyramid** is a solid figure, contained by planes, which is constructed from one plane to one point.

13. A **prism** is a solid figure contained by planes two of which, namely those which are opposite, are equal, similar and parallel, while the rest are parallelograms.

14. When, the diameter of a semicircle remaining fixed, the semicircle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **sphere**.

15. The **axis of the sphere** is the straight line which remains fixed and about which the semicircle is turned.

16. The **center of the sphere** is the same as that of the semicircle.

17. A **diameter of the** sphere is any straight line drawn through the center and terminated in both directions by the surface of the sphere.

18. When, one side of those about the right angle in a right-angled triangle remaining fixed, the triangle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **cone**.

And, if the straight line which remains fixed be equal, to the remaining side about the right angle which is carried round, the cone will be **right-angled**; if less, **obtuse-angled**; and if greater, **acute-angled**.

19. The **axis of the cone** is the straight line which remains fixed and about which the triangle is turned.

20. And the **base** is the circle described by the straight line which is carried round.

21. When, one side of those about the right angle in a rectangular parallelogram remaining fixed, the parallelogram is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a **cylinder**.

22. The **axis of the cylinder** is the straight line which remains fixed and about which the parallelogram is turned.

23. And the **bases** are the circles described by the two sides opposite to one another which are carried round.

24. **Similar cones and cylinders** are those in which the axes and the diameters of the bases are proportional.

25. A **cube** is a solid figure contained by six equal squares.

26. An **octahedron** is a solid figure contained by eight equal and equilateral triangles.

27. An **icosahedron** is a solid figure contained by twenty equal and equilateral triangles.

28. A **dodecahedron** is a solid figure contained by twelve equal, equilateral, and equiangular pentagons.

Book 1. Propositions.

Book 1 Proposition 1.

On a given line to construct an equilateral triangle.

Book 1 Proposition 2.

To construct at a given point, a line equal to a given line.

Book 1 Proposition 3.

Given two unequal lines, to subtract the less from the greater.

Book 1 Proposition 4.

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal lines equal, they will, also, have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

Book 1 Proposition 5.

In isosceles triangles the angles at the base are equal to one another, and, if the equal lines be produced further, the angles under the base will be equal to one another.

Book 1 Proposition 6.

If in a triangle two angles be equal to one another, the sides which subtend the equal angles will, also, *be equal to one another.*

Book 1 Proposition 7.

Given two lines constructed on a line (*from its extremities*) *and meeting in a point, there cannot be constructed on the same line* (*from its extremities*), *and on the same side of it, two other lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.*

Book 1 Proposition 8.

If two triangles have the two sides equal to two sides respectively, and have, also, *the base equal to the base, they will,* also, *have the angles equal which are contained by the equal lines.*

Book 1 Proposition 9.

To bisect a given angle.

Book 1 Proposition 10.

To bisect a given line.

Book 1 Proposition 11.

To draw a line at right angles to a given line from a given point on it.

Book 1 Proposition 12.

To a given line, from a given point which is not on it, to draw a perpendicular line.

Book 1 Proposition 13.

If a line set up on a line make angles, it will make either two right angles or angles equal to two right angles.

Book 1 Proposition 14.

If with any line, and at a point on it, two lines not lying on the same side make the adjacent angles equal to two right angles, the two lines will be in a line with one another.

Book 1 Proposition 15.

If two lines divide one another, they make the vertical angles equal to one another.

Book 1 Proposition 16.

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.

Book 1 Proposition 17.

In any triangle, two angles taken together in any manner are less than two right angles.

Book 1 Proposition 18.

In any triangle the greater side subtends the greater angle.

Book 1 Proposition 19.

In any triangle the greater angle is subtended by the greater side.

Book 1 Proposition 20.

In any triangle two sides taken together in any manner are greater than the remaining one.

Book 1 Proposition 21.

If on one of the sides of a triangle, from its extremities, there be constructed two lines meeting within the triangle, the lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater angle.

Book 1 Proposition 22.

Out of three lines, which are equal to three given lines, to construct a triangle: thus, it is necessary that two of the lines taken together in any manner should be greater than the remaining one.

Book 1 Proposition 23.

On a given *line and at a point on it to construct an angle equal to* a given *angle.*

Book 1 Proposition 24.

If two triangles have the two sides equal to two sides respectively, but have the one of the angles contained by the equal lines greater than the other, they will, also, *have the base greater than the base.*

Book 1 Proposition 25.

If two triangles have the two sides equal to two sides respectively, but have the base greater than the base, they will, also, *have the one of the angles contained by the equal lines greater than the other.*

Book 1 Proposition 26.

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will, also, *have the remaining sides equal to the remaining sides and the remaining angle to the remaining angle.*

Book 1 Proposition 27.

If a line dividing two lines make the alternate angles equal to one another, the lines will be parallel to one another.

Book 1 Proposition 28.

If a line dividing two lines make the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the lines will be parallel to one another.

Book 1 Proposition 29.

A line dividing parallel lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

Book 1 Proposition 30.

Lines parallel to the same line are, also, *parallel to one another.*

Book 1 Proposition 31.

Through a given *point to draw a line parallel to* a given *line.*

Book 1 Proposition 32.

*In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angle*s.

Book 1 Proposition 33.

The lines joining equal and parallel lines (*at the extremities which are*) *in the same directions* (*respectively*) *are themselves*, also, *equal and parallel.*

Book 1 Proposition 34.

In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.

Book 1 Proposition 35.

Parallelograms which are on the same base and in the same parallels are equal to one another.

Book 1 Proposition 36.

Parallelograms which are on equal bases and in the same parallels are equal to one another.

Book 1 Proposition 37.

Triangles which are on the same base and in the same parallels are equal to one another.

Book 1 Proposition 38.

Triangles which are on equal bases and in the same parallels are equal to one another.

Book 1 Proposition 39.

Equal triangles which are on the same base and on the same side are, also, *in the same parallels.*

Book 1 Proposition 40.

Equal triangles which are on equal bases and on the same side are, also, *in the same parallels.*

Book 1 Proposition 41.

If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle.

Book 1 Proposition 42.

To construct in a given angle a parallelogram equal to a given triangle.

Book 1 Proposition 43.

In any parallelogram the complements of the parallelograms about the diameter are equal to one another.

Book 1 Proposition 44.

To a given line to apply, in a given angle, a parallelogram equal to a given triangle.

Book 1 Proposition 45.

To construct, in a given angle, a parallelogram equal to a given rectilineal figure.

Book 1 Proposition 46.

On a given line to describe a square.

Book 1 Proposition 47.

In right-angled triangles the square, on the side subtending the right angle is equal to the squares on the sides containing the right angle.

Book 1 Proposition 48.

If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.

Book 2. Propositions.

Book 2 Proposition 1.

If there be two lines, and one of them be divided into any line of segments whatever, the rectangle contained by the two lines is equal to the rectangles contained by the undivided line and each of the segments.

Book 2 Proposition 2.

If a line be divided at random, the rectangle contained by the whole and both of the segments is equal to the square on the whole.

Book 2 Proposition 3.

If a line be divided at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.

Book 2 Proposition 4.

If a line be divided at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

Book 2 Proposition 5.

If a line be divided into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the line between the points of section is equal to the square on the half.

Book 2 Proposition 6.

If a line be bisected and a line be added to it in a line, the rectangle contained by the whole with the added line and the added line together with the square on the half is equal to the square on the line made up of the half and the added line.

Book 2 Proposition 7.

If a line be divided at random, the square on the whole and that on one of the segments both together are equal to twice the rectangle contained by the whole and the said segment and the square on the remaining segment.

Book 2 Proposition 8.

If a line be divided at random, four times the rectangle contained by the whole and one of the segments together with the square on the remaining segment is equal to the square described on the whole and the aforesaid segment as on one line.

Book 2 Proposition 9.

If a line be divided into equal and unequal segments, the squares on the unequal segments of the whole are double of the square, on the half and of the square on the line between the points of section.

Book 2 Proposition 10.

If a line be bisected, and a line be added to it in a line, the square, on the whole with the added line and the square, on the added line both together are double of the square, on the half and of the square described on the line made up of the half and the added line as on one line.

Book 2 Proposition 11.

To divide a given line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

Book 2 Proposition 12.

In obtuse-angled triangles, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the line subtract outside by the perpendicular towards the obtuse angle.

Book 2 Proposition 13.

In acute-angled triangles the square, on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the line subtract within by the perpendicular towards the acute angle.

Book 2 Proposition 14.

To construct a square equal to a given *rectilineal figure.*

Book 3. Propositions.

Book 3 Proposition 1.

To find the center of a given circle.

Book 3 Proposition 2.

If on the circumference of a circle two points be taken at random, the line joining the points will fall within the circle.

Book 3 Proposition 3.

If in a circle, a line through the center bisects a line not through the center, it, also, *divides it at right angles; and if it divides it at right angles, it,* also, *bisects it.*

Book 3 Proposition 4.

If in a circle two lines divide one another which are not through the center, they do not bisect one another.

Book 3 Proposition 5.

If two circles divide one another, they will not have the same center.

Book 3 Proposition 6.

If two circles have one point in common with one another, they will not have the same center.

Book 3 Proposition 7.

If, on the diameter of a circle, a point be taken which is not the center of the circle, and from the point lines fall upon the circle, that will be greatest on which the center is, the remainder of the same diameter will be least, and of the rest the nearer to the line through the center is always greater than the more remote, and only two equal lines will fall from the point on the circle, one on each side of the least line.

Book 3 Proposition 8.

If a point be taken outside a circle and from the point lines be drawn through to the circle, one of which is through the center and the others are drawn at random, then, of the lines which fall on the concave circumference, that through the center is greatest, while of the rest the nearer to that through the center is always greater than the more remote, but, of the lines dividing the convex circumference, that between the point and the diameter is least, while of the rest

the nearer to the least is always less than the more remote, and only two equal lines will fall on the circle from the point, one on each side of the least.

Book 3 Proposition 9.

If a point be taken within a circle, and more than two equal lines fall from the point on the circle, the point taken is the center of the circle.

Book 3 Proposition 10.

A circle does not divide a circle at more points than two.

Book 3 Proposition 11.

If two circles have one point in common with one another internally, and their centers be taken, the line joining their centers, if it be, also, *produced, will fall on the point of contact of the circles.*

Book 3 Proposition 12.

If two circles have one point in common with one another externally, the line joining their centers will pass through the point of contact.

Book 3 Proposition 13.

A circle does not have one point in common with a circle at more points than one, whether it have one point in common with it internally or externally.

Book 3 Proposition 14.

In a circle, equal lines are equally distant from the center, and those which are equally distant from the center are equal to one another.

Book 3 Proposition 15.

Of lines in a circle the diameter is greatest, and of the rest the nearer to the center is always greater than the more remote.

Book 3 Proposition 16.

The line drawn at right angles to the diameter of a circle *from its extremity will fall outside the circle, and into the space between the line and the circumference another line cannot be interposed; further the angle of the semicircle is greater, and the remaining angle less, than any acute angle.*

Book 3 Proposition 17.

From a given point to draw a line touching a given circle.

Book 3 Proposition 18.

If a line has one point in common with a circle, and a line be described from the center to the point of contact, the line so described will be perpendicular to the tangent.

Book 3 Proposition 19.

If a line has one point in common with a circle, and from the point of contact a line be drawn at right angles to the tangent, the center of the circle will be on the line so drawn.

Book 3 Proposition 20.

In a circle the angle at the center is double of the angle at the circumference, when the angles have the same circumference as base.

Book 3 Proposition 21.

In a circle, the angles in the same segment are equal to one another.

Book 3 Proposition 22.

The opposite angles of quadrilaterals in circles are equal to two right angles.

Book 3 Proposition 23.

On the same line there cannot be constructed two similar and unequal segments of circles on the same side.

Book 3 Proposition 24.

Similar segments of circles on equal lines are equal to one another.

Book 3 Proposition 25.

Given a segment of a circle, to describe the complete circle of which it is a segment.

Book 3 Proposition 26.

In equal circles equal angles stand on equal circumferences, whether they stand at the centers or at the circumferences.

Book 3 Proposition 27.

In equal circles, angles standing on equal circumferences are equal to one another, whether they stand at the centers or at the circumferences.

Book 3 Proposition 28.

In equal circles equal lines subtract equal circumferences, the greater equal to the greater and the less to the less.

Book 3 Proposition 29.

In equal circles equal circumferences are subtended by equal lines.

Book 3 Proposition 30.

To bisect a given circumference.

Book 3 Proposition 31.

In a circle, the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle; and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.

Book 3 Proposition 32.

If a line have one point in common with a circle, and from the point of contact there be drawn across, in the circle, a line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.

Book 3 Proposition 33.

On a given line to describe a segment of a circle admitting an angle equal to a given angle.

Book 3 Proposition 34.

From a given circle to subtract a segment admitting an angle equal to a given angle.

Book 3 Proposition 35.

If in a circle two lines divide one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other.

Book 3 Proposition 36.

If a point be taken outside a circle and from it there fall on the circle two lines, and if one of them divide the circle and the other have one point in common with it, the rectangle contained by the whole of the line which cuts the circle and the

line intercepted on it outside between the point and the convex circumference will be equal to the square, on the tangent.

Book 3 Proposition 37.

If a point be taken outside a circle and from the point there fall on the circle two lines, if one of them divide the circle, and the other fall on it, and if further the rectangle contained by the whole of the line which cuts the circle and the line intercepted on it outside between the point and the convex circumference be equal to the square, on the line which falls on the circle, the line which falls on it will have one point in common with the circle.

If a correlative is asserted outside of a circle, and from that point we construct two lines to the circle, one of them dividing it and the other tangent to it, and if the rectangle contained by the whole of the line which divides the circle and the tangent line from their common origin, etc.,

Book 4. Propositions

Book 4 Proposition 1.

Into a given circle copy a line equal to a given line which is not greater than the diameter of the circle.

Book 4 Proposition 2.

In a given circle to inscribe a triangle equiangular with a given triangle.

Book 4 Proposition 3.

About a given circle to circumscribe a triangle equiangular with a given triangle.

Book 4 Proposition 4.

In a given triangle to inscribe a circle.

Book 4 Proposition 5.

About a given triangle to circumscribe a circle.

Book 4 Proposition 6.

In a given circle to inscribe a square.

Book 4 Proposition 7.

About a given circle to circumscribe a square.

Book 4 Proposition 8.

In a given square to inscribe a circle.

Book 4 Proposition 9.

About a given square to circumscribe a circle.

Book 4 Proposition 10.

To construct an isosceles triangle having each, of the angles at the base double of the remaining one.

Book 4 Proposition 11.

In a given circle to inscribe an equilateral and equiangular pentagon.

Book 4 Proposition 12.

About a given circle to circumscribe an equilateral and equiangular pentagon.

Book 4 Proposition 13.

In a given pentagon, to inscribe a circle.

Book 4 Proposition 14.

About a given pentagon, to circumscribe a circle.

Book 4 Proposition 15.

In a given circle to inscribe hexagon.

Book 4 Proposition 16.

Divide a circle, into a fifteen-angled figure which shall be both equilateral and equiangular.

Book 5. Propositions.

Book 5 Proposition 1.

If there be any line of lines whatever which are, respectively, equimultiples of any lines equal in multitude, then, whatever multiple one of the lines is of one, that multiple, also, *will all be of all.*

Book 5 Proposition 2.

If a first line be the same multiple of a second that a third is of a fourth, and a fifth, also, *be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will,* also, *be the same multiple of the second that the sum of the third and sixth is of the fourth.*

Book 5 Proposition 3.

If a first line be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then, also, equally *the lines taken will be equimultiples respectively, the one of the second and the other of the fourth.*

Book 5 Proposition 4.

If a first line has to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will, also, *have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.*

Book 5 Proposition 5.

If a line be the same multiple of a line that a part subtracted is of a part subtracted, the remainder will, also, *be the same multiple of the remainder that the whole is of the whole.*

Book 5 Proposition 6.

If two lines be equimultiples of two lines, and any lines subtracted from them be equimultiples of the same, the remainders, also, *are either equal to the same or equimultiples of them.*

Book 5 Proposition 7.

Equal lines have to the same the same ratio, as, also, *has the same to equal lines.*

Book 5 Proposition 8.

Of unequal lines, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater.

Book 5 Proposition 9.

Lines which have the same ratio to the same are equal to one another; and lines to which the same has the same ratio are equal.

Book 5 Proposition 10.

Of lines which have a ratio to the same, that which has a greater ratio is greater; and that to which the same has a greater ratio is less.

Book 5 Proposition 11.

Ratios which are the same with the same ratio are, also, *the same with one another.*

Book 5 Proposition 12.

If any number of lines be proportional as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents.

Book 5 Proposition 13.

If a first line have to a second the same ratio as a third to a fourth, and the third have to the fourth a greater ratio than a fifth has to a sixth, the first will, also, *have to the second a greater ratio than the fifth to the sixth.*

Book 5 Proposition 14.

If a first line have to a second the same ratio as a third has to a fourth, and the first be greater than the third, the second will, also, *be greater than the fourth; if equal, equal; and if less, less.*

Book 5 Proposition 15.

Parts have the same ratio as the same multiples of them taken in corresponding order.

Book 5 Proposition 16.

If four lines be proportional, they will, also, *be proportional alternately.*

Book 5 Proposition 17.

If lines be proportional taken jointly, *they will,* also, *be proportional* taken separately.

Book 5 Proposition 18.

If lines be proportional taken separately, *they will,* also, *be proportional* taken jointly.

Book 5 Proposition 19.

If, as a whole is to a whole, so is a part subtracted to a part subtracted, the remainder will, also, *be to the remainder as whole to whole.*

Book 5 Proposition 20.

If there be three lines, and others equal to them in multitude, which taken two and two are in the same ratio, and if equally *the first be greater than the third, the fourth will,* also, *be greater than the sixth; if equal, equal; and, if less, less.*

Book 5 Proposition 21.

If there be three lines, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, then, if equally *the first line is greater than the third, the fourth will,* also, *be greater than the sixth; if equal, equal; and if less, less.*

Book 5 Proposition 22.

If there be any line of lines whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will, also, *be in the same ratio* equally.

Book 5 Proposition 23.

If there be three lines, and others equal to them in multitude, which taken two and two together are in the same ratio, and the proportion of them be perturbed, they will, also, *be in the same ratio* equally.

Book 5 Proposition 24:

If a first line have to a second the same ratio as a third has to a fourth, and, also, *a fifth have to the second the same ratio as a sixth to the fourth, the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.*

Book 5 Proposition 25:

If four lines be proportional, the greatest and the least are greater than the remaining two.

Book 6. Propositions.

Book 6 Proposition 1.

Triangles and parallelograms which are under the same height are to one another as their bases.

Book 6 Proposition 2.

If a line be drawn parallel to one of the sides of a triangle, it will divide the sides of the triangle proportionally; and, if the sides of the triangle be divided

proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.

Book 6 Proposition 3.

If an angle of a triangle be bisected and the line cutting the angle divide the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the line joined from the vertex to the point of section will bisect the angle of the triangle.

Book 6 Proposition 4.

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

Book 6 Proposition 5.

If two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Book 6 Proposition 6.

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Book 6 Proposition 7.

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.

Book 6 Proposition 8.

If, in a right-angled triangle, a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.

Book 6 Proposition 9.

From a given line to subtract a prescribed part.

Repeat of proposition Book 1 Proposition 3.

Book 6 Proposition 10.

To divide a given uncut line similarly to a given divide line.

Book 6 Proposition 11.

To two given lines to find a third proportional.

Book 6 Proposition 12.

To three given lines to find a fourth proportional.

Book 6 Proposition 13.

To two given lines to find a mean proportional.

Book 6 Proposition 14.

In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Book 6 Proposition 15.

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

Book 6 Proposition 16.

If four lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four lines will be proportional.

Book 6 Proposition 17.

If three lines be proportional, the rectangle contained by the extremes is equal to the square, on the mean; and, if the rectangle contained by the extremes be equal to the square, on the mean, the three lines will be proportional.

Book 6 Proposition 18.

On a given line to describe a rectilineal figure similar and similarly situated to a given rectilineal figure.

Book 6 Proposition 19.

Similar triangles are to one another in the duplicate ratio of the corresponding sides.

Book 6 Proposition 20.

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

Book 6 Proposition 21.

Figures which are similar to the same rectilineal figure are, also, *similar to one another.*

Book 6 Proposition 22.

If four lines be proportional, the rectilineal figures similar and similarly described upon them will, also, *be proportional*; *and, if the rectilineal figures similar and similarly described upon them be proportional, the lines will themselves*, also, *be proportional.*

Book 6 Proposition 23.

Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides.

Book 6 Proposition 24.

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

Book 6 Proposition 25.

To construct one and the same figure similar to a given rectilineal figure and equal to another given rectilineal figure.

Book 6 Proposition 26.

If from a parallelogram there be taken away a parallelogram similar and similarly situated to the whole and having a common angle with it, it is about the same diameter with the whole.

Book 6 Proposition 27.

Of all the parallelograms applied to the same line and deficient by parallelogrammic figures similar and similarly situated to that described on the half of the line, that parallelogram is greatest which is applied to the half of the line and is similar to the defect.

Book 6 Proposition 28.

To a given line to apply a parallelogram equal to a given rectilineal figure and deficient by a parallelogrammic figure similar to a given one: *thus, the given rectilineal figure must not be greater than the parallelogram described on the half of the line and similar to the defect.*

Book 6 Proposition 29.

To a given line to apply a parallelogram equal to a given rectilineal figure and exceeding by a parallelogrammic figure similar to a given one.

Book 6 Proposition 30.

To divide a given line in extreme and mean ratio.

See Book 2 proposition 11, duplicate proposition.

Book 6 Proposition 31.

In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

Book 6 Proposition 32.

If two triangles having two sides proportional to two sides be placed together at one angle so that their corresponding sides are, also, *parallel, the remaining sides of the triangles will be in a line.*

Book 6 Proposition 33.

In equal circles, angles have the same ratio as the circumferences on which they stand, whether they stand at the centers or at the circumferences.

Now we have a conflict of terminology. Are angles, as defined a ratio, or do they have a ratio? Bad grammar. It is an attempted repeat.

Book 7. Propositions.

Book 7 Proposition 1.

Two unequal lines being set out, and the less being continually subtracted in turn from the greater, if the line which is left never measures the one before it until a unit is left, the original lines will be prime to one another.

Book 7 Proposition 2.

Given two lines not prime to one another, to find their greatest common measure.

Book 7 Proposition 3.

Given three lines not prime to one another, to find their greatest common measure.

Book 7 Proposition 4.

Any line is either a part or parts of any line, the less of the greater.

Book 7 Proposition **5.**

If a line be a part of a line, and another be the same part of another, the sum will, also, *be the same part of the sum that the one is of the one.*

Book 7 Proposition 6.

If a line be parts of a line, and another be the same parts of another, the sum will, also, *be the same parts of the sum that the one is of the one.*

Book 7 Proposition 7.

If a line be that part of a line, which a line subtracted is of a line subtracted, the remainder will, also, *be the same part of the remainder that the whole is of the whole.*

Book 7 Proposition 8.

If a line be the same parts of a line that a line subtracted is of a line subtracted, the remainder will, also, *be the same parts of the remainder that the whole is of the whole.*

Book 7 Proposition 9.

If a line be a part of a line, and another be the same part of another, alternately also, whatever part or parts the first is of the third, the same part, or the same parts, will the second, also, *be of the fourth.*

Book 7 Proposition 10.

If a line be parts of a line, and another be the same parts of another, alternately also, whatever parts or part the first is of the third, the same parts or the same part will the second, also, *be of the fourth.*

Book 7 Proposition 11.

If, as whole is to whole, so is a line subtracted to a line subtracted, the remainder will, also, *be to the remainder as whole to whole.*

Book 7 Proposition 12.

If there be as many lines as we please in proportion, then, as one of the antecedents is to one of the consequents, so are all the antecedents to all the consequents.

Book 7 Proposition 13.

If four lines be proportional, they will, also, *be proportional alternately.*

Book 7 Proposition 14.

If there be as many lines as we please, and others equal to them in multitude, which taken two and two are in the same ratio, they will, also, *be in the same ratio* equally.

Book 7 Proposition 15.

If a unit measure any line, and another line measure any other line the same line of times, alternately also, the unit will measure the third line the same line of times that the second measures the fourth.

Book 7 Proposition 16.

If two lines, by multiplying one another, make certain lines, the lines so produced will be equal to one another.

Book 7 Proposition 17.

If a line, by multiplying two lines, make certain lines, the lines so produced will have the same ratio as the lines multiplied.

Book 7 Proposition 18.

If two lines, by multiplying any line, make certain lines, the lines so produced will have the same ratio as the multipliers.

Book 7 Proposition 19.

If four lines be proportional the line produced from the first and fourth will be equal to the line produced from the second and third; and, if the line produced from the first and fourth be equal to that produced from the second and third, the four lines will be proportional.

Book 7 Proposition 20.

The least lines of those which have the same ratio with them measure those which have the same ratio the same line of times, the greater the greater and the less the less.

Book 7 Proposition 21.

Lines prime to one another are the least of those which have the same ratio with them.

Book 7 Proposition 22.

The least lines of those which have the same ratio with them are prime to one another.

Book 7 Proposition 23.

If two lines be prime to one another, the line which measures the one of them will be prime to the remaining line.

Book 7 Proposition 24.

If two lines be prime to any line, their product, also, *will be prime to the same.*

Book 7 Proposition **25.**

If two lines be prime to one another, the product of one of them into itself will be prime to the remaining one.

Book 7 Proposition 26.

If two lines be prime to two lines, both to each, their products, also, *will be prime to one another.*

Book 7 Proposition 27.

If two lines be prime to one another, and each, by multiplying itself, make a certain line, the products will be prime to one another; and, if the original lines, by multiplying the products, make certain lines, the latter will, also, *be prime to one another* [*and this is always the case with the extremes*].

Book 7 Proposition 28.

If two lines be prime to one another, the sum will, also, *be prime to each, of them; and, if the sum to two lines be prime to any one of them, the original lines will,* also, *be prime to one another.*

Book 7 Proposition 29.

Any prime line is prime to any line which it does not measure.

Book 7 Proposition 30.

If two lines, by multiplying one another, make some line, and any prime line measure the product, it will, also, *measure one of the original lines.*

Book 7 Proposition 31.

Any composite line is measured by some prime line.

Book 7 Proposition 32.

Any line either is prime or is measured by some prime line.

Book 7 Proposition 33.

Given as many lines as we please, to find the least of those which have the same ratio with them.

Book 7 Proposition 34.

Given two lines, to find the least line which they measure.

Book 7 Proposition 35.

If two lines measure any line, the least line measured by them will, also, *measure the same.*

Book 7 Proposition 36.

Given three lines, to find the least line which they measure.

Book 7 Proposition 37.

If a line be measured by any line, the line which is measured will have a part called by the same name as the measuring line.

Book 7 Proposition 38.

If a line have any part whatever, it will be measured by a line called by the same name as the part.

Book 7 Proposition 39.

To find the line which is the least that will have given parts.

Book 8. Propositions.

Book 8 Proposition 1.

If there be as many lines as we please in continued proportion, and the extremes of them be prime to one another, the lines are the least of those which have the same ratio with them.

Book 8 Proposition 2.

To find lines in continued proportion, as many as may be prescribed, and the least that are in a given ratio.

Book 8 Proposition 3.

If as many lines as we please in continued proportion be the least of those which have the same ratio with them, the extremes of them are prime to one another.

Book 8 Proposition 4.

Given as many ratios as we please in least lines, to find lines in continued proportion which are the least in the given ratios.

Book 8 Proposition 5.

Plane figures have to one another the ratio compounded of the ratios of their sides.

Book 8 Proposition 6.

If there be as many lines as we please in continued proportion, and the first do not measure the second, neither will any other measure any other.

Book 8 Proposition 7.

If there be as many lines as we please in continued proportion and the first measure the last, it will measure the second also.

Book 8 Proposition 8.

If between two lines there fall lines in continued proportion with them, then, however many lines fall between them in continued proportion, so many will, also, *fall in continued proportion between the lines which have the same ratio with the original lines.*

Book 8 Proposition 9.

If two lines be prime to one another, and lines fall between them in continued proportion then however many lines fall between them in continued proportion, so many will, also, *fall between each, of them and an unit in continued proportion.*

Book 8 Proposition 10.

If lines fall between each, of two lines and an unit in continued proportion, however many lines fall between each, of them and an unit in continued

proportion, so many, also, *will fall between the lines themselves in continued proportion.*

Book 8 Proposition 11.

Between two square figures there is one mean proportional line, and the square has to the square the ratio duplicate of that which the side has to the side.

Book 8 Proposition 12.

Between two cube figures there are two mean proportional lines, and the cube has to the cube the ratio triplicate of that which the side has to the side.

Book 8 Proposition 13.

If there be as many lines as we please in continued proportion, and each, by multiplying itself, make some line, the products will be proportional; and, If the original lines, by multiplying the products, make certain lines, the latter will, also, *be proportional.*

Book 8 Proposition 14.

If a square measure a square, the side will, also, *measure the side; and, if the side measure the side, the square will*, also, *measure the square.*

Book 8 Proposition 15.

If a cube figure measure a cube figure, the side will, also, *measure the side; and, if the side measure the side, the cube will*, also, *measure the cube.*

Book 8 Proposition 16.

If a square figure does not measure a square figure, neither will the side measure the side; and, if the side do not measure the side, neither will the square measure the square.

Book 8 Proposition 17.

If a cube figure does not measure a cube figure, neither will the side measure the side; and, if the side do not measure the side, neither will the cube measure the cube.

Book 8 Proposition 18.

Between two similar plane figures there is one mean proportional line; and the plane figure has to the plane figure the ratio duplicate of that which the corresponding side has to the corresponding side.

Book 8 Proposition 19.

Between two similar solid figures there fall two mean proportional lines; *and the solid line has to the similar the ratio triplicate of that which the corresponding side has to the corresponding side.*

Book 8 Proposition 20.

If one mean proportional line fall between two lines, the lines will be similar plane figures.

Book 8 Proposition 21.

If two mean proportionals fall between two mean proportionals, the two mean proportionals are similar solid figures.

Book 8 Proposition 22.

If three lines be in continued proportion, and the first be square, the third will, also, *be square.*

Book 8 Proposition 23.

If four lines be in continued proportion, and the first be cube, the fourth will, also, *be cube.*

Book 8 Proposition 24.

If two lines have to one another the ratio which a square figure has to a square figure, and the first be square, the second will, also, *be square.*

Book 8 Proposition 25.

If two lines have to one another the ratio which a cube figure has to a cube figure, and the first be cube, the second will, also, *be cube.*

Book 8 Proposition 26.

Similar plane figures have to one another the ratio which a square figure has to a square figure.

Book 8 Proposition 27.

Similar solid figures have to one another the ratio which a cube figure has to a cube figure.

Book 9. Propositions.

Book 9 Proposition 1.

If two similar plane figures, by multiplying one another, make some line, the product will be square.

Book 9 Proposition 2.

If two lines, by multiplying one another, make a square figure, they are similar plane figures.

Book 9 Proposition 3.

If a cube figure, by multiplying itself, make some line, the product will be cube.

Book 9 Proposition 4.

If a cube figure, by multiplying a cube figure, make some line, the product will be cube.

Book 9 Proposition 5.

If a cube figure, by multiplying any line, make a cube figure, the multiplied line will, also, *be cube.*

Book 9 Proposition 6.

If a line, by multiplying itself, make a cube figure, it will itself, also, *be cube.*

Now, here is a proposition which means absolutely nothing. A line, being divided, by itself so as to make any exponent, the line, itself, will already be that exponent, which means, nothing at all is inferred about the line, but about operations with them.

This also indicates that there is no such thing as a rectangle number, a square number, etc., the operations perform with a thing is not predicable of that thing.

Book 9 Proposition 7.

If a composite line, by multiplying any line, make some line, the product will be solid.

Book 9 Proposition 8.

If as many lines as we please beginning from an unit be in continued proportion, the third from the unit will be square, as will, also, *those which successively leave out one; the fourth will be cube, as will,* also, *all those which leave out two; and the seventh will be at once cube and square, as will,* also, *those which leave out five.*

Book 9 Proposition 9.

If as many lines as we please, beginning from an unit, be in continued proportion, and the line after the unit be square, all the rest will, also, *be square. And, if the line after the unit be cube, all the rest will,* also, *be cube.*

Book 9 Proposition 10.

If as many lines as we please beginning from an unit be in continued proportion, and the line after the unit be not square, neither will any other be square except the third from the unit and all those which leave out one. And if the line after the unit be not cube, neither will any other be cube except the fourth from the unit and all those which leave out two.

Book 9 Proposition 11.

If as many lines as we please beginning from an unit be in continued proportion, the less measures the greater according to some one of the lines which have place among the proportional lines.

Book 9 Proposition 12.

If as many lines as we please beginning from an unit be in continued proportion, by however many prime lines the last is measured, the next to the unit will, also, *be measured by the same.*

Book 9 Proposition 13.

If as many lines as we please beginning from an unit be in continued proportion, and the line after the unit be prime, the greatest will not be measured by any except those which, have a place among the proportional lines.

Book 9 Proposition 14.

If a line be the least that is measured by prime lines, it will not be measured by any other prime line except those originally measuring it.

Book 9 Proposition 15.

If three lines in continued proportion be the least of those which have the same ratio with them, any two whatever added together will be prime to the remaining line.

Book 9 Proposition 16.

If two lines be prime to one another, the second will not be to any other line as the first is to the second.

Book 9 Proposition 17.

If there be as many lines as we please in continued proportion, and the extremes of them be prime to one another, the last will not be to any other line as the first to the second.

Book 9 Proposition 18.

Given two lines, to investigate whether it is possible to find a third proportional to them.

Book 9 Proposition 19.

Given three lines, to investigate when it is possible to find a fourth proportional to them.

Book 9 Proposition 20.

Prime lines are more than any assigned multitude of prime lines.

Book 9 Proposition 21.

If as many even lines as we please be added together, the whole is even.

Book 9 Proposition 22.

If as many odd lines as we please be added together, and their multitude be even, the whole will be even.

Book 9 Proposition 23.

If as many odd lines as we please be added together, and their multitude be odd, the whole will, also, *be odd.*

Book 9 Proposition 24.

If from an even line an even line be subtracted, the remainder will be even.

Book 9 Proposition 25.

If from an even line an odd line be subtracted, the remainder will be odd.

Book 9 Proposition 26.

If from an odd line an odd line be subtracted, the remainder will be even.

Book 9 Proposition 27.

If from an odd line an even line be subtracted, the remainder will be odd.

Book 9 Proposition 28.

If an odd line, by multiplying an even line, make some line, the product will be even.

Book 9 Proposition 29.

If an odd line, by multiplying an odd line, make some line, the product will be odd.

Book 9 Proposition 30.

If an odd line measure an even line, it will, also, *measure the half of it.*

Book 9 Proposition 31.

If an odd line be prime to any line, it will, also, *be prime to the double of it.*

Book 9 Proposition 32.

Each, of the lines which are continually doubled beginning from a dyad is even-times even only.

Book 9 Proposition 33.

If a line have its half odd, it is even-times odd only.

Book 9 Proposition 34.

If a line neither be one of those which are continually doubled from a dyad, nor have its half odd, it is both even-times even and even-times odd.

Book 9 Proposition 35.

If as many lines as we please be in continued proportion, and there be subtracted from the second and the last lines equal to the first, then, as the excess of the second is to the first, so will the excess of the last be to all those before it.

Book 9 Proposition 36.

If as many lines as we please beginning from an unit be set out continuously in double proportion, until the sum of all becomes prime, and if the sum multiplied into the last make some line, the product will be perfect.

Book 10. Propositions.

Book 10 Proposition 1.

Two unequal lines being set out, if from the greater there be subtracted a line greater than its half and from that which is left a line greater than its half and if this process be repeated continually there will be left some line which will be less than the lesser line set out.

Book 10 Proposition 2.

If when the less of two unequal lines is continually subtracted in turn from the greater, that which is left never measures the one before it, the lines will be incommensurable.

Book 10 Proposition 3.

Given two commensurable lines, to find their greatest common measure.

Book 10 Proposition 4.

Given three commensurable lines, to find their greatest common measure.

Book 10 Proposition 5.

Commensurable lines have to one another the ratio which a line has to a line.

Book 10 Proposition 6.

If two lines have to one another the ratio which a line has to a line, the lines will be commensurable.

Book 10 Proposition 7.

Incommensurable lines have not to one another the ratio which a line has to a line.

Book 10 Proposition 8.

If two lines have not to one another the ratio which a line has to a line, the lines will be incommensurable.

Book 10 Proposition 9.

The squares on lines commensurable in length have to one another the ratio which a square figure has to a square figure; and squares which *have to one another the ratio which a square figure has to a square figure will*, also, *have their sides commensurable in length. But the squares on lines incommensurable in length have not to one another the ratio which a square figure has to a square*

figure; *and squares which have not to one another the ratio which a square figure has to a square figure will not have their sides commensurable in length either.*

Book 10 Proposition 10.

To find two lines incommensurable, the one in length only, and the other in square also, with an assigned line.

Book 10 Proposition 11.

If four lines be proportional, and the first be commensurable with the second, the third will, also, *be commensurable with the fourth*; *and, if the first be incommensurable with the second, the third will,* also, *be incommensurable with the fourth.*

Book 10 Proposition 12.

lines commensurable with the same line are commensurable with one another also.

Book 10 Proposition 13.

If two lines be commensurable, and the one of them be incommensurable with any line, the remaining one will, also, *be incommensurable with the same.*

Book 10 Proposition 14.

If four lines be proportional, and the square, on the first be greater than the square, on the second by the square, on a line commensurable with the first, the square, on the third will, also, *be greater than the square, on the fourth by the square, on a line commensurable with the third.*

And, if the square, on the first be greater than the square, on the second by the square, on a line incommensurable with the first, the square, on the third will, also, *be greater than the square, on the fourth by the square, on a line incommensurable with the third.*

Book 10 Proposition 15.

If two commensurable lines be added together, the whole will, also, *be commensurable with each, of them*; *and, if the whole be commensurable with one of them, the original lines will,* also, *be commensurable.*

Book 10 Proposition 16.

If two incommensurable lines be added together, the whole will, also, *be incommensurable with each, of them*; *and, if the whole be incommensurable with one of them, the original lines will,* also, *be incommensurable.*

Book 10 Proposition 17.

If there be two unequal lines, and to the greater there be applied a parallelogram equal to the fourth part of the square, on the less and deficient by a square figure, and if it divide it into parts which are commensurable in length, then the square, on the greater will be greater than the square, on the less by the square, on a line commensurable with the greater.

And, if the square, on the greater be greater than the square, on the less by the square, on a line commensurable with the greater, and if there be applied to the greater a parallelogram equal to the fourth part of the square, on the less and deficient by a square figure, it will divide it into parts which are commensurable in length.

Book 10 Proposition 18.

If there be two unequal lines, and to the greater there be applied a parallelogram equal to the fourth part of the square, on the less and deficient by a square figure, and if it divide it into parts which are incommensurable, the square, on the greater will be greater than the square, on the less by the square, on a line incommensurable with the greater.

And, if the square, on the greater be greater than the square, on the less by the square, on a line incommensurable with the greater, and if there be applied to the greater a parallelogram equal to the fourth part of the square, on the less and deficient by a square figure, it divides it into parts which are incommensurable.

Book 10 Proposition 19.

The rectangle, contained by rational lines commensurable in length, is rational.

Book 10 Proposition 20.

If a rational area be applied to a rational line, it produces as breadth a line rational and commensurable, in length, with the line to which it is applied.

Book 10 Proposition 21.

The rectangle contained by rational lines commensurable in square, only is irrational, and the side of the square equal to it is irrational. Let the latter be called **medial**.

Book 10 Proposition 22.

The square, on a medial line, if applied to a rational line, produces as breadth a line rational and incommensurable, in length, with that to which it is applied.

Book 10 Proposition 23.

A line commensurable with a medial line is medial.

Book 10 Proposition 24.

The rectangle contained by medial lines commensurable in length is medial.

Book 10 Proposition 25.

The rectangle contained by medial lines commensurable in square, only is either rational or medial.

Book 10 Proposition 26.

A medial area does not exceed a medial area by a rational area.

Book 10 Proposition 27.

To find medial lines commensurable in square, only which contain a rational rectangle.

Book 10 Proposition 28.

To find medial lines commensurable, in square, only which contain a medial rectangle.

Book 10 Proposition 29.

To find two rational lines commensurable, in square, only and such that the square, on the greater is greater than the square, on the less by the square, on a line commensurable in length with the greater.

Book 10 Proposition 30.

To find two rational lines commensurable in square, only and such that the square, on the greater is greater than the square, on the less by the square, on a line incommensurable in length with the greater.

Book 10 Proposition 31.

To find two medial lines commensurable in square, only, containing a rational rectangle, and such that the square, on the greater is greater than the square, on the less by the square, on a line commensurable in length with the greater.

Book 10 Proposition 32.

To find two medial lines commensurable, in square, only, containing a medial rectangle, and such that the square, on the greater is greater than the square, on the less by the square, on a line commensurable with the greater.

Book 10 Proposition 33.

To find two lines incommensurable, in square, which make the sum of the squares on them rational but the rectangle contained by them medial.

Book 10 Proposition 34.

To find two lines incommensurable, in square, which make the sum of the squares on them medial but the rectangle contained by them rational.

Book 10 Proposition 35.

To find two lines incommensurable, in square, which make the sum of the squares on them medial and the rectangle contained by them medial and moreover incommensurable with the sum of the squares on them.

Book 10 Proposition 36.

If two rational lines commensurable, in square, only be added together, the whole is irrational; and let it be called **binomial**.

Book 10 Proposition 37.

If two medial lines commensurable, in square, only and containing a rational rectangle be added together, the whole is irrational; and let it be called a **first bimedial** *line.*

Book 10 Proposition 38.

If two medial lines commensurable, in square, only and containing a medial rectangle be added together, the whole is irrational; and let it be called a **second bimedial** *line.*

Book 10 Proposition 39.

If two lines incommensurable, in square, which make the sum of the squares on them rational, but the rectangle contained by them medial, be added together, the whole line is irrational; and let it be called **major**.

Book 10 Proposition 40.

If two lines incommensurable, in square, which make the sum of the squares on them medial, but the rectangle contained by them rational, be added together, the whole line is irrational; and let it be called the **side of a rational plus a medial area**.

Book 10 Proposition 41.

If two lines incommensurable, in square, which make the sum of the squares on them medial, and the rectangle contained by them medial and, also, *incommensurable with the sum of the squares on them, be added together, the whole line is irrational; and let it be called the* **side of the sum of two medial areas**.

Book 10 Proposition 42.

A binomial line is divided into its terms at one point only.

Book 10 Proposition 43.

A first bimedial line is divided at one point only.

Book 10 Proposition 44.

A second bimedial line is divided at one point only.

Book 10 Proposition 45.

A major line is divided at one and the same point only.

Book 10 Proposition 46.

The side of a rational plus a medial area is divided at one point only.

Book 10 Proposition 47.

The side of the sum of two medial areas is divided at one point only.

Book 10 Proposition 48.

To find the first binomial line.

Book 10 Proposition 49.

To find the second binomial line.

Book 10 Proposition 50.

To find the third binomial line.

Book 10 Proposition 51.

To find the fourth binomial line.

Book 10 Proposition 52.

To find the fifth binomial line.

Book 10 Proposition 53.

To find the sixth binomial line.

Book 10 Proposition 54.

If an area be contained by a rational line and the first binomial, the "side", of the area is the irrational line which is called binomial.

Book 10 Proposition 55.

If an area be contained by a rational line and the second binomial the "side", of the area is the irrational line which is called a first bimedial.

Book 10 Proposition 56.

If an area be contained by a rational line and the third binomial, the "side", of the area, is the irrational line called a second bimedial.

Book 10 Proposition 57.

If an area be contained by a rational line and the fourth binomial, the "side", of the area, is the irrational line called major.

Book 10 Proposition 58.

If an area be contained by a rational line and the fifth binomial, the "side", of the area, is the irrational line called the side of a rational plus a medial area.

Book 10 Proposition 59.

If an area be contained by a rational line and the sixth binomial, the "side", of the area, is the irrational line called the side of the sum of two medial areas.

Book 10 Proposition 60.

The square, on the binomial line applied to a rational line produces as breadth the first binomial.

Book 10 Proposition 61.

The square, on the first bimedial line applied to a rational line produces as breadth the second binomial.

Book 10 Proposition 62.

The square, on the second bimedial line applied to a rational line produces as breadth the third binomial.

Book 10 Proposition 63.

The square, on the major line applied to a rational line produces as breadth the fourth binomial.

Book 10 Proposition 64.

The square, on the side of a rational plus a medial area applied to a rational line produces as breadth the fifth binomial.

Book 10 Proposition 65.

The square, on the side of the sum of two medial areas applied to a rational line produces as breadth the sixth binomial.

Book 10 Proposition 66.

A line commensurable in length with a binomial line is itself, also, *binomial and the same, in order.*

Book 10 Proposition 67.

A line commensurable in length with a bimedial line is itself, also, *bimedial and the same, in order.*

Book 10 Proposition 68.

A line commensurable with a major line is itself, also, *major.*

Book 10 Proposition 69.

A line commensurable with the side of a rational plus a medial area is itself, also, *the side of a rational plus a medial area.*

Book 10 Proposition 70.

A line commensurable with the side of the sum of two medial areas is the side of the sum of two medial areas.

Book 10 Proposition 71.

If a rational and a medial area be added together, four irrational lines arise, namely a binomial or a first bimedial or a major or a side of a rational plus a medial area.

Book 10 Proposition 72.

If two medial areas incommensurable with one another be added together the remaining two irrational lines arise, namely either a second bimedial or a side of the sum of two medial areas.

Book 10 Proposition 73.

If from a rational line there be subtracted a rational line commensurable with the whole, in square, only, the remainder is irrational; and let it be called an **apotome**.

Book 10 Proposition 74.

If from a medial line there be subtracted a medial line which is commensurable with the whole, in square, only, and which contains with the whole a rational rectangle, the remainder is irrational. And let it be called a **first apotome of a medial** *line.*

Book 10 Proposition 75.

If from a medial line there be subtracted a medial line which is commensurable with the whole, in square, only, and which contains with the whole a medial rectangle, the remainder is irrational; and let it be called a **second apotome of a medial** *line.*

Book 10 Proposition 76.

If from a line there be subtracted a line which is incommensurable, in square, with the whole and which with the whole makes the squares on them added together rational, but the rectangle contained by them medial, the remainder is irrational; and let it be called **minor**.

Book 10 Proposition 77.

If from a line there be subtracted a line which is incommensurable, in square, with the whole, and which with the whole makes the sum of the squares on them medial, but twice the rectangle contained by them rational, the remainder is irrational: and let it be called **that which produces with a rational area a medial whole**.

Book 10 Proposition 78.

If from a line there be subtracted a line which is incommensurable, in square, with the whole and which with the whole makes the sum of the squares on them medial, twice the rectangle contained by them medial, and further the squares on them incommensurable with twice the rectangle contained by them, the remainder

is irrational; *and let it be called* **that which produces**, **with a medial area**, **a medial whole**.

Book 10 Proposition 79.

To an apotome only one rational line can be annexed which is commensurable with the whole, in square, only.

Book 10 Proposition 80.

To a first apotome of a medial line only one medial line can be annexed which is commensurable with the whole, in square, only and which contains with the whole a rational rectangle.

Book 10 Proposition 81.

To a second apotome of a medial line only one medial line can be annexed which is commensurable with the whole, in square, only, and which contains with the whole a medial rectangle.

Book 10 Proposition 82.

To a minor line only one line can be annexed which is incommensurable, in square, with the whole and which makes, with the whole, the sum of the squares on them rational, but twice the rectangle contained by them, medial.

Book 10 Proposition 83.

To a line which produces with a rational area a medial whole only one line can be annexed which is incommensurable, in square, with the whole line and which with the whole line makes the sum of the squares on them medial, but twice the rectangle contained by them rational.

Book 10 Proposition 84.

To a line which produces with a medial area a medial whole only one line can be annexed which is incommensurable, in square, with the whole line and which with the whole line makes the sum of the squares on them medial and twice the rectangle contained by them both medial and, also, *incommensurable with the sum of the squares on them.*

Book 10 Proposition 85.

To find the first apotome.

Book 10 Proposition 86.

To find the second apotome.

Book 10 Proposition 87.

To find the third apotome.

Book 10 Proposition 88.

To find the fourth apotome.

Book 10 Proposition 89.

To find the fifth apotome.

Book 10 Proposition 90.

To find the sixth apotome.

Book 10 Proposition 91.

If an area be contained by a rational line and a first apotome, the "side", of the area, is an apotome.

Book 10 Proposition 92.

If an area be contained by a rational line and a second apotome, the "side", of the area, is a first apotome of a medial line.

Book 10 Proposition 93.

If an area be contained by a rational line and a third apotome, the "side", of the area, is a second apotome of a medial line.

Book 10 Proposition 94.

If an area be contained by a rational line and a fourth apotome, the "side", of the area, is minor.

Book 10 Proposition 95.

If an area be contained by a rational line and a fifth apotome, the "side", of the area, is a line which produces with a rational area a medial whole.

Book 10 Proposition 96.

If an area be contained by a rational line and a sixth apotome, the "side", of the area, is a line which produces with a medial area a medial whole.

Book 10 Proposition 97.

The square, on an apotome applied to a rational line produces as breadth a first apotome.

Book 10 Proposition 98.

The square, on a first apotome of a medial line applied to a rational line produces as breadth a second apotome.

Book 10 Proposition 99.

The square, on a second apotome of a medial line applied to a rational line produces as breadth a third apotome.

Book 10 Proposition 100.

The square, on a minor line applied to a rational line produces as breadth a fourth apotome.

Book 10 Proposition 101.

The square, on the line which produces with a rational area a medial whole, if applied to a rational line, produces as breadth a fifth apotome.

Book 10 Proposition 102.

The square, on the line which produces with a medial area a medial whole, if applied to a rational line, produces as breadth a sixth apotome.

Book 10 Proposition 103.

A line commensurable, in length, with an apotome is an apotome and the same, in order.

Book 10 Proposition 104.

A line commensurable with an apotome of a medial line is an apotome of a medial line and the same, in order.

Book 10 Proposition 105.

A line commensurable with a minor line is minor.

Book 10 Proposition 106.

A line commensurable with that which produces with a rational area a medial whole is a line which produces with a rational area a medial whole.

Book 10 Proposition 107.

A line commensurable with that which produces, with a medial area, a medial whole, is itself, also, *a line which produces, with a medial area, a medial whole.*

Book 10 Proposition 108.

If from a rational area a medial area be subtracted, the "side", of the remaining area, becomes one of two irrational lines, either an apotome or a minor line.

Book 10 Proposition 109.

If, from a medial area, a rational area be subtracted, there arise two other irrational lines, either a first apotome of a medial line or a line which produces with a rational area a medial whole.

Book 10 Proposition 110.

If from a medial area there be subtracted a medial area incommensurable with the whole, the two remaining irrational lines arise, either a second apotome of a medial line or a line which produces with a medial area a medial whole.

Book 10 Proposition 111.

The apotome is not the same with the binomial line.

Book 10 Proposition 112.

The square, on a rational line applied to the binomial line produces as breadth an apotome the terms of which are commensurable with the terms of the binomial and moreover in the same ratio; and further the apotome so arising will have the same order as the binomial line.

Book 10 Proposition 113.

The square, on a rational line, if applied to an apotome, produces as breadth the binomial line the terms of which are commensurable with the terms of the apotome and in the same ratio; and further the binomial so arising has the same order as the apotome.

Book 10 Proposition 114.

If an area be contained by an apotome and the binomial line the terms of which are commensurable with the terms of the apotome and in the same ratio, the "side", of the area, is rational.

Book 10 Proposition 115.

From a medial line there arise irrational lines infinite in line, and none of them is the same as any of the preceding.

Book 11. Propositions.

Book 11 Proposition 1.

A part of a line cannot be in the plane of reference and a part in a plane more elevated.

Book 11 Proposition 2.

If two lines divide one another, they are in one plane, and every triangle is in one plane.

Book 11 Proposition 3.

If two planes divide one another, their common section is a line.

Book 11 Proposition 4.

If a line be set up at right angles to two lines which divide one another, at their common point of section, it will, also, *be at right angles to the plane through them.*

Book 11 Proposition 5.

If a line be set up at right angles to three lines which meet one another, at their common point of section, the three lines are in one plane.

Book 11 Proposition 6.

If two lines be at right angles to the same plane, the lines will be parallel.

Book 11 Proposition 7.

If two lines be parallel and points be taken at random on each, of them, the line joining the points is in the same plane with the parallel lines.

Book 11 Proposition 8.

If two lines be parallel, and one of them be at right angles to any plane, the remaining one will, also, *be at right angles to the same plane.*

Book 11 Proposition 9.

Lines which are parallel to the same line and are not in the same plane with it are, also, *parallel to one another.*

Book 11 Proposition 10.

If two lines meeting one another be parallel to two lines meeting one another not in the same plane, they will contain equal angles.

Book 11 Proposition 11.

From a given elevated point to draw a line perpendicular to a given plane.

Book 11 Proposition 12.

To set up a line at right angles to a given plane from a given point in it.

Book 11 Proposition 13.

From the same point two lines cannot be set up at right angles to the same plane on the same side.

Book 11 Proposition 14.

Planes to which the same line is at right angles will be parallel.

Book 11 Proposition 15.

If two lines meeting one another be parallel to two lines meeting one another, not being in the same plane, the planes through them are parallel.

Book 11 Proposition 16.

If two parallel planes be divided by any plane, their common sections are parallel.

Book 11 Proposition 17.

If two lines be divided by parallel planes, they will be divided in the same ratios.

Book 11 Proposition 18.

If a line be at right angles to any plane, all the planes through it will, also, *be at right angles to the same plane.*

Book 11 Proposition 19.

If two planes which divide one another be at right angles to any plane, their common section will, also, *be at right angles to the same plane.*

Book 11 Proposition 20.

If a solid angle be contained by three plane angles, any two, taken together in any manner, are greater than the remaining one.

Book 11 Proposition 21.

Any solid angle is contained by plane angles less than four right angles.

Book 11 Proposition 22.

If there be three plane angles of which two, taken together in any manner, are greater than the remaining one, and they are contained by equal lines, it is possible to construct a triangle out of the lines joining the extremities of the equal lines.

Book 11 Proposition 23.

To construct a solid angle out of three plane angles two of which, taken together in any manner, are greater than the remaining one: thus the three angles must be less than four right angles.

Book 11 Proposition 24.

If a solid be contained by parallel planes, the opposite planes in it are equal and parallelogrammic.

Book 11 Proposition 25.

If a parallelepipedal solid be divided by a plane which is parallel to the opposite planes, then, as the base is to the base, so will the solid be to the solid.

Book 11 Proposition 26.

On a given line, and at a given point on it, to construct a solid angle equal to a given solid angle.

Book 11 Proposition 27.

On a given line to describe a parallelepipedal solid similar and similarly situated to a given parallelepipedal solid.

Book 11 Proposition 28.

If a parallelepipedal solid be divided by a plane through the diagonals of the opposite planes, the solid will be bisected by the plane.

Book 11 Proposition 30.

Parallelepipedal solids which are on the same base and of the same height, and in which the extremities of the sides which stand up are not on the same lines, are equal to one another.

Book 11 Proposition 31.

Parallelepipedal solids which are on equal bases and of the same height are equal to one another.

Book 11 Proposition 32.

Parallelepipedal solids which are of the same height are to one another as their bases.

Book 11 Proposition 33.

Similar parallelepipedal solids are to one another in the triplicate ratio of their corresponding sides.

Book 11 Proposition 34.

In equal parallelepipedal solids the bases are reciprocally proportional to the heights; and those parallelepipedal solids in which the bases are reciprocally proportional to the heights are equal.

Book 11 Proposition 35.

If there be two equal plane angles, and on their vertices there be set up elevated lines containing equal angles with the original lines respectively, if on the elevated lines points be taken at random and perpendiculars be drawn from them to the planes in which the original angles are, and if from the points so arising in the planes lines be described to the vertices of the original angles, they will contain, with the elevated lines, equal angles.

Book 11 Proposition 36.

If three lines be proportional, the parallelepipedal solid formed out of the three is equal to the parallelepipedal solid on the mean which is equilateral, but equiangular with the aforesaid solid.

Book 11 Proposition 37.

If four lines be proportional, the parallelepipedal solids on them which are similar and similarly described will, also, *be proportional; and, if the parallelepipedal solids on them which are similar and similarly described be proportional, the lines will themselves,* also, *be proportiona*l.

Book 11 Proposition 38.

If the sides of the opposite planes of a cube be bisected, and planes be carried through the points of section, the common section of the planes and the diameter of the cube bisect one another.

Book 11 Proposition 39.

If there be two prisms of equal height, and one have a parallelogram as base and the other a triangle, and if the parallelogram be double of the triangle, the prisms will be equal.

Book 12. Propositions.

Book 12 Proposition 1.

Similar polygons inscribed in circles are to one another as the squares on the diameters.

Book 12 Proposition 2.

Circles are to one another as the squares on the diameters.

Book 12 Proposition 3.

Any pyramid which has a triangular base is divided into two pyramids equal and similar to one another, similar to the whole and having triangular bases, and into two equal prisms; *and the two* prisms *are greater than the half of the whole pyramid.*

Book 12 Proposition 4.

If there be two pyramids of the same height which have triangular bases, and each, of them be divided into two pyramids equal to one another and similar to the whole, and into two equal prisms, then, as the base of the one pyramid is to the base of the other pyramid, so will all the prisms in the one pyramid be to all the prisms, being equal in multitude, in the other pyramid.

Book 12 Proposition 5.

Pyramids which are of the same height and have triangular bases are to one another as the bases.

Book 12 Proposition 6.

Pyramids which are of the same height and have polygonal bases are to one another as the bases.

Book 12 Proposition 7.

Any prism which has a triangular base is divided into three pyramids equal to one another which have triangular bases.

Book 12 Proposition 8.

Similar pyramids which have triangular bases are in the triplicate ratio of their corresponding sides.

Book 12 Proposition 9.

In equal pyramids which have triangular bases the bases are reciprocally proportional to the heights; and those pyramids in which the bases are reciprocally proportional to the heights are equal.

Book 12 Proposition 10.

Any cone is a third part of the cylinder which has the same base with it and equal height.

Book 12 Proposition 11.

Cones and cylinders which are of the same height are to one another as their bases.

Book 12 Proposition 12.

Similar cones and cylinders are to one another in the triplicate ratio of the diameters in their bases.

Book 12 Proposition 13.

If a cylinder be divided by a plane which is parallel to its opposite planes, then, as the cylinder is to the cylinder, so will the axis be to the axis.

Book 12 Proposition 14.

Cones and cylinders which are on equal bases are to one another as their heights.

Book 12 Proposition 15.

In equal cones and cylinders the bases are reciprocally proportional to the heights; and those cones and cylinders in which the bases are reciprocally proportional to the heights are equal.

Book 12 Proposition 16.

Given two circles about the same center, to inscribe in the greater circle an equilateral polygon with an even line of sides which does not have one point in common with the lesser circle.

Book 12 Proposition 17.

Given two spheres about the same center, to inscribe in the greater sphere a polyhedral solid which does not have one point in common with the lesser sphere at its surface.

Book 12 Proposition 18.

Spheres are to one another in the triplicate ratio of their respective diameters.

Book 13. Propositions.

Book 13 Proposition 1.

If a line be divided in extreme and mean ratio, the square, on the greater segment added to the half of the whole is five times the square, on the half.

Book 13 Proposition 2.

If the square, on a line be five times the square, on a segment of it, then, when the double of the said segment is divide in extreme and mean ratio, the greater segment is the remaining part of the original line.

Book 13 Proposition 3.

If a line be divided in extreme and mean ratio, the square, on the lesser segment added to the half of the greater segment is five times the square, on the half of the greater segment.

Book 13 Proposition 4.

If a line be divided in extreme and mean ratio, the square, on the whole and the square, on the lesser segment together are triple of the square, on the greater segment.

Book 13 Proposition 5.

If a line be divided in extreme and mean ratio, and there be added to it a line equal to the greater segment, the whole line has been divide in extreme and mean ratio, and the original line is the greater segment.

Book 13 Proposition 6.

If a rational line be divided in extreme and mean ratio, each, of the segments is the irrational line called apotome.

Book 13 Proposition 7.

If three angles of an equilateral pentagon, taken either in order or not in order, be equal, the pentagon will be equiangular.

Book 13 Proposition 8.

If in an equilateral and equiangular pentagon lines subtend two angles taken in order, they divide one another in extreme and mean ratio, and their greater segments are equal to the side of the pentagon.

Book 13 Proposition 9.

If the side of the hexagon and that of the decagon inscribed in the same circle be added together, the whole line has been divide in extreme and mean ratio, and its greater segment is the side of the hexagon.

Book 13 Proposition 10.

If an equilateral pentagon be inscribed in a circle, the square, on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.

Book 13 Proposition 11.

If in a circle which has its diameter rational an equilateral pentagon be inscribed, the side of the pentagon is the irrational line called minor.

Book 13 Proposition 12.

If an equilateral triangle be inscribed in a circle, the square, on the side of the triangle is triple of the square, on the radius of the circle.

Book 13 Proposition 13.

To construct a pyramid, to comprehend it in a given sphere, and to prove that the square, on the diameter of the sphere is one and a half times the square, on the side of the pyramid.

Book 13 Proposition 14.

To construct an octahedron and comprehend it in a sphere, as in the preceding case; and to prove that the square, on the diameter of the sphere is double of the square, on the side of the octahedron.

Book 13 Proposition 15.

To construct a cube and comprehend it in a sphere, like the pyramid; and to prove that the square, on the diameter of the sphere is triple of the square, on the side of the cube.

Book 13 Proposition 16.

To construct an icosahedron and comprehend it in a sphere, like the aforesaid figures; and to prove that the side of the icosahedron is the irrational line called minor.

Book 13 Proposition 17.

To construct a dodecahedron and comprehend it in a sphere, like the aforesaid figures, and to prove that the side of the dodecahedron is the irrational line called apotome.

Book 13 Proposition 18.

To set out the sides of the five figures and to compare them with one another.

A World At Risk

By John Clark

A World at Risk Introduction

Saturday, June 10, 2023

A long time ago, I made a stupid mistake. Normally, such a mistake would end one's life. However, something decided that I might be useful, and I was put through what most would consider an impossible experience. I considered it impossible, not only impossible, but ridiculously improbable. I was shown very advanced communication ability and what it could do. The experience, however, was so far out of my realm of imagination, I spent decades in a state of cognitive dissonance.

However, I was also asked to teach what I eventually understood, how to save our lives through doing our own work as a mind. What I did not know at the time of my very odd experience, I now know with certainty. Everything depends upon us doing our own work as a functional life support system called Mind. I have been asked to show you that mankind is endowed with the possibility of perfect judgment and Total Recall and how it is factually and simply possible for a great many of us. In other words, I am here to help bring about Judgment Day, the day that mankind has learnt judgment. My work is not performed using vain words, but words, the likes of which, prove themselves, all one has to do is learn to read them, and speak with them. Those words are designed to trace cause and effect to perfection. The Grammar is called Geometry, learning how the hand can teach the mind the truth of all things.

I simply failed with my own hand, and was asked to try again, learn what I considered wholly impossible, now I see, it is the only thing possible.

So, you can believe I was touched by God, or you can believe I had a true alien encounter, where everything I learn is proven, or you can believe that I speak bull-shit, that is until you examine the Geometry I discovered, the likes of which has never existed on earth before. I really do not care what you believe, I only care about what we factually can do, and can learn by it. Our behavior, the behavior of everything, is only an exhibition of advanced applications of Geometry. So, we are to become as children again, learning how to speak and think in provable truth.

Once a geometric figure is constructed, it produces its result perfectly and independent of any processing time, i.e., independent even of the speed of light. You really want truth, learn the grammar, or shut the hell up about wanting to know anything. That which does not strive to live, is simply dead.

Like any life support system, we either do our job and live, or fail and die. This work is a proof of life.

I was presented with, what I believed to be an impossible perceptible experience, and I have been ask to present to you, what your educators have claimed to be an impossible intelligible experience. Do you really know how many things have been claimed impossible to demonstrate in Geometry; some of those have even gone unnoticed, simply missing from what has been passed down to us. You will find more than you know what to do with here, in the companion

DVD, called Library Dialectic. And like my decades of cognitive dissonance, you too will experience decades of discovering that your ignorance of simple fact has placed you in a state of unrecognized cognitive dissonance, and hopefully, you too will get over it.

A Nation at Risk:

The Imperative for Educational Reform

A Report to the Nation
and the Secretary of Education
United States Department of Education
by
The National Commission on Excellence in Education

April 1983

April 26, 1983

Honorable T. H. Bell
Secretary of Education
U.S. Department of Education
Washington, D.C. 20202

Dear Mr. Secretary:

On August 26, 1981, you created the National Commission on Excellence in Education and directed it to present a report on the quality of education in America to you and to the American people by April of 1983.

It has been my privilege to chair this endeavor and on behalf of the members of the Commission it is my pleasure to transmit this report, *A Nation at Risk: The Imperative for Educational Reform*.

Our purpose has been to help define the problems afflicting American education and to provide solutions, not search for scapegoats. We addressed the main issues as we saw them, but have not attempted to treat the subordinate matters in any detail. We were forthright in our discussions and have been candid in our report regarding both the strengths and weaknesses of American education.

The Commission deeply believes that the problems we have discerned in American education can be both understood and corrected if the people of our country, together with those who have public responsibility in the matter, care enough and are courageous enough to do what is required.

Each member of the Commission appreciates your leadership in having asked this diverse group of persons to examine one of the central issues which will define our Nation's future. We especially welcomed your confidence throughout the course of our deliberations and your anticipation of a report free of political partisanship.

It is our collective and earnest hope that you will continue to provide leadership in this effort by assuring wide dissemination and full discussion of this report, and by encouraging appropriate action throughout the country. We believe that materials compiled by the Commission in the course of its work constitute a major resource for all persons interested in American education.

The other Commissioners and I sincerely appreciate the opportunity to have served our country as members of the National Commission on Excellence in Education, and on their behalf I remain,

Respectfully,

David Pierpont Gardner
Chairman

Members of the National Commission on Excellence in Education

David P. Gardner (Chair)
President
University of Utah and
President-Elect, University of California
Salt Lake City, Utah

Yvonne W. Larsen (Vice-Chair)
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A Nation At Risk - April 1983

Introduction

Secretary of Education T. H. Bell created the National Commission on Excellence in Education on August 26, 1981, directing it to examine the quality of education in the United States and to make a report to the Nation and to him within 18 months of its first meeting. In accordance with the Secretary's instructions, this report contains practical recommendations for educational improvement and fulfills the Commission's responsibilities under the terms of its charter.

The Commission was created as a result of the Secretary's concern about "the widespread public perception that something is seriously remiss in our educational system." Soliciting the "support of all who care about our future," the Secretary noted that he was establishing the Commission based on his "responsibility to provide leadership, constructive criticism, and effective assistance to schools and universities."

The Commission's charter contained several specific charges to which we have given particular attention. These included:

- assessing the quality of teaching and learning in our Nation's public and private schools, colleges, and universities;
- comparing American schools and colleges with those of other advanced nations;
- studying the relationship between college admissions requirements and student achievement in high school;
- identifying educational programs which result in notable student success in college;
- assessing the degree to which major social and educational changes in the last quarter century have affected student achievement; and
- defining problems which must be faced and overcome if we are successfully to pursue the course of excellence in education.

The Commission's charter directed it to pay particular attention to teenage youth, and we have done so largely by focusing on high schools. Selective attention was given to the formative years spent in elementary schools, to higher education, and to vocational and technical programs. We refer those interested in the need for similar reform in higher education to the recent report of the American Council on Education, *To Strengthen the Quality of Higher Education*.

In going about its work the Commission has relied in the main upon five sources of information:

- papers commissioned from experts on a variety of educational issues;
- administrators, teachers, students, representatives of professional and public groups, parents, business leaders, public officials, and scholars who testified at eight meetings of the full Commission, six public hearings, two panel discussions,

a symposium, and a series of meetings organized by the Department of Education's Regional Offices;
- existing analyses of problems in education;
- letters from concerned citizens, teachers, and administrators who volunteered extensive comments on problems and possibilities in American education; and
- descriptions of notable programs and promising approaches in education.

To these public-minded citizens who took the trouble to share their concerns with us--frequently at their own expense in time, money, and effort--we extend our thanks. In all cases, we have benefited from their advice and taken their views into account; how we have treated their suggestions is, of course, our responsibility alone. In addition, we are grateful to the individuals in schools, universities, foundations, business, government, and communities throughout the United States who provided the facilities and staff so necessary to the success of our many public functions.

The Commission was impressed during the course of its activities by the diversity of opinion it received regarding the condition of American education and by conflicting views about what should be done. In many ways, the membership of the Commission itself reflected that diversity and difference of opinion during the course of its work. This report, nevertheless, gives evidence that men and women of good will can agree on common goals and on ways to pursue them.

The Commission's charter, the authors and topics of commissioned papers, a list of the public events, and a roster of the Commission's staff are included in the appendices which complete this volume.

A Nation At Risk

All, regardless of race or class or economic status, are entitled to a fair chance and to the tools for developing their individual powers of mind and spirit to the utmost. This promise means that all children by virtue of their own efforts, competently guided, can hope to attain the mature and informed judgement needed to secure gainful employment, and to manage their own lives, thereby serving not only their own interests but also the progress of society itself.

Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world. This report is concerned with only one of the many causes and dimensions of the problem, but it is the one that undergirds American prosperity, security, and civility. We report to the American people that while we can take justifiable pride in what our schools and colleges have historically accomplished and contributed to the United States and the well-being of its people, the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people. What was unimaginable a generation ago has begun to occur--others are matching and surpassing our educational attainments.

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge. Moreover, we have dismantled essential support systems which helped make those gains possible. We have, in effect, been committing an act of unthinking, unilateral educational disarmament.

Our society and its educational institutions seem to have lost sight of the basic purposes of schooling, and of the high expectations and disciplined effort needed to attain them. This report, the result of 18 months of study, seeks to generate reform of our educational system in fundamental ways and to renew the Nation's commitment to schools and colleges of high quality throughout the length and breadth of our land.

That we have compromised this commitment is, upon reflection, hardly surprising, given the multitude of often conflicting demands we have placed on our Nation's schools and colleges. They are routinely called on to provide solutions to personal, social, and political problems that the home and other institutions either will not or cannot resolve. We must understand that these demands on our schools and colleges often exact an educational cost as well as a financial one.

On the occasion of the Commission's first meeting, President Reagan noted the central importance of education in American life when he said: "Certainly there are few areas of American life as important to our society, to our people, and to our families as our schools and colleges." This report, therefore, is as much an open letter to the American

people as it is a report to the Secretary of Education. We are confident that the American people, properly informed, will do what is right for their children and for the generations to come.

The Risk

History is not kind to idlers. The time is long past when American's destiny was assured simply by an abundance of natural resources and inexhaustible human enthusiasm, and by our relative isolation from the malignant problems of older civilizations. The world is indeed one global village. We live among determined, well-educated, and strongly motivated competitors. We compete with them for international standing and markets, not only with products but also with the ideas of our laboratories and neighborhood workshops. America's position in the world may once have been reasonably secure with only a few exceptionally well-trained men and women. It is no longer.

The risk is not only that the Japanese make automobiles more efficiently than Americans and have government subsidies for development and export. It is not just that the South Koreans recently built the world's most efficient steel mill, or that American machine tools, once the pride of the world, are being displaced by German products. It is also that these developments signify a redistribution of trained capability throughout the globe. Knowledge, learning, information, and skilled intelligence are the new raw materials of international commerce and are today spreading throughout the world as vigorously as miracle drugs, synthetic fertilizers, and blue jeans did earlier. If only to keep and improve on the slim competitive edge we still retain in world markets, we must dedicate ourselves to the reform of our educational system for the benefit of all--old and young alike, affluent and poor, majority and minority. Learning is the indispensable investment required for success in the "information age" we are entering.

Our concern, however, goes well beyond matters such as industry and commerce. It also includes the intellectual, moral, and spiritual strengths of our people which knit together the very fabric of our society. The people of the United States need to know that individuals in our society who do not possess the levels of skill, literacy, and training essential to this new era will be effectively disenfranchised, not simply from the material rewards that accompany competent performance, but also from the chance to participate fully in our national life. A high level of shared education is essential to a free, democratic society and to the fostering of a common culture, especially in a country that prides itself on pluralism and individual freedom.

For our country to function, citizens must be able to reach some common understandings on complex issues, often on short notice and on the basis of conflicting or incomplete evidence. Education helps form these common understandings, a point Thomas Jefferson made long ago in his justly famous dictum:

I know no safe depository of the ultimate powers of the society but the people themselves; and if we think them not enlightened enough to exercise their control with a

wholesome discretion, the remedy is not to take it from them but to inform their discretion.
Part of what is at risk is the promise first made on this continent: All, regardless of race or class or economic status, are entitled to a fair chance and to the tools for developing their individual powers of mind and spirit to the utmost. This promise means that all children by virtue of their own efforts, competently guided, can hope to attain the mature and informed judgment needed to secure gainful employment, and to manage their own lives, thereby serving not only their own interests but also the progress of society itself.

Indicators of the Risk

The educational dimensions of the risk before us have been amply documented in testimony received by the Commission. For example:

- International comparisons of student achievement, completed a decade ago, reveal that on 19 academic tests American students were never first or second and, in comparison with other industrialized nations, were last seven times.
- Some 23 million American adults are functionally illiterate by the simplest tests of everyday reading, writing, and comprehension.
- About 13 percent of all 17-year-olds in the United States can be considered functionally illiterate. Functional illiteracy among minority youth may run as high as 40 percent.
- Average achievement of high school students on most standardized tests is now lower than 26 years ago when Sputnik was launched.
- Over half the population of gifted students do not match their tested ability with comparable achievement in school.
- The College Board's Scholastic Aptitude Tests (SAT) demonstrate a virtually unbroken decline from 1963 to 1980. Average verbal scores fell over 50 points and average mathematics scores dropped nearly 40 points.
- College Board achievement tests also reveal consistent declines in recent years in such subjects as physics and English.
- Both the number and proportion of students demonstrating superior achievement on the SATs (i.e., those with scores of 650 or higher) have also dramatically declined.
- Many 17-year-olds do not possess the "higher order" intellectual skills we should expect of them. Nearly 40 percent cannot draw inferences from written material; only one-fifth can write a persuasive essay; and only one-third can solve a mathematics problem requiring several steps.
- There was a steady decline in science achievement scores of U.S. 17-year-olds as measured by national assessments of science in 1969, 1973, and 1977.
- Between 1975 and 1980, remedial mathematics courses in public 4-year colleges increased by 72 percent and now constitute one-quarter of all mathematics courses taught in those institutions.
- Average tested achievement of students graduating from college is also lower.
- Business and military leaders complain that they are required to spend millions of dollars on costly remedial education and training programs in such basic skills as

reading, writing, spelling, and computation. The Department of the Navy, for example, reported to the Commission that one-quarter of its recent recruits cannot read at the ninth grade level, the minimum needed simply to understand written safety instructions. Without remedial work they cannot even begin, much less complete, the sophisticated training essential in much of the modern military.

These deficiencies come at a time when the demand for highly skilled workers in new fields is accelerating rapidly. For example:

- Computers and computer-controlled equipment are penetrating every aspect of our lives--homes, factories, and offices.
- One estimate indicates that by the turn of the century millions of jobs will involve laser technology and robotics.
- Technology is radically transforming a host of other occupations. They include health care, medical science, energy production, food processing, construction, and the building, repair, and maintenance of sophisticated scientific, educational, military, and industrial equipment.

Analysts examining these indicators of student performance and the demands for new skills have made some chilling observations. Educational researcher Paul Hurd concluded at the end of a thorough national survey of student achievement that within the context of the modern scientific revolution, "We are raising a new generation of Americans that is scientifically and technologically illiterate." In a similar vein, John Slaughter, a former Director of the National Science Foundation, warned of "a growing chasm between a small scientific and technological elite and a citizenry ill-informed, indeed uninformed, on issues with a science component."

But the problem does not stop there, nor do all observers see it the same way. Some worry that schools may emphasize such rudiments as reading and computation at the expense of other essential skills such as comprehension, analysis, solving problems, and drawing conclusions. Still others are concerned that an over-emphasis on technical and occupational skills will leave little time for studying the arts and humanities that so enrich daily life, help maintain civility, and develop a sense of community. Knowledge of the humanities, they maintain, must be harnessed to science and technology if the latter are to remain creative and humane, just as the humanities need to be informed by science and technology if they are to remain relevant to the human condition. Another analyst, Paul Copperman, has drawn a sobering conclusion. Until now, he has noted:

Each generation of Americans has outstripped its parents in education, in literacy, and in economic attainment. For the first time in the history of our country, the educational skills of one generation will not surpass, will not equal, will not even approach, those of their parents.

It is important, of course, to recognize that *the average citizen* today is better educated and more knowledgeable than the average citizen of a generation ago--more literate, and exposed to more mathematics, literature, and science. The positive impact of this fact on the well-being of our country and the lives of our people cannot be overstated.

Nevertheless, *the average graduate* of our schools and colleges today is not as well-educated as the average graduate of 25 or 35 years ago, when a much smaller proportion of our population completed high school and college. The negative impact of this fact likewise cannot be overstated.

Hope and Frustration

Statistics and their interpretation by experts show only the surface dimension of the difficulties we face. Beneath them lies a tension between hope and frustration that characterizes current attitudes about education at every level.

We have heard the voices of high school and college students, school board members, and teachers; of leaders of industry, minority groups, and higher education; of parents and State officials. We could hear the hope evident in their commitment to quality education and in their descriptions of outstanding programs and schools. We could also hear the intensity of their frustration, a growing impatience with shoddiness in many walks of American life, and the complaint that this shoddiness is too often reflected in our schools and colleges. Their frustration threatens to overwhelm their hope.

What lies behind this emerging national sense of frustration can be described as both a dimming of personal expectations and the fear of losing a shared vision for America.

On the personal level the student, the parent, and the caring teacher all perceive that a basic promise is not being kept. More and more young people emerge from high school ready neither for college nor for work. This predicament becomes more acute as the knowledge base continues its rapid expansion, the number of traditional jobs shrinks, and new jobs demand greater sophistication and preparation.

On a broader scale, we sense that this undertone of frustration has significant political implications, for it cuts across ages, generations, races, and political and economic groups. We have come to understand that the public will demand that educational and political leaders act forcefully and effectively on these issues. Indeed, such demands have already appeared and could well become a unifying national preoccupation. This unity, however, can be achieved only if we avoid the unproductive tendency of some to search for scapegoats among the victims, such as the beleaguered teachers.

On the positive side is the significant movement by political and educational leaders to search for solutions--so far centering largely on the nearly desperate need for increased support for the teaching of mathematics and science. This movement is but a start on what we believe is a larger and more educationally encompassing need to improve teaching and learning in fields such as English, history, geography, economics, and foreign languages. We believe this movement must be broadened and directed toward reform and excellence throughout education.

Excellence in Education

We define "excellence" to mean several related things. At the level of the *individual learner*, it means performing on the boundary of individual ability in ways that test and push back personal limits, in school and in the workplace. Excellence characterizes a *school or college* that sets high expectations and goals for all learners, then tries in every way possible to help students reach them. Excellence characterizes a *society* that has adopted these policies, for it will then be prepared through the education and skill of its people to respond to the challenges of a rapidly changing world. Our Nation's people and its schools and colleges must be committed to achieving excellence in all these senses.

We do not believe that a public commitment to excellence and educational reform must be made at the expense of a strong public commitment to the equitable treatment of our diverse population. The twin goals of equity and high-quality schooling have profound and practical meaning for our economy and society, and we cannot permit one to yield to the other either in principle or in practice. To do so would deny young people their chance to learn and live according to their aspirations and abilities. It also would lead to a generalized accommodation to mediocrity in our society on the one hand or the creation of an undemocratic elitism on the other.

Our goal must be to develop the talents of all to their fullest. Attaining that goal requires that we expect and assist all students to work to the limits of their capabilities. We should expect schools to have genuinely high standards rather than minimum ones, and parents to support and encourage their children to make the most of their talents and abilities.

The search for solutions to our educational problems must also include a commitment to life-long learning. The task of rebuilding our system of learning is enormous and must be properly understood and taken seriously: Although a million and a half new workers enter the economy each year from our schools and colleges, the adults working today will still make up about 75 percent of the workforce in the year 2000. These workers, and new entrants into the workforce, will need further education and retraining if they--and we as a Nation--are to thrive and prosper.

The Learning Society

In a world of ever-accelerating competition and change in the conditions of the workplace, of ever-greater danger, and of ever-larger opportunities for those prepared to meet them, educational reform should focus on the goal of creating a Learning Society. At the heart of such a society is the commitment to a set of values and to a system of education that affords all members the opportunity to stretch their minds to full capacity, from early childhood through adulthood, learning more as the world itself changes. Such a society has as a basic foundation the idea that education is important not only because of what it contributes to one's career goals but also because of the value it adds to the general quality of one's life. Also at the heart of the Learning Society

are educational opportunities extending far beyond the traditional institutions of learning, our schools and colleges. They extend into homes and workplaces; into libraries, art galleries, museums, and science centers; indeed, into every place where the individual can develop and mature in work and life. In our view, formal schooling in youth is the essential foundation for learning throughout one's life. But without life-long learning, one's skills will become rapidly dated.

In contrast to the ideal of the Learning Society, however, we find that for too many people education means doing the minimum work necessary for the moment, then coasting through life on what may have been learned in its first quarter. But this should not surprise us because we tend to express our educational standards and expectations largely in terms of "minimum requirements." And where there should be a coherent continuum of learning, we have none, but instead an often incoherent, outdated patchwork quilt. Many individual, sometimes heroic, examples of schools and colleges of great merit do exist. Our findings and testimony confirm the vitality of a number of notable schools and programs, but their very distinction stands out against a vast mass shaped by tensions and pressures that inhibit systematic academic and vocational achievement for the majority of students. In some metropolitan areas basic literacy has become the goal rather than the starting point. In some colleges maintaining enrollments is of greater day-to-day concern than maintaining rigorous academic standards. And the ideal of academic excellence as the primary goal of schooling seems to be fading across the board in American education.

Thus, we issue this call to all who care about America and its future: to parents and students; to teachers, administrators, and school board members; to colleges and industry; to union members and military leaders; to governors and State legislators; to the President; to members of Congress and other public officials; to members of learned and scientific societies; to the print and electronic media; to concerned citizens everywhere. America is at risk.

We are confident that America can address this risk. If the tasks we set forth are initiated now and our recommendations are fully realized over the next several years, we can expect reform of our Nation's schools, colleges, and universities. This would also reverse the current declining trend--a trend that stems more from weakness of purpose, confusion of vision, underuse of talent, and lack of leadership, than from conditions beyond our control.

The Tools at Hand

It is our conviction that the essential raw materials needed to reform our educational system are waiting to be mobilized through effective leadership:

- the natural abilities of the young that cry out to be developed and the undiminished concern of parents for the well-being of their children;
- the commitment of the Nation to high retention rates in schools and colleges and to full access to education for all;

- the persistent and authentic American dream that superior performance can raise one's state in life and shape one's own future;
- the dedication, against all odds, that keeps teachers serving in schools and colleges, even as the rewards diminish;
- our better understanding of learning and teaching and the implications of this knowledge for school practice, and the numerous examples of local success as a result of superior effort and effective dissemination;
- the ingenuity of our policymakers, scientists, State and local educators, and scholars in formulating solutions once problems are better understood;
- the traditional belief that paying for education is an investment in ever-renewable human resources that are more durable and flexible than capital plant and equipment, and the availability in this country of sufficient financial means to invest in education;
- the equally sound tradition, from the Northwest Ordinance of 1787 until today, that the Federal Government should supplement State, local, and other resources to foster key national educational goals; and
- the voluntary efforts of individuals, businesses, and parent and civic groups to cooperate in strengthening educational programs.

These raw materials, combined with the unparalleled array of educational organizations in America, offer us the possibility to create a Learning Society, in which public, private, and parochial schools; colleges and universities; vocational and technical schools and institutes; libraries; science centers, museums, and other cultural institutions; and corporate training and retraining programs offer opportunities and choices for all to learn throughout life.

The Public's Commitment

Of all the tools at hand, the public's support for education is the most powerful. In a message to a National Academy of Sciences meeting in May 1982, President Reagan commented on this fact when he said:

This public awareness--and I hope public action--is long overdue.... This country was built on American respect for education. . . Our challenge now is to create a resurgence of that thirst for education that typifies our Nation's history.

The most recent (1982) Gallup Poll of the *Public's Attitudes Toward the Public Schools* strongly supported a theme heard during our hearings: People are steadfast in their belief that education is the major foundation for the future strength of this country. They even considered education more important than developing the best industrial system or the strongest military force, perhaps because they understood education as the cornerstone of both. They also held that education is "extremely important" to one's future success, and that public education should be the top priority for additional Federal funds. Education occupied first place among 12 funding categories considered in the survey--above health care, welfare, and military defense, with 55 percent selecting public education as one of their first three choices. Very clearly, the public understands the primary importance of education as the foundation for a satisfying life, an enlightened and civil society, a strong economy, and a secure Nation.

At the same time, the public has no patience with undemanding and superfluous high school offerings. In another survey, more than 75 percent of all those questioned believed every student planning to go to college should take 4 years of mathematics, English, history/U.S. government, and science, with more than 50 percent adding 2 years each of a foreign language and economics or business. The public even supports requiring much of this curriculum for students who do not plan to go to college. These standards far exceed the strictest high school graduation requirements of any State today, and they also exceed the admission standards of all but a handful of our most selective colleges and universities.

Another dimension of the public's support offers the prospect of constructive reform. The best term to characterize it may simply be the honorable word "patriotism." Citizens know intuitively what some of the best economists have shown in their research, that education is one of the chief engines of a society's material well-being. They know, too, that education is the common bond of a pluralistic society and helps tie us to other cultures around the globe. Citizens also know in their bones that the safety of the United States depends principally on the wit, skill, and spirit of a self-confident people, today and tomorrow. It is, therefore, essential--especially in a period of long-term decline in educational achievement--for government at all levels to affirm its responsibility for nurturing the Nation's intellectual capital.

And perhaps most important, citizens know and believe that the meaning of America to the rest of the world must be something better than it seems to many today. Americans like to think of this Nation as the preeminent country for generating the great ideas and material benefits for all mankind. The citizen is dismayed at a steady 15-year decline in industrial productivity, as one great American industry after another falls to world competition. The citizen wants the country to act on the belief, expressed in our hearings and by the large majority in the Gallup Poll, that education should be at the top of the Nation's agenda.

Findings

We conclude that declines in educational performance are in large part the result of disturbing inadequacies in the way the educational process itself is often conducted. The findings that follow, culled from a much more extensive list, reflect four important aspects of the educational process: content, expectations, time, and teaching.

Findings Regarding Content

By content we mean the very "stuff" of education, the curriculum. Because of our concern about the curriculum, the Commission examined patterns of courses high school students took in 1964-69 compared with course patterns in 1976-81. On the basis of these analyses we conclude:

- Secondary school curricula have been homogenized, diluted, and diffused to the point that they no longer have a central purpose. In effect, we have a cafeteria

style curriculum in which the appetizers and desserts can easily be mistaken for the main courses. Students have migrated from vocational and college preparatory programs to "general track" courses in large numbers. The proportion of students taking a general program of study has increased from 12 percent in 1964 to 42 percent in 1979.

- This curricular smorgasbord, combined with extensive student choice, explains a great deal about where we find ourselves today. We offer intermediate algebra, but only 31 percent of our recent high school graduates complete it; we offer French I, but only 13 percent complete it; and we offer geography, but only 16 percent complete it. Calculus is available in schools enrolling about 60 percent of all students, but only 6 percent of all students complete it.
- Twenty-five percent of the credits earned by general track high school students are in physical and health education, work experience outside the school, remedial English and mathematics, and personal service and development courses, such as training for adulthood and marriage.

Findings Regarding Expectations

We define expectations in terms of the level of knowledge, abilities, and skills school and college graduates should possess. They also refer to the time, hard work, behavior, self-discipline, and motivation that are essential for high student achievement. Such expectations are expressed to students in several different ways:

- by grades, which reflect the degree to which students demonstrate their mastery of subject matter;
- through high school and college graduation requirements, which tell students which subjects are most important;
- by the presence or absence of rigorous examinations requiring students to demonstrate their mastery of content and skill before receiving a diploma or a degree;
- by college admissions requirements, which reinforce high school standards; and
- by the difficulty of the subject matter students confront in their texts and assigned readings.

Our analyses in each of these areas indicate notable deficiencies:

- The amount of homework for high school seniors has decreased (two-thirds report less than 1 hour a night) and grades have risen as average student achievement has been declining.
- In many other industrialized nations, courses in mathematics (other than arithmetic or general mathematics), biology, chemistry, physics, and geography start in grade 6 and are required of *all* students. The time spent on these subjects, based on class hours, is about three times that spent by even the most science-oriented U.S. students, i.e., those who select 4 years of science and mathematics in secondary school.

- A 1980 State-by-State survey of high school diploma requirements reveals that only eight States require high schools to offer foreign language instruction, but none requires students to take the courses. Thirty-five States require only 1 year of mathematics, and 36 require only 1 year of science for a diploma.
- In 13 States, 50 percent or more of the units required for high school graduation may be electives chosen by the student. Given this freedom to choose the substance of half or more of their education, many students opt for less demanding personal service courses, such as bachelor living.
- "Minimum competency" examinations (now required in 37 States) fall short of what is needed, as the "minimum" tends to become the "maximum," thus lowering educational standards for all.
- One-fifth of all 4-year public colleges in the United States must accept every high school graduate within the State regardless of program followed or grades, thereby serving notice to high school students that they can expect to attend college even if they do not follow a demanding course of study in high school or perform well.
- About 23 percent of our more selective colleges and universities reported that their general level of selectivity declined during the 1970s, and 29 percent reported reducing the number of specific high school courses required for admission (usually by dropping foreign language requirements, which are now specified as a condition for admission by only one-fifth of our institutions of higher education).
- Too few experienced teachers and scholars are involved in writing textbooks. During the past decade or so a large number of texts have been "written down" by their publishers to ever-lower reading levels in response to perceived market demands.
- A recent study by Education Products Information Exchange revealed that a majority of students were able to master 80 percent of the material in some of their subject-matter texts before they had even opened the books. Many books do not challenge the students to whom they are assigned.
- Expenditures for textbooks and other instructional materials have declined by 50 percent over the past 17 years. While some recommend a level of spending on texts of between 5 and 10 percent of the operating costs of schools, the budgets for basal texts and related materials have been dropping during the past decade and a half to only 0.7 percent today.

Findings Regarding Time

Evidence presented to the Commission demonstrates three disturbing facts about the use that American schools and students make of time: (1) compared to other nations, American students spend much less time on school work; (2) time spent in the classroom and on homework is often used ineffectively; and (3) schools are not doing enough to help students develop either the study skills required to use time well or the willingness to spend more time on school work.

- In England and other industrialized countries, it is not unusual for academic high school students to spend 8 hours a day at school, 220 days per year. In the United States, by contrast, the typical school day lasts 6 hours and the school year is 180 days.
- In many schools, the time spent learning how to cook and drive counts as much toward a high school diploma as the time spent studying mathematics, English, chemistry, U.S. history, or biology.
- A study of the school week in the United States found that some schools provided students only 17 hours of academic instruction during the week, and the average school provided about 22.
- A California study of individual classrooms found that because of poor management of classroom time, some elementary students received only one-fifth of the instruction others received in reading comprehension.
- In most schools, the teaching of study skills is haphazard and unplanned. Consequently, many students complete high school and enter college without disciplined and systematic study habits.

Findings Regarding Teaching

The Commission found that not enough of the academically able students are being attracted to teaching; that teacher preparation programs need substantial improvement; that the professional working life of teachers is on the whole unacceptable; and that a serious shortage of teachers exists in key fields.

- Too many teachers are being drawn from the bottom quarter of graduating high school and college students.
- The teacher preparation curriculum is weighted heavily with courses in "educational methods" at the expense of courses in subjects to be taught. A survey of 1,350 institutions training teachers indicated that 41 percent of the time of elementary school teacher candidates is spent in education courses, which reduces the amount of time available for subject matter courses.
- The average salary after 12 years of teaching is only $17,000 per year, and many teachers are required to supplement their income with part-time and summer employment. In addition, individual teachers have little influence in such critical professional decisions as, for example, textbook selection.
- Despite widespread publicity about an overpopulation of teachers, severe shortages of certain kinds of teachers exist: in the fields of mathematics, science, and foreign languages; and among specialists in education for gifted and talented, language minority, and handicapped students.
- The shortage of teachers in mathematics and science is particularly severe. A 1981 survey of 45 States revealed shortages of mathematics teachers in 43 States, critical shortages of earth sciences teachers in 33 States, and of physics teachers everywhere.
- Half of the newly employed mathematics, science, and English teachers are not qualified to teach these subjects; fewer than one-third of U. S. high schools offer physics taught by qualified teachers.

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- Half of the newly employed mathematics, science, and English teachers are not qualified to teach these subjects; fewer than one-third of U. S. high schools offer physics taught by qualified teachers.

Appendix A: Charter--National Commission on Excellence in Education

Authority

20 U.S.C. 1233a. The Commission is governed by the provisions of Part D of the General Education Provisions Act (P.L. 90-247 as amended; 20 U.S.C. 1233 *et seq*.) and the Federal Advisory Committee Act (P.L. 92-463; 5 U.S.C Appendix I) which set forth standards for the formation and use of advisory committees.

Purpose and Functions

The Commission advises and makes recommendations to the nation and to the Secretary of Education. To carry out this mission the Commission is charged with the following responsibilities:

1. To review and synthesize the data and scholarly literature on the quality of learning and teaching in the nation's schools, colleges, and universities, both public and private, with special concern for the educational experience of teen-age youth;
2. To examine and to compare and contrast the curricula, standards, and expectations of the educational systems of several advanced countries with those of the United States;
3. To study a representative sampling of university and college admission standards and lower division course requirements with particular reference to the impact upon the enhancement of quality and the promotion of excellence such standards may have on high school curricula and on expected levels of high school academic achievement;
4. To review and to describe educational programs that are recognized as preparing students who consistently attain higher than average scores in college entrance examinations and who meet with uncommon success the demands placed on them by the nation's colleges and universities;
5. To review the major changes that have occurred in American education as well as events in society during the past quarter century that have significantly affected educational achievement;
6. To hold hearings and to receive testimony and expert advice on efforts that could and should be taken to foster higher levels of quality and academic excellence in the nation's schools, colleges, and universities;
7. To do all other things needed to define the problems of and the barriers to attaining greater levels of excellence in American education; and
8. To report and to make practical recommendations for action to be taken by educators, public officials, governing boards, parents, and others having a vital interest in American education and a capacity to influence it for the better.

Structure

The Commission consists of at least 12, but not more than 19, public members appointed by the Secretary. The Secretary shall designate a chairperson from among the members. Among its members the Commission includes persons who are knowledgeable about educational programs at various levels and are familiar with views of the public, of employers, of educators, and of leaders of a range of professions regarding the status of education today, requirements for the future, and ways the quality of education for all Americans can be improved.

A quorum of the Commission is a majority of appointed members.

Terms of service of members end with the termination of the Commission.

Hearings on behalf of the Commission may be held by one or more members with the authorization of the chairperson.

The Commission may establish standing committees composed exclusively of its members. Each standing committee complies with the requirements of applicable statutes and Departmental regulations. Each committee presents to the Commission findings and recommendations for action by the full Commission. Timely notification of the establishment of a committee and any change therein, including its charge, membership, and frequency of meetings, will be made in writing to the Committee Management Officer. All committees act under the policies established by the Commission as a whole.

Management and staff services are provided by the Executive Director who serves as the Designated Federal Official to the Commission and by the National Institute of Education.

Meetings

The Commission meets approximately four times a year at the call of the Chairperson, with the advance approval of the Secretary or the Designated Federal Official who approves the agenda and is present or represented at all meetings. Standing committees meet as required at the call of their Chairperson with the concurrence of the Commission Chairperson. All meetings are open to the public except as determined otherwise by the Assistant Secretary for Educational Research and Improvement. Notice of all meetings shall be given to the public. Meetings are conducted, and records of proceedings kept, in accordance with applicable laws and Department regulations.

Compensation

In accordance with the General Education Provisions Act and other applicable laws, Commission members shall be entitled to an honorarium of $100 per day for official

business of the Commission. Their per diem and travel expenses will be paid in accordance with Federal Travel Regulations.

Annual Cost Estimate

Estimate of the direct cost for operating the Commission, including compensation and travel expenses for members as well as costs for studies, but excluding staff support, is $332,000. Estimate of annual person-years of staff required is 16. Estimate of direct annual costs for administrative support, staff and staff per diem and travel expenses is $453,000. The National Institute of Education will provide additional administrative and research assistance to the Commission.

Reports

In addition to its final report, which is expected eighteen months from the initial meeting, the Commission submits to the Congress by March 31 of each year an annual report which contains as a minimum a list of the names and business addresses of the members, a list of the dates and places of the meetings, the functions of the Commission, and a summary of Commission activities and recommendations made during the year. Such report is transmitted with the Secretary's annual report to Congress. The Commission makes such other reports or recommendations as may be appropriate. A copy of the annual report and other reports is provided to the Committee Management Officer.

Termination Date

It is estimated that the time necessary for the Commission to complete its activities and report is at least 18 months. Therefore, to insure the completion of the report, the Secretary determines that this Commission terminates not later than two years from the date of this Charter.

APPROVED:

August 5, 1981
T.H. Bell, Secretary of Education

Appendix B: Schedule of the Commission's Public Events

In addition to these public events, the Commission members also attended a number of subcommittee meetings and worksessions over the course of 18 months.

Event: Full Commission Meeting
Dates: October 9-10, 1981
Place: Washington, D.C.

Event: Full Commission Meeting
Date: December 7, 1981
Place: Washington, D.C.

Event: Full Commission Meeting
Date: February 25, 1982
Place: Washington, D.C.

Event: Hearing--Science, Mathematics, and Technology Education
Date: March 11, 1982
Place: Stanford University, Stanford, California
Hosts: *Donald Kennedy*, President, Stanford University; and *J. Myron Atkin*, Dean, Graduate School of Education, Stanford University

Event: Hearing--Language and Literacy: Skills for Academic Learning
Date: April 16, 1982
Place: Houston Independent School District, Houston, Texas
Hosts: *Raymon Bynum*, Texas State Commissioner of Education; and *Billy R. Reagan*, General Superintendent, Houston Independent School District

Event: Panel Discussion--Performance Expectations in American Education
Date: April 30, 1982
Place: The University of Pennsylvania, Philadelphia, Pennsylvania
Host: *Thomas Erlich*, Provost, The University of Pennsylvania

Event: Hearing--Teaching and Teacher Education
Date: May 12, 1982
Place: Gerogia State University, Atlanta, Georgia
Hosts: *Alonzo Crim*, Superintendent, Atlanta Public Schools; *Sherman Day*, Dean, School of Education, Georgia State University; and *Barbara Hatton*, Dean, School of Education, Atlanta University

Event: Full Commission Meeting
Date: May 25, 1982
Place: Washington, D.C.

Event: Hearing--College Admissions and the Transition to Postsecondary Education
Date: June 23, 1982
Place: Roosevelt University, Chicago, Illinois
Hosts: *Rolf Weil*, President, Roosevelt University; and *John Corbally*, President, John D. and Catherine T. MacArthur Foundation, Chicago

Event: Symposium--The Student's Role in Learning
Date: July 30, 1982
Place: San Diego State University, California
Hosts: *Thomas Day*, President, San Diego State University; and *Richard Atkinson*, Chancellor, University of California, San Diego

Event: Panel Discussion--College Curriculum: Shape, Influence, and Assessment
Date: August 27, 1982
Place: University of Rhode Island, Kingston, Rhode Island
Host: *Frank Newman*, President, University of Rhode Island

Event: Hearing--Education for a Productive Role in a Productive Society
Date: September 16, 1982
Place: St. Cajetan's Center, Denver, Colorado
Host: *Robert Andringa*, Executive Director, Education Commission of the States, Denver

Event: Full Commission Meeting
Dates: September 28-29, 1982
Place: New York, New York
Host: *Robert Payton*, President, Exxon Education Foundation, Exxon Corporation, New York, New York

Event: Hearing--Education for the Gifted and Talented
Date: October 15, 1982
Place: Harvard University, Cambridge Massachusetts
Hosts: *Derek Bok*, President, Harvard University; and *Patricia Albjerg Graham*, Dean, Harvard Graduate School of Education

Event: Full Commission Meeting
Dates: November 15-16, 1982
Place: Washington, D.C.

Event: Full Commission Meeting
Dates: January 21-22, 1983
Place: Washington, D.C.

Event: Full Commission Meeting
Dates: April 26, 1983
Place: Washington, D.C.

Appendix C: Commissioned Papers

Joseph Adelson, The University of Michigan, Ann Arbor
"Twenty-Five years of American Education: An Interpretation"
Catherine P. Ailes and Francis W. Rushing, SRI International, Arlington, Virginia
"A Summary Report on the Educational Systems of the United States and the Soviet Union: Comparative Analysis"
Alexander W. Astin, University of California, Los Angeles
"Excellence and Equity in American Education"
Alexander W. Astin, University of California, Los Angeles
"The American Freshman, 1966-1981: Some Implications for Educational Policy and Practice"
Herman Blake, University of California, Santa Cruz
"Demographic Change and Curriculum: New Students in Higher Education
Richard I. Brod, The Modern Language Association, New York, New York
Nicholas Farnham, The International Council on the Future of the University, New York, New York
William V. Mayer, Biological Sciences Curriculum Study, Boulder, Colorado
Robert A. McCaughey, Barnard College, New York, New York
"University Entrance Examinations and Performance Expectations"
Barbara B. Burn and Christopher H. Hurn, University of Massachusetts, Amherst
"An Analytic Comparison of Educational Systems"
Philip Cusick, Michigan State University, East Lansing
"Secondary Public Schools in America"
Paul DeHart Hurd, Stanford University, California
"An Overview of Science Education in the United States and Selected Foreign Countries"
Walter Doyle, University of Texas at Austin
"Academic Work"
Kenneth Duckworth, University of Oregon, Eugene
"Some Ideas About Student Cognition, Motivation and Work" (A Critique of the Symposium on *The Student's Role in Learning*)
Max A. Eckstein, Queens College/City of New York Flushing
Susanne Shafer, Arizona State University, Tempe
Kenneth Travers, University of Illinois, Champaign-Urbana
"A Comparative Review of Curriculum: Mathematics and International Studies in the Secondary Schools of Five Countries"
Eleanor Farrar, The Huron Institute, Cambridge, Massachusetts
Matthew B. Miles, Center for Policy Research, New York, New York
Barbara Neufeld, The Huron Institute, Cambridge, Massachusetts
"A Review of Effective Schools Research: Implications for Practice and Research"
Zelda Gamson, University of Michigan, Ann Arbor
"A Little Light on the Subject: Keeping General and Liberal Education Alive"
William E. Gardner, University of Minnesota, Minneapolis
John R. Palmer, University of Wisconsin, Madison

"Certification and Accreditation: Background, Issue Analysis, and Recommendations"

Thomas L. Good, University of Missouri-Columbia
"What Is Learned in Schools: Responding to School Demands, Grades K-6"

Thomas L. Good and Gail M. Hinkel, University of Missouri-Columbia
"Schooling in America: Some Descriptive and Explanatory Statements"

Donald B. Holsinger, State University of New York, Albany
"Time, Content and Expectations as Predictors of School Achievement in the U.S.A. and Other Developed Countries: A Review of IEA Evidence"

Kenneth R. Howey, University of Minnesota, Minneapolis
"Charting Directions for Preservice Teacher Education"

Torsten Husen, University of Stockholm, Sweden
"A Cross-National Perspective on Assessing the Quality of Learning"

Nancy Karweit, Johns Hopkins University, Baltimore, Maryland
"Time on Task: A Reserch Review"

Howard London, Bridgewater State College, Massachusetts
"Academic Standards in the American Community College: Trends and Controversies"

Martin L. Maehr, University of Illinois, Champaign-Urbana
"Motivational Factors in School Achievement"

Matthew B. Miles, Center for Policy Research, New York, New York
Eleanor Farrar and Barbara Neufeld, The Huron Institute Cambridge, Massachusetts
"The Extent of Adoption of Effective Schools Programs"

Barbara Neufeld and Eleanor Farrar, The Huron Institute Cambridge, Massachusetts
Matthew B. Miles, Center for Policy Research, New York, New York
"A Review of Effective Schools Research: The Message for Secondary Schools"

William Neumann, Syracuse University, New York
"College Press and Student Fit"

C. Robert Pace, University of California, Los Angeles
"Achievement and Quality of Student Effort"

Harvey L. Prokop, San Diego Unified School District California
"Intelligence, Motivation and the Quantity and Quality of Academic Work and Their Impacts on the Learning of Students: A Practitioner's Reaction" (A Critique of the Symposium on *The Student's Role in Learning*)

Lauren B. Resnick, University of Pittsburgh, Pennsylvania
Daniel P. Resnick, Carnegie-Mellon University Pittsburgh, Pennsylvania
"Standards, Curriculum, and Performance: An Historical and Comparative Perspective"

Frederick Rudolph, Williams College, Williamstown, Massachusetts
"Educational Excellence--The Secondary School-College Connection and Other Matters: An Historical Assessment"

Clifford Sjogren, University of Michigan, Ann Arbor
"College Admissions and the Transition to Postsecondary Education: Standards and Practices"

Richard E. Snow, Stanford University, California

"Intelligence, Motivation and Academic Work" (A Critique of the Symposium on *The Student's Role in Learning*)

Robert J. Sternberg and Richard Wagner, Yale University, New Haven, Connecticut

"Understanding Intelligence: What's in It for Educators?"

Deborah Stipek, University of California, Los Angeles

"Motivating Students to Learn: A Lifelong Perspective"

Judith Torney-Purta, University of Maryland, College Park

John Schwille, Michigan State University, East Lansing

"The Values Learned in School: Policy and Practice in Industralized Countries"

Beatrice Ward, John R. Mergendoller, and Alexis L. Mitman, Far West Laboratory for Educational Research and Development, San Francisco, California

"The Years Between Elementary School and High School: What Schooling Experiences Do Students Have?"

Jonathan Warren, Educational Testing Service, Berkeley, California

"The Faculty Role in Educational Excellence"

Dean K. Whitla, Harvard University, Cambridge, Massachusetts

"Value Added and Ohter Related Matters"

Sam J. Yarger, Syracuse University, New York

"Inservice Education"

Herbert Zimiles, Bank Street College of Education, New York, New York

"The Changing American Child: The Perspective of Educators"

Commissioned papers will be available in the ERIC system after July 1983 (See Ordering Information).

Also available through the ERIC system after July 1983:

Clifford Adelman, National Institute of Education, Washington, D.C.

"A Study of High School Transcripts, 1964-1981"

Available through the ERIC system after August 1983:

Fast Response Survey System, National Center for Education Statistics, Washington, D.C.

"School District Survey of Academic Requirements and Achievement"

Fast Response Survey System, National Center for Education Statistics, Washington, D.C.

"Survey of Schools of Teacher Education: Perceptions of Methods for Improvement"

Service Delivery Assessment, Office of Management, U.S. Department of Education, Washington, D.C.

"Study Skills Instruction"

Appendix D: Hearing Testimony

Science, Mathematics, and Technology Education

Without a deep, sturdy science and technology foundation, U.S. needs cannot be satisfied. The base of the foundation is education in science and mathematics from grade school through high school. But the evidence is all about us of our recent neglect and the strong possibility of a further downgrading of the national importance of such education.

Simon Ramo the TRW-Fujitsu Company
Redondo Beach, California

H. Guyford Stever, National Academy of Sciences, Washington, D.C.
Bernard M. Oliver, Hewlett-Packard Company, Palo Alto, California
Henry L. Alder, University of California, Davis, representing the Council of Scientific Society Presidents
Sarah E. Klein, Roton Middle School, Norwalk, Connecticut, representing the National Science Teachers Association
Harold D. Taylor, Hillsdale High School, San Mateo, California, representing the National Council of Teachers of Mathematics

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John Martin, Palo Alto Unified School District, California
Ruth Willis, Hamilton Junior High School, Oakland, California
Sarn Dederian, San Francisco Unified School District, California
Leroy Finkel, San Mateo County Office of Education, California
Olivia Martinez, San Jose Unified School District, California
Robert Bell, General Electric Company, San Jose, California
Judith Hubner, representing the Governor's Office, State of California
Robert W Walker, De Anza-Foothill Community College District, Californla
Nancy Kreinberg, Lawrence Hall of Science, Berkeley, California
Robert Finnell, Lawrence Hall of Science, Berkeley, California
Marian E. Koshland, University of California, Berkeley, representing the National Science Board
Alan M. Portis, University of California, Berkeley, representing the Education Committee of the American Physical Society
Leon Henkin, University of California, Berkeley, representing the U.S. Commission on Mathematical Instruction
John Pawson, Edison High School, Huntington Beach, California
Alan Fibish, Lowell High School, San Francisco, California
Juliet R. Henry, representing the California Teachers Association
Jess Bravin, Board of Education, Los Angeles, California

Frank Oppenheimer, Exploratorium, San Francisco, California
Leigh Burstein, University of California, Los Angeles

Judy Chamberlain, Cupertino Unified School District, California
Michael Summerville, Fremont Unified High School District, California
Ted Perry, San Juan Unified School District, California
Paul DeHart Hurd, Stanford University, California
Elizabeth Karplus, Campolindo High School, Moraga, California
Louis Fein, Palo Alto Learners Association, California
Bob McFarland, representing the California Math Council
Katherine Burt, Cupertino Elementary School District, California
Leo Ruth, California Engineering Foundation

Gordon M. Ambach, State Education Department, Albany, New York
James L. Casey, State Department of Education, Oklahoma City, Oklahoma
Carolyn Graham, Jefferson Elementary School, Burbank, California
Marcy Holteen, Ambler, Pennsylvania
Howard C. Mel and *Kay Fairwell*, Lawrence Hall of Science, Berkeley, California
Jean Phillips, Thousand Oaks, California
Simon Ramo, the TRW-Fujitsu Company, Redondo Beach, California
Gerhardt W. Reidel, University of West Los Angeles, Culver City, California
Carl L. Riehm, Virginia State Department of Education, Richmond, Virginia
John H. Saxon, Norman, Oklahoma
Thomas O. Sidebottom, Interactive Sciences, Inc., Palo Alto, California
Karl Weiss, Northeastern University, Boston, Massachusetts
Jan West, Oroville, California

Related Activities in the Bay Area

Site Visit

Lawrence Hall of Science
University of California, Berkeley
Howard C. Mel, Director

Tour of the Paul and Jean Hanna Collection on the Role of Education and the Archives and Library at the Hoover Institution, Stanford University

Dinner with business, education, and community leaders

Sponsored by the Chamber of Commerce of the United States, Western Regional Office, and the William and Flora Hewlett Foundation

Language and Literacy: Skills for Academic Learning

Writing as an activity is not honored by the American public in the opinion of the students. They see a surface picture, dominated by television, film, and radio, in which the acts of writing and reading are not viewed as important or even relevant. The cultural heroes are athletes, actresses, actors, politicians and big business tycoons. None seemingly need reading or writing to achieve their stature.

James Kinneavy University of Texas, Austin

Richard C. Anderson, University of Iltinois, Champaign-Urbana
Margaret Smith-Burke, New York University, New York
Donaid Graves, University of New Hampshire, Durham
Eileen Lundy, University of Texas, San Antonio
Ray Clifford, Defense Language Institute, Presidio of Monterey, California
Lity Wong-Fillmore, University of California, Berkeley

Victoria Bergin, Texas Education Agency, Austin
Alan C. Purves, University of Illinois, Champaign-Urbana
Delia Pompa, Houston Independent School District, Texas
Olivia Munoz, Houston Independent School District, Texas
James Kinneavy, University of Texas, Austin
Betty Von Maszewski, Deer Park Independent School District, Texas
Claire E. Weinstein, University of Texas, Austin
Patricia Sturdivant, Houston Independent School District, Texas

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June Dempsey, University of Houston, Texas, representing the American Association of Community and Junior Colleges, the Western College Reading Association, and the National Association for Remedial and Developmental Studies in Postsecondary Education
Jane Porter, College Board, Austin, Texas
Kay Bell, Texas Classroom Teachers Association, Austin, Texas
Judy Walker de Felix, University of Houston, Texas
Barbara Glave, University of Houston, Texas, representing the Houston Area Teachers of Foreign Language
Dora Scott, Houston Independent School District, Texas, representing the National Education Association and the Texas State Teachers Association, Houston
Georgette Sullins, Spring Independent School District, Texas
Renate Donovan, Spring Branch Independent School District, Texas

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Gordon M. Ambach, State Education Department, Albany, New York
Jo Bennett and *Jean Parochetti*, Alvin Community College, Texas
Sharon Robinson, National Education Association, Washington, D.C.
Donald L. Rubin, University of Georgia, Athens, representing the Speech Communication Association
Robert N. Schwartz, University of Houston, Texas
Ralph C. Staiger, International Reading Association, Newark, Delaware
Helen Warriner-Burke and *Carl L. Riehm*, Department of Education, Richmond, Virginia
William Work, Speech Communication Association, Annandale, Virginia
Daryl R. Yost, East Allen County Schools, New Haven, Indiana

Related Activities in Houston

Site visits coordinated by the Office of the General Superintendent of the Houston Independent School District

Briargrove Elementary School
Wilson Elementary School
Clifton Middle School
Bellaire High School
High School for Engineering Professions
High School for Health Professions
High School for Performing and Visual Arts

Teaching and Teacher Education

Realizing aptitudes and performance expectations early in the training program will force the teacher education student to determine if he or she will survive in a profession where effective members are those who believe all students can learn and take the responsibility upon themselves to see that they do.
Robert Fortenberry Jackson City Schools, Mississippi

Gary Sykes, National Institute of Education, Washington, D.C.
Gary Fenstermacher, Virginia Polytechnic Institute and State University, Blacksburg
David G. Imig, American Association of Colleges for Teacher Education, Washington, D.C.
Anne Flowers, Georgia Southern University, Statesboro
Barbara Peterson, Seven Oaks Elementary School, Columbia, South Carolina
Eva Galumbos, Southern Regional Education Board, Atlanta, Georgia
Robert Scanlon, Pennsylvania State Department of Education, Harrisburg
Ralph Turlington, Florida State Department of Education, Tallahassee
Gail MacColl, National Institute of Education, Washington, D.C.
Kathy Jones, Roan State Community College, Harriman, Tennessee, representing the National Education Association
Mary Lou Romaine, Atlanta Federation of Teachers, Georgia, representing the American Federation of Teachers
Janet Towslee-Collier, Georgia State University, Atlanta, representing the Association of Teacher Educators
Robert Fortenberry, Jackson City Schools, Mississippi, representing the American Association of School Administrators
Nicholas Hobar, West Virginia Department of Education, Charleston, representing the National Association of State Directors of Teacher Education and Certification
Fred Loveday, Georgia Private Education Council, Smyrna, representing the Council for American Private Education
James Lowden, Alabama Christian Education Association, Prattville, representing the American Association of Christian Schools
J.L. Grant, Florida State University, Tallahassee, representing the American Association for Colleges of Teacher Education
Carolyn Huseman, Georgia State Board of Education, representing the National Association of State Boards of Education

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Robert Fontenot, University of Southwestern Louisiana, LaFayette
Nancy Ramseur, Camden High School, South Carolina
Eugene Kelly, George Washington University, Washington, D.C.
Richard Hodges, Decatur, Georgia
James Gray, University of California, Berkeley
Robert Dixon, Institute for Research, Development and Engineering in Nuclear Energy, Atlanta, Georgia
Pat Woodall, Columbus, Georgia
Wayne Wheatley, Furman University, Greenville, South Carolina, representing the Council for Exceptional Children
Joe Hasenstab, Project Teach, Westwood, New Jersey
William Drummond, University of Florida, Gainesville
Debbie Yoho, Southeastern Regional Teacher Center, Columbia, South Carolina
Donald Gallehr, Virginia Writing Project, Fairfax
James Collins, National Council of States on In-service Education, Syracuse, New York
Ann Levy, Project New Adventure in Learning, Tallahassee, Florida
Bill Katzenmeyer, University of South Florida, Tampa
Walt Mika, Virginia Education Association
Eunice Sims, Georgia Writing Project, Atlanta

Gordon M. Ambach, State Education Department, Albany, New York
Elaine Banks and *Sam Sava*, National Association of Elementary School Principals, Reston, Virginia
Aladino A. Burchianti, Masontown, Pennsylvania
Roy Edelfelt, Washington, D.C.
Ed Foglia, California Teachers Association, Burlingame
June Johnson, New Adventure in Learning, Tallahassee, Florida
Richard A. Krueger, Staples Teacher Center, Minnesota
Clare Miezio, Eagle Forum Education Committee, Schaumburg, Illinois
Donald L. Rubin, University of Georgia, Athens, representing the Speech Communication Association Committee on Assessment and Testing
Daryl R. Yost, East Allen County Schools, New Haven, Indiana

Related Activities in Atlanta

Site Visits
Douglas High School
    L.W. Butts, Principal
Mays High School
    Thomas E. Wood, Jr., Principal
Lunch with local dignitaries hosted by Georgia State University
Dinner with business, education, and community leaders
    Coordinated by the Atlanta Partnership of Business and Education
    Sponsored by FABRAP Architects, Inc., and the Coca-Cola Company

College Admissions and the Transition to Postsecondary Education

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We're in the student learning business, and if we're going to have effectiveness in terms of student learning we've got to have good teachers, and we've got to have sound management.

Ralph Turlington Florida State Department of Education Tallahassee

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Clifford Sjogren, University of Michigan, Ann Arbor
Ralph McGee, New Trier Township High School, Winnetka, Illinois
Alice Cox, University of California Systemwide Administration, Berkeley
George Stafford, Prairie View A&M University, Texas
Fred Hargadon, Stanford University, California
Margaret MacVicar, Massachusetts Institute of Technology, Cambridge

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Lois Mazzuca, National Association of College Admissions Counselors, Rolling Meadows, Illinois
Ora McConnor, Chicago Public Schools, Illinois
Theodore Brown, Hales Franciscan High School, Chicago, Illinois
Charles D. O'Connell, University of Chicago, Illinois
Oscar Shabat, Chicago Community College System, Illinois
Arnold Mitchum, Marquette University, Milwaukee, Wisconsin
Michael Kean, Educational Testing Service, Midwestern Regional Office, Evanston, Illinois
John B. Vaccaro, The College Board, Midwestern Regional Office, Evanston, Illinois

William Kinnison, Wittenberg University, Springfield, Ohio

William J. Pappas, Northview High School, Grand Rapids, Michigan
Carmelo Rodriguez, ASPIRA of Illinois, Chicago
Jeffrey Mallow, Loyola University, Chicago, Illinois
Carol Elder, Local 4100 of American Federation of Teachers, Chicago, Illinois
Bettye J. Lewis, Michigan Alliance of Families
Rachel Ralya, Michigan Alliance of Families
Austin Doherty, Alverno College, Milwaukee, Wisconsin

Gordon M. Ambach, State Education Department, Albany, New York
Gordon C. Godbey, Pennsylvania Association for Adult Continuing Education
Daryl R. Yost, East Allen County Schools, New Haven, Indiana

Related Activities in Chicago

Site Visits
Standard Oil of Indiana

Gene E. Cartwright, Manager of Employee Relations
Joseph Feeney, Director, Training and Personnel Planning
Continental Illinois Bank
Jennifer Olsztynski, Personnel Manager
De Paul University
Rev. John T. Richardson, President
David Justice, Dean, School for New Learning
Luncheon with leaders of higher education institutions
Sponsored by the John D. and Catherine T. MacArthur Foundation
Dinner with business, education, and community leaders
Sponsored by the John D. and Catherine T. MacArthur Foundation
Chaired by Stanley O. Ikenberry, President, University of Illinois

Education for a Productive Role in a Productive Society

Fortunately for my students, I have found a school district where teachers are considered valuable professionals and where professional development is taken seriously.

Debbie Yoho Southeastern Regional Teacher Center Columbia, South Carolina

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Daniel Saks, Brookings Institution, Washington, D.C.
Roy Forbes, Education Commission of the States, Denver, Colorado
Sol Hurwitz, Committee for Economic Development, New York, New York
Martha Brownlee, Naval Education and Training for Research and Development, Pensacola, Florida
Norman Pledger, Colorado AFL-CIO, Denver

Lucretia James, Storage Technology, Inc., Louisville, Colorado
Kathy Collins Smith, American Institute of Banking, Denver, Colorado
Wade Murphree, Denver Institute of Technology, Colorado
Calvin Frazier, State Department of Education, Denver, Colorado
Robert Taylor, The Ohio State University, Columbus
John Peper, Jefferson County Schools, Lakewood, Colorado
Michael A. MacDowell, Joint Council on Economic Education, New York, New York
Larry Brown, 70001, Inc., Washington, D.C.
Robert Stewart, University of Missouri, Columbia
Gordon Dickinson, Colorado Community College and Vocational Education Board, Sterling
Karl Weiss, Northeastern University, Boston, Massachusetts
Donald Schwartz, University of Colorado, Colorado Springs

Patricia Brevik, Auraria Library and Media Center, Denver, Colorado
John Dromgoole, National Commission on Cooperative Education, Boston, Massachusetts

Faith Hamre, Littleton Public Schools, Ohio
Vernon Broussard, National Council on Vocational Education, Culver City, California
David Terry, Utah System of Higher Education, Salt Lake City
Georgia Van Adestine, Western Michigan University, Kalamazoo, Michigan
Gordon E. Heaton, Colorado Education Association, Aurora, Colorado
Young Jay Mulkey, American Institute for Character Education, San Antonio, Texas
George P. Rusteika, Far West Laboratory for Educational Research and Development, San Francisco, California

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Gordon M. Ambach, State Education Department, Albany, New York
Donald Clark, National Association for Industry-Education Cooperation, Buffalo, New York
Jacqueline Danzberger, Youth-Work, Inc., Washington, D.C.
Charles Davis, Education Clinics, Inc., Seattle, Washington
Dennis A. Dirksen, San Diego State University, California
Ben Lawrence, National Center for Higher Education Management Systems, Boulder, Colorado
Bill Rosser and *Jennie Sanchez*, Chicano Education Project, Denver, Colorado
Sandra K. Squires, University of Nebraska, Omaha

Related Activities in the Denver Area

Site Visits
Warren Occupational Technology Center, Golden
    Byron Tucker, Principal
Mountain Bell Education and Training Center, Lakewood
    Fred Wells, Director
Career Education Center, Denver
    John Astuno, Principal
Emily Griffith Opportunity School, Denver
    Butch Thomas, Principal
Luncheon discussion with Robert Worthington, Assistant Secretary for Vocational and Adult Education, U. S. Department of Education, Washington, D.C.
Dinner discussion with Willard Wirtz, National Institute for Work and Learning, Washington, D.C., and Henry David, National Institute of Education, Washington, D.C.
Dinner with business, education, and community leaders
    Sponsored by the Education Commission of the States
    Chaired by Calvin Frazier, Commissioner of Education, Colorado

Education for the Gifted and Talented

Our greatest resource--and the greatest resource of any nation--is the education of its people.

Norman Pledger Colorado AFL-CIO
Denver

James J. Gallagher, University of North Carolina, Chapel Hill
Marcel Kinsbourne, Eunice Kennedy Shriver Center, Waltham, Massachusetts
Joseph Renzulli, University of Connecticut, Storrs
David Feldman, Tufts University, Medford, Massachusetts

William Durden, Johns Hopkins University, Baltimore, Maryland
Connie Steele, Texas Technical University, Lubbock
Isa Kaftal Zimmerman, Lexington Public Schools, Massachusetts
Alexinia Baldwin, State University of New York, Albany

Arthur Pontarelli, Rhode Island State Department of Education, Providence
Armand E. Bastastini, Jr., Rhode Island State Legislature, Providence
William R. Holland, Narragansett School District, Rhode Island
Melissa Lawton, Bristol School District, Rhode Island
Rachel Christina, Bristol School District, Rhode Island
Catherine Valentino, North Kingstown School District, Rhode Island
Marie Friedel, National Foundation for Gifted and Creative Children, Providence, Rhode Island
Marsha R. Berger, Rhode Island Federation of Teachers, Providence
Sidney Rollins, Rhode Island College, Providence
David Laux, State Advocates for Gifted Education, Providence, Rhode Island
James A. Di Prete, Coventry High School, Rhode Island
Harold Raynolds, Maine State Department of Education, Augusta
June K. Goodman, Connecticut State Board of Education, Hartford
Mary Hunter Wolfe, Connecticut State Task Force on Gifted and Talented Education, Hartford
Paul Regnier, speaking on behalf of Gordon Ambach, State Education Department, Albany, New York
Benson Snyder, Massachusetts Institute of Technology, Cambridge, Massachusetts
June Cox, Sid Richardson Foundation, Fort Worth, Texas
Loretta L. Frissora, Needham Public Schools, Massachusetts, representing the National Education Association
Patricia O'Connell, Augusta, Maine, representing the Council of State Directors for Programs for the Gifted

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Gloria Duclos, University of Southern Maine, Portland
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Rhoda Spear, New Haven Schools, Connecticut
Judith Grunbaum, Southeastern Massachusetts University, North Dartmouth
Vincent Hawes, American Association of State Colleges and Universities, Washington, D.C.
Dorothy Moser, Mortar Board, Inc., Columbus, Ohio
Wendy Mareks, Chelmsford Association for Talented and Gifted, Massachusetts
James DeLisle, University of Connecticut, Storrs
Naomi Zymelman, Charles E. Smith Jewish Day School, Rockville, Maryland

Sherry Earle, Connecticut Association for the Gifted, Danbury
C. Grey Austin, University of Georgia, Athens
Sally Reis, Council for Exceptional Children, Talented and Gifted Division, Reston, Virginia
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Roberta McHardy, Louisiana Department of Education, Baton Rouge
Felicity Freund, Gifted Child Society, Oakland, New Jersey
Lydia Smith, Simmons College, Boston, Massachusetts
Betsy Buchbinder, Massachusetts Association for Advancement of Individual Potential, Milton
Artemis Kirk, Simmons College, Boston, Massachusetts, representing the Association of College and Research Libraries

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Elizabeth F. Abbott, Governor's Program for Gifted and Talented, Gainesville, Florida
James Alvino, Gifted Child Newsletter, Sewell, New Jersey
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Philip J. Burke and *Karen A. Verbeke*, University of Maryland, College Park
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California Association for the Gifted, Downey
Carolyn M. Callahan, The Association for the Gifted
Anne B. Crabbe, Coe College, Cedar Rapids, Iowa
Roxanne H. Cramer, American Mensa, Arlington, Virginia
Neil Daniel, Texas Christian University, Fort Worth
Sue Ellen Duggan and *Mary Lou Fernandes*, Lackawanna City School District, New York
John F. Feldhusen, Purdue University, West Lafayette, Indiana *Frank F. Fowle, III*, Clayton, Missouri
Joseph Harrington, College Academy, Stoughton, Massachusetts
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Betty Johnson, Minnesota Council for the Gifted and Talented, Minneapolis
Nancy Kalajian, Sommerville, Massachusetts
John Lawson, Massachusetts Department of Education, Quincy
Barbara Lindsey, Southwest Iowans for Talented and Gifted, Council Bluffs
Diane Modest, Framingham Public Schools, Massachusetts
Jack L. Omond, Office for the Gifted, Port Elizabeth, South Africa
Arthur Purcell, Resource Policy Institute, Washington, D.C.
Annette Raphel, Milton Academy, Massachusetts
Susanne Richert, Educational Improvement Center, Sewell, New Jersey
Carl L. Riehm, Virginia State Department of Education, Richmond
Terry Ruby, Raynham Public Schools, Massachusetts
Barbara Moore Schuch, San Diego City Schools, California
Dorothy Sisk, University of South Florida, Tampa
Mercedes Smith, Gifted Association of Missouri, Springfield
Christopher L. Sny, Janesville Public Schools, Wisconsin
Julian C. Stanley, SMPY, Department of Psychology, Johns Hopkins University,

Baltimore, Maryland
Jo Thomason and *Frederick J. Weintraub*, Council for Exceptional Children, Reston, Virginia
Jo Anne Welch, Mississippi Association for the Talented and Gifted

Related Activities in the Boston Area

Site Visits
Buckingham, Brown and Nichols School, Carnbridge
Peter Gunness, Headmaster
Brookline High School, Brookline
Robert McCarthy, Headmaster

Secretary's Regional Representatives

...within any human group, any ethnic or socio-economic sample, there will be people of high intellectual potential but none of them will realize their potential unless they are also afforded the opportunity to do so.
Marcel Kinsbourne Eunice Kennedy Shriver Center Waltham, Massachusetts

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The Secretary's Regional Representatives held their own conferences or hearings for educators in their regions in order to provide additional testimony to the Commission. In addition to these events, they also supported the hearings the Commission sponsored in their regions.

Region I, *Wayne Roberts*
Boston, Massachusetts
Forum on Effective Schools, September 16, 1982

Region II, *Lorraine Colville*
New York, New York
Forum on Excellence, October 21, 1982

Region III, *Joseph Ambrosino*
Philadelphia, Pennsylvania
Hearing/Conference on Cooperative Education, October 11, 1982

Region IV, *Ted B. Freeman*
Atlanta, Georgia
Public Meeting on Excellence in Education, October 22, 1982

Region V, *Harold Wright*
Chicago, Illinois
Excellence in Education: Preparation for the Transition to Higher Education, October 6, 1982

Region VI, *Scott Tuxhorn*
Dallas, Texas
Public Hearing on Excellence in Education, October 4, 1982

Region VII, *Cynthia A. Harris*
Kansas City, Missouri
Rural and Small Schools Excellence, October 26, 1982

Region VIII, *Tom Tancredo*
Denver, Colorado
Conference on Excellence in Education, November 12-13, 1982

Region IX, *Eugene Gonzales*
San Francisco, California
The Teacher: Key to Excellence in the Classroom, October 18, 1982

Region X, *George Hood*
Seattle, Washington
Public Hearing, June 25, 1982, August 27, 1982
(Hearing Officer: Hyrum M. Smith)

Transcripts of the preceding hearings sponsored by and for the Commission will be available in the ERIC System (See Ordering Information).

In addition to these hearings sponsored by and for the Commission, Commission members participated in a series of site visits and a public hearing focusing on Excellence in Rural Education. These events took place on April 23-24, 1982, in Kentucky. The hearing was held at the University of Kentucky-Somerset Community College.

Appendix E: Other Presentations to the Commission

Adrienne Bailey, The College Board, New York, New York
Stephen Bailey, Harvard Graduate School of Education, Cambridge, Massachusetts
Irene Bandy, Ohio Department of Education, Columbus
Elias Blake, Clark College, Atlanta, Georgia
Lewis M. Branscomb, National Science Board, Washington, D.C.
David Burnett, University of Pennsylvania, Philadelphia
Lawrence Cremin, Teachers College, Columbia University, New York, New York
James V. Gaddy, New Rochelle High School, New York
John Goodlad, University of California, Los Angeles
Elaine Hairston, Ohio Board of Regents, Columbus
John Hurley, INA Corporation (Now CIGNA), Philadelphia, Pennsylvania
Edward Kelly, State University of New York at Albany
Robert McMillan, University of Rhode Island, Kingston
Edward Pellegrino, Georgetown Medical Center, Washington, D.C.
Francis Roberts, National Endowment for the Humanities, Washington, D.C.
David S. Seeley, Staten Island, New York
John Sprott, U.S. Department of State, Washington, D.C.
Carol Stoel, U.S. Department of Education, Washington, D.C.
Abraham Tannenbaum, Teachers College, Columbia University, New York, New York
Harold Tragash, Xerox Corporation, Stamford, Connecticut

Appendix F: Notable Programs

Institutions Which Submitted Profiles of Programs

With the assistance of a variety of organizations, the Commission conducted four searches for examples of notable programs and promising approaches to specific problems in American education. Our purpose was to understand better how schools, school districts, colleges, and other education organizations were defining and addressing these problems. Where the evidence was convincing, we also sought to learn what made successful programs work in different settings.

The Commission's procedure in these four searches was to solicit original profiles of these programs and approaches, profiles that would answer a number of key questions concerning their purpose, content, organization, impact, and transferability.

Evidence of program success was provided wholly by the institution submitting the profile. The Commission is, thus, in no position to validate these programs or to claim any of them to be "exemplary."

Over 200 schools, school districts, colleges, and other educational organizations responded to our solicitations. They sent in profiles and other descriptions of nearly 300 programs. Due to the specific problems on which we were seeking information (e. g., the transition from secondary to postsecondary education, the use of educational technology, mathematics education, cooperative educational ventures with business and industry), most of the respondents were postsecondary institutions. But many of the profiles submitted by colleges involved programs developed for or with elementary and/or secondary schools and are in operation in many school districts.

For their assistance in the efforts to identify and solicit this information, we are particularly grateful to the American Council on Education, the American Association for Higher Education, the American Association of State Colleges and Universities, the American Association of Community and Junior Colleges, the National Association of Secondary School Principals, the Academy for Educational Development, the Council on American Private Education, and the Fund for the Improvement of Postsecondary Education.

The following document will be available in the ERIC System sometime after July, 1983, (See Ordering Information):

Clifford Adelman, National Institute of Education, Washington, D.C.
Elaine Reuben, Elaine Reuben Associates, Washington, D.C.
"Notable Programs in American Postsecondary Education: Selected Analytical Abstracts"

Appendix G: Acknowledgments

We want to express particular appreciation to the Commission staff which, under the leadership of Executive Director Milton Goldberg, assisted us in our work and helped prepare this report. The staff included:

Betty S. Baten
Stella Carol Foley
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Jarnes Harvey
Arnetta D. LaGrone
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Mollie Shannahan MacAdarns
Penny S. McDonald
Shelia L. Sarn
Haroldie K. Spriggs
Tommy M. Tornlinson
Susan Traiman
Patricia A. Welch

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Metaphysics

Monday, June 5, 2023

One can say that Metaphysics is the education required for learning the elements of mapping a good future. This education is a biological imperative, required for human survival. That man would eventually discover this fact was written about thousands of years ago, and therefore, impossible to be of human origins. A mind is designed to learn how to do its job, and therefore could not function until a cusp point in evolution, a time that mankind would be intelligent enough to recognize it; and as being designed to learn, it must first know the elements of its job. There is one, and only one path to that end; your ability to learn it depends on you: You have to learn exactly what one is, and how to use it to survive and this is an unescapable biological fact.

The first principle concerning purpose, reasoning can be stated thus: What may be predicated of any-thing is wholly determined by the definition of that thing. It is a statement of the Law of Identity over every system of grammar.

The thing, the first principles, which we should be of most interested in, is ourselves. In order to learn about ourself we have to become proficient in the ability to ask and answer questions, especially concerning ourselves. What are we, and why are we?

We are a mind whose job is to perform judgment concerning our every behavior such that that behavior maintains and promotes life. Life is a never-ending job, and we have a perfect method of doing that job: binary recursion. Reasoning is a process which cannot be seen, so we do and can learn, how to manage the perceptible via the invisible. One part of a binary cannot be seen called the intelligible, but it is the part which manages the perceptible. Intelligence is proportional to the Principles of Language which are functionally resident in a mind as a Grammar Matrix derived from binary recursion; binary meaning the two parts of a thing, also known as the Intelligible Elements. Metaphysics is a map of the Intelligible, invisible, Grammar Matrix which affords a sapient species life.

A brain is one of the life support systems of the body. One of its products is called Mind, we are that mind. As a mind is a biological product a mind has a well-defined biologically determined job to perform and well-defined physically determined means of doing that job. As Plato stated, the obvious, our concern is doing our own work and no one can do it, if they are even ignorant of what they are, why they are, and how to do their biologically defined job. Metaphysics presents us the ability to learn the fundamentals of doing our own work. Our job is predictive behavior, the utilization of memory by which we can formulate recursive life supporting behavior. All of the fundamentals are capable of demonstrating Total Recall and application. Correct memory management is called judgment, and the application of that judgment affords us life.

It is a very simple fact, doing our own work as a mind and we live, and live more abundantly or not doing our biologically defined job and we die. Look around you, man is destroying the earth, dying is not a pleasant experience.

Grammar, whose function is judgment, is the currency by which sapient behavior is exchanged for life.

All we can do, as a mind, is to read, using our body called perception, process what we have read in order to produce judgments in response to the environment, called conception and judgment, and write back to the body the results of those judgments in order to produce our behavior, called will. There is nothing mysterious about the general statement, however, most people do not know the particulars of information processing using a system of perceptible paradigms each of which shows proven mental operations.

Our foundation is then judgment afforded by literacy which we use to regulate behavior so that we can have life, and have it more abundantly, and as our purpose and our abilities are well defined, we order and lay out those principles and call this foundation Metaphysics.

We process the memory of things and their interactions. In order to understand which part of a thing we can change; we have to comprehend what a thing is in the complete abstract. A thing is a binary construct of a relative within correlatives. A mind functions by parsing and ordering information. Parsing is the application of limits, boundaries, i.e., the correlative. As Aristotle noted, the verb, the relative is a given; it cannot be produced. One can say that we shape material to our advantage, so too the mind. Total recall is not about the particulars, it is about the methods of parsing information by which we gain advantage. Thus, in the following figure we can only add the correlatives:

The relative is a given. We do not create the material difference, a sculptor does not create the stone he shapes; he only shapes what is given; we sculpt our own behavior, by the definition of mind, and this is physically factual. By fact and by design we do not create anything except order by effecting judgment as to where a correlative is required. One can view what they see between the two correlatives is a record of memory, the memory of the motion of our own hand.

Judgment is the ability to apply correlatives in order to accomplish a given result, in the Universal, that result is, we get to live. Parroting gibberish is not judgment.

As we can see with our visible paradigm, one can also say that a thing is called a standard of behavior for as standard is to the correlative so too the relative is to behavior. Memory is then, a record of our own behavior, our parsing of sense perception. By biological fact, we, as mind, are responsible for the production of standards of life supporting behavior and the only method available to us is through binary recursion effected by asserting, or denying, correlatives to some given relative. By biological fact, we are craftsmen, designed

to impose standards of behavior acrost our own environment, but we must first impose it upon ourselves.

Grammar is then, no more than recorded memory used for playback in order to pass on that memory to our fellow man. This cannot be done, when man is still an illiterate savage.

As one may guess, all the mental operations performable are determined also as a function of binary recursion. This led Plato to formulate an observable fact, that there are two kinds of knowledge, the perceptible brought to us by sense perception, and the inborn ability to compare them for equality and inequality from which all operations upon information can be performed called the intelligible. Therefore, we have the visible, afforded by perception or the perceptible, and the invisible afforded by the intelligible, just like our binary an effect we notice when one relative ends and another begins. Metaphysics, grammar systems, are formulated to build on that ability, i.e., methods of effecting binary recursion. Thus, grammar systems are focused on how to denote a binary, and how to effect recursion with that given system of notation. Thus, it should have been obvious that a mind functions wholly by managing correlatives.

Common Notions

We discern limits when we observe a change in relatives. These changes are best discerned by the eye, followed by tactile sense. The senses are designed for elementary parsing of information. Change in odor, for smell, change in air pressure for sound, change in tactile qualities for touch, and changes in light. Thus, there are common notions, the ability to judge equal and not equal as greater and less in accordance with some sense. This ability is a given. As Aristotle noted, the loss of a sense perception is the loss of all the knowledge it affords us. Common Notions are all derived from this form of knowledge and that is why they are called common, they are common to all forms of life. The mind, then, is measured by how we can use this information for survival. And when someone is recognized not to be able to take advantage of this biological ability, they are said not to have common sense. Common sense affords us common notions.

Thus, senses are designed for the elementary parsing of the perceptible. A mind is designed to predict the results of these changes and this is called rationalization, judgment, and even prophecy.

We learn intelligibly how to take advantage of sense perception by producing what has been called mathematical operations. These operations are also common to all members of our Grammar Matrix.

Thus, survival of the most fit, starts with sense perception followed by our ability to judge that information for being bad or good. Our function is designed to achieve the good, i.e., life. The next step in survival of the most fit is memory, how long can one use that information for the good? When one recognizes this third step, then one knows what the function of grammar systems are for, the recursion of memory, itself.

As we are a life support system, it is hard to imagine that people would call the good anything else than doing our own work to attain to it, life unless they are incredibly dim witted.

Common Notions then led to statements about equal, not equal, greater, less, and the results being developed into addition, subtraction, multiplication, and division, or four basic mathematical operations designed for predictive behavior. One will notice that these four are also a product of two squared. Two perceptible, times two intelligible. They are performed on both the relative and correlative meaning, arithmetic, and geometric recursion. In short, they are applicable to both parts of a thing, also known as the literal and the metaphorical. As the literal is to the correlative, i.e., noun, so too, the metaphorical is to the relative, i.e., verb.

How many methods of binary recursion are there for constructing grammar systems? Or, in the simple, how do we describe the two parts of a binary as an intelligible multiplied by the two parts of grammar which is perceptible and what is the result? What is two times two?

Grammar Matrix

Grammar is also a particular thing which consists of two parts and all we have to do is name those parts and establish them in a one-to-one correspondence with our universal binary.

Every system of grammar is composed of symbol sets, which are absolute, and methods of managing those symbols which is our relative behavior with them.

In Common Grammar we have symbols which are constructed with many motions of the hand, while the management of those symbols are also many. Common Grammar is optimized for non-organized information and when one does not learn how to order one's words, then one often spout words which are factually gibberish. One can see, then, why it is preferred by those who use it for the recursion of emotion. Most people only use Common Grammar by rote, which, unfortunately, is the same way it is taught by those who may actually believe that they are speaking sense. Operations in Common Grammar and the application of words as to their being literal or metaphorical is never clearly distinguished.

Nothing is worse than a senseless teacher save, when one has no teacher at all. Thus, even a bad teacher is better than no teacher at all.

As by biological fact, grammar systems are used to teach the fundamental behaviors of our Universal Binary Unit, a grammar book which does not instruct you as to how to manage that behavior of a thing, as a universal, is not a grammar book at all.

As we name both relatives and correlatives in Common Grammar, then there is no method of discerning when one is used as the other except through memory. As Plato noted, it takes rigorous mental exercises to keep from making mistakes in Common Grammar and a retentive memory. We have to learn to

instill in memory, nouns and verbs and how to use them, recursively in order to effect the basic management of fact. Part of that process involves dictionaries, which cannot be produced by an illiterate culture such as the one we have today. Common Grammar demands an extensive system of dictionaries and socially preserved, in books, and in practice; however, the fundamentals that all grammar systems are composed of, are relatively simple, after all, we have two, and only two concepts to master. The invention of new words happens often because a dictionary is not available, or someone is too stupid to know how to use a dictionary or because a sociopath is playing a shell game with words. Yet there are no grammar books today, socially used, which teaches our biologically given Grammar Matrix, nor are there any dictionaries today, which are founded on a convention of naming and its preservation save basic weights and measures of perceptible goods.

In Arithmetic, again, we have symbols which are constructed with many motions of the hand, while the management of those symbols is constrained or ordered by what is called place value notation.

However, since the Arithmetic Naming Convention is set up to denote a unit standard which is literal, results may either be rational or irrational, meaning that Arithmetic cannot always produce an exact name for some result. This condition is named irrational, the inability to provide an exact name. The results of Arithmetic management is called a number. Arithmetic is then tailored to the noun.

In Algebra we use both symbol sets, where Common Grammar can afford it more control over the verb, thus, for the most part, eliminating the irrational, but not the irrational behavior of mathematicians. Algebra gives us names which are independent of the noun, thus it is better equipt than arithmetic for use in exact results; however, it also requires a more extensive system of dictionaries which explain the grammar of the Common Grammar symbols.

In Algebra, as we use both symbol sets, then one can always write the grammar without the limitations of Arithmetic irrationality. So, no, an Algebraic name is not a number. The result of Algebraic management is called an equation, however, converting it to an Arithmetic number, the result cannot always be rational. Algebra is refined for the verb.

So far, we have started with common grammar, and then divided it into Arithmetic, i.e., recursion of the one, and Algebra, recursion of the many.

Geometry uses one symbol, the correlative, and we note the memory of the correlative called a loci, or line and again we use this to effect the memory of binary recursion by these loci, or lines. Each line is then a record of one relative, or many relatives, commonly called a line and a circle or curve. One can say that we have a binary and binary functions as the result of binary recursion.

Geometry can, as Plato noted, afford us visible paradigms, or Total Recall, of rational thought. It can visually present us with things metaphorically, and afford us visual memory of the operations on them, factual operations over all things. The result of Geometric management is always exact, converting the

result into any logical grammar is subject to that particular grammar's limitations. However, geometry also requires the greatest intelligence as it is wholly metaphorical; simple minds cannot see the metaphor at all or too dim wittedly.

We then start with a grammar; Common Grammar, whose naming convention does not stress the distinction between the noun and the verb, nor the literal and the metaphorical. To aid in reasoning we then develop Arithmetic aimed at the literal, while Algebra stresses the metaphorical but all of these retain symbol sets which are relative.

Geometry, the symbol set is arithmetic. With it we recognize only the application of the correlative, and its path displays the relative. This produces a grammar which is wholly metaphorical.

Every possible grammar is a method of utilizing binary recursion, be that as it may, binary recursion can only produce a binary result, and it is not possible for binary to contradict binary recursion; we contradict due to our level of illiteracy.

Summing all of this up, there are countless imbeciles who cannot multiply two times two, and claim that there are countless systems of grammar, for each of our four members, however, this only means, they do not even comprehend the notion of the unit, of one. Some of these people even construct new symbols, claiming that they are special, magical, but they are provably deluded.

The Logical Grammars, are then produced with many symbols, the Analogical Grammar of geometry, with only one, the correlative while the relative is recorded as a loci, or line. Therefore, in this stage of binary recursion, Geometry is the One over the many, Common Grammar, Arithmetic and Algebra, which agrees with the definition of mind, whose job is the regulation of behavior in order to maintain and promote life. We have made our way through the confusion of the many we were born into, into a single idea of our own behavior. One should also now see, why geometry, which actually is exhibited today as interactive Geometry, is becomes truly representative, for otherwise we would have to draw, as in the old days of cartoons, every step one frame at a time.

Straightedge and Compass

Like most of what Plato wrote, his words went right over people's head. Plato taught Dialectic, grammar in accordance with the two parts of a thing, correlative and relative, noun and verb. Another expression he used was the one and the many. And, if you did not know, it was Plato, not Euclid, who set the standard for the writing of geometry, and Plato set it, because he knew no other tools are required: the straightedge for producing the one, and the compass for producing the many of that particular unit. Now, people are rather simple minded, starting with a line segment, we go on to produce segments proportional to it, we do not claim that a figure has to be made up of unit segments. We do, however claim, that all of computation is afforded by a standard unit, or our starting binary. Binary is the first principle of existence and we can state;

Language is Universal and Intelligible afforded by a binary construct, while Grammar is Particular and Perceptible afforded by the intelligence of a form of life. And in recognition of the evolution of intelligence; Psychology is commensurate with the principles of Language which are functionally resident in a mind as Grammar Systems. As a mind functions wholly by binary recursion, the only two parts of a thing, this produces a fact that there are four, and only four, possible systems of Grammar which we can name as: Common Grammar, Arithmetic, Algebra and Geometry, or the two intelligible parts of a thing, multiplied by the two behaviors of the hand.

Another way to put the words 'the one and the many' is binary recursion, not curves which are produced by a circle. One can actually draw a circle with only straight lines. Or again, a simple segment implies both arithmetic and geometric recursion, or again, literal and metaphorical judgment, each of which are fully demonstrable in Geometry. The key is memory management which is the function of grammar to begin with. It is also a fact, and demonstrable, that a circle is simply the reciprocal of a straight line. Every possible grammar is a method of effecting binary recursion, and certainly, it has never mattered which two items one used to produce their result. The fact of the matter has always been, in geometry one has never needed more than these two tools, but it has never meant that a loci, produced during binary recursion had to be the result of only one kind of curved line. Binary recursion always produces a binary result, and what every grammar uses to produce that result is only the application of the correlative which is not actually limited by the compass or straightedge. One is not limited by the tools, but in their intelligence on how to use them. In information processing we can only parse information, we can neither create nor destroy it. We can, however, create and destroy its memory within us, and its availability through grammar for others.

If one does not understand grammar basics, they will find a lot of things impossible, that is what it means to be stupid. Therefore, the study of Metaphysics is not about the topics you find in a dictionary, those topics are only part of what one learns, Metaphysics is about biological fact, doing your own work in effecting binary recursion using a Grammar Matrix of Common Grammar, Arithmetic, Algebra and Geometry while using Geometry to construct visual paradigms in order to proof your comprehension. If you are looking to parrot words from works titled Metaphysics, ask yourself one question, which of them proved their words, and which of them showed how to prove yours? One can confuse themselves and others in the logical grammars, which only denotes the inability to keep one's agreement with random symbols, however, in geometry one can construct the very same things and visually examine any result. Geometry is the grammar of truth between body and mind, and it has the ability to standardize agreements in naming via logical systems of grammar, and the provable result of those agreements. The ability to give and keep those agreements are a measure of your intelligence, i.e., determine if you have a good brain or not.

Metaphysics is how to use the invisible to manage the visible, it is not about vain words. If one actually does understand this, then they can draw the map, they can demonstrate, via our Grammar Matrix, the same, concerning the same, in all four systems of grammar.

Evolution of Man

As man is defined by the Mind, it stands to reason that the really important evolution to comprehend is that of its ability to do its own work. Thus, our evolution is mirrored by our grammar systems. Psychology is commensurate with the principles of Language which are resident in the mind as grammar systems. First there is Geometry, the behaviors of mankind codified for socially organized survival. Then Common Grammar, Arithmetic, and Algebra. Grammar started with Geometry, and it will be the last formalized. Only when they are first used, and continue to be used can one trace their individual evolution and their evolution relative to each other.

I recommend, that before you try to tackle my work, you read A Nation at Risk a companion file to this one. You might understand why, in the 50's a state I.Q. tester expecting a mentally challenged student as the school officials informed him, he almost pissed his pants when I was tested. It was said that I was even smarter than the schools known genius. I am not bragging, but I do not know why people find simple ideas so hard other than evolution. I was not doing the work, because all the books I read I could see the flaws and they were extensive. I lived in a world of functionally illiterate people and that is scary, but, at the same time, I knew I was ignorant and it would take a long time to fix that problem on my own. An I.Q. tests potential, but without teachers, how could I develop it? How does one find their teachers if they do not even look? For a long time, I read. Eventually, I could see a hole in the mind of man. He could discern greater and less in terms of the body, pleasure and pain, but not of his own intelligence, the result is that he imagined that every intelligent form of life had some expression of his own mental limitations. Man could not see his own future. Our future can be understood as: What may be predicated of any thing is wholly determined by the definition of that thing, therefore, man is ignorant of what he is and thus why he is. A person claiming to know themselves, and not know their own work, is clearly simple and delusional.

I have been banned on almost every website, for my attempts to explain Metaphysics. I am shunned by organized educational, and even NASA websites because I can prove fact. I still live today among strangers in a very strange land. I was, however, asked a long time ago, to learn judgment and to help mankind in changing this world for one that is not strange at all, one that is a product which maintains the Garden of God. For the intelligible, God, the only power a mind can know, is binary recursion, from the perceptible to the intelligible. We simply learn to pass it on.

The time to go from one place to another is inversely proportional to the length of your legs so, we better get a move on.

The Unit

Friday, June 2, 2023

There are two concepts afforded by our unit which early Greeks had a lot of fun with. Is a thing motionless, or is a thing in motion? i.e., is a thing a noun or a verb. It may sound like nonsense, but it gives us two kinds of units, which depend upon which you are trying to effectively recurse. One is called Arithmetic and the other Geometric. In short, are we effecting induction, i.e., adding, or deduction, i.e., dividing?

In order to demonstrate our knowledge about any thing we first must acquire that knowledge. The ability to manage information is only possible after the acquisition of information and one's ability to demonstrate what they have learnt is a display of that person's intelligence. Even grammar must first be learnt and used prior to being able to organize it, sift and sort it, comprehend the common ideas among all of it, before actually taking up a serious study of it. Many who write dictionaries have constructed them too early in their life, and late in life, there is not enough time left to make such a product.

It seems that dictionary writers have never studied Plato, for if you look up the word thing in a dictionary, you do not get a definition, but a whole lot of gibberish. Plato demonstrated this in his dialogs, ask someone to defined some intelligible thing, and the first reaction is often a heap of words involving particular things. The confusion is the inability to comprehend the distinction between the intelligible and the perceptible. However, if one has the least bit of common sense, they know that in terms of grammar, we must first have systems of grammar, all of which rely on the naming of things and the parts of things learnt by, as Aristotle learnt from Plato, induction, i.e., perception and as everyone should have learnt as a child, by pairing intelligible names with perceptible objects. One must learn the evolution of grammar by using grammar for correcting it. This is done by finding first the most accurate, and applying it to the simpler grammars first to learn them.

Synonyms for 'a thing' are 'a unit' and even 'a group.' So, let us provide for ourself a perceptible paradigm for all of these synonyms.

Therefore, by grammatical and biological fact, if someone wanted to know the definition of a thing, the simplest thing to show anyone is the simplest thing which we can produce with our own hand. And, as learnt from Plato, establish our relationships so that they are clear in our mind, viz. correlative is to the noun as relative is to verb; or again, correlative : noun :: relative : verb. And, repeating as Aristotle was taught, we have the matter, i.e. verb, the form, i.e. correlative, and the combination of these, the concrete whole. Thus, we find the definition of a thing in Aristotle, yet nobody seems to know it.

Many current drawing programs emphasize the fact of the limits, i.e., something which looks like Whimpie's barbells. Suffice it to say, in Geometry we define a thing by showing how to construct an example of the simplest possible

thing. It is called also a line-segment, however, when you use a particular name for a particular thing, people often never make it intelligible and their mind tends to spin off into the ozone layer of mythology. One cannot impose any behavior upon the given figure which would make it simpler. During the evolution of the brain, the ability to correlate information is often impossible for that mind.

A line segment is then the First Principle thing in geometry.

Now, there is a bit of a complicated metaphor for the above figure suggested perhaps by the Bible, consider it a walking-cane used to aid the blind for guiding one's steps. If you think about it, the mind is blind, the information reaching the mind is at the behest of other life support systems. A mind parses information, and once the information is parsed, it can be ordered, i.e., organized. So, our First Principle thing in Geometry is a walking-cane for the blind and we have to first construct it, pick it up, and learn how to wield it for walking and for defense.

What does the above mean? Well, by biological fact, we are a form of life and every form of life is composed of a number of life support systems. A life support system is that system of a living organism which must acquire things from the environment so that it can process those things for products which maintain and promote its life. A mind differs in one respect to the other life support systems, for each of them take advantage of particular things in the environment, where a mind must learn intelligible things exampled by everything. Thus, our body is feeding the brain, information expecting, in return, for its contribution to life. A brain is encased in a shell, and a stupid person is no better than a turtle without the contents of its shell.

How many parts of a thing are there?

How do we count the parts? Is the limit at one end different from the limit at the other? And is that difference the difference physically between them? i.e., are they different only by geographical location? Is common grammar, for example, limited to speaking about geographical locations? Or is arithmetic, and algebra so limited? But if that is what you may claim, did anyone ask for a difference of geographical location, or a different part of a thing? Is the difference between the limits of the line, any particular difference whatsoever? Does the word difference differ from itself? If the difference between terms is predicable of either term, what then would you call that difference? The gist is, is the perceptible difference the same as the intelligible difference? Perceptibly we focus on the wrong topic in regard to the question, intelligibly we do not get this distraction. So, one must learn that the limit of a thing is not the thing of which it is a limit, nor is the relative of a thing, relative to the thing of which it is a relative. Many people in the past have considered geometry something they make out of what is perceptible, literal. Geometry is wholly metaphorical. If one asserted that the difference belonged to either relative, then one would be saying that there are factually two relatives, which is not true.

The relative difference between terms is not predicable of either term.

Thus, people differentiate themselves not by what they are, but by some chance relative difference such as color, geographical location, beliefs, gender, religion, etc., but none of those things are defining characteristics of Man. Such claims are made by the mentally dysfunctional.

A predicate are the two parts of a thing, grouped by the recursion of a correlative acting as a container for that group, i.e., Noun equals noun and verb: Viz., Jack is a man. It is called a definitive sentence. The grouping is done intelligibly, something no other sense of the body can do. This means that a definitive sentence is a geometric ratio, or again, metaphorical. The other type of sentence is literal, used for denoting synonyms, or members of a class. Jack is red, is not the same as Jack is a color. A literal statement is not definitive, it is arithmetic. Jack is red translates into Jack + red and then we have to ask, how was this addition made? By blushing? At any rate, Jack is now a data set, which does not, in anywise, change the definition of Jack as a man, red is not a defining characteristic of a man, nor is gender, beliefs, etc. A literal sentence, equates either two nouns or two verbs or both. Jack is red, is divisible into A man is A color. Fast is good. All of this should be obvious, however we have to learn how to parse in common grammar, and we should learn first using Arithmetic, Algebra and Geometry for it will take time to apply these notions into sentence parsing.

I can draw Jack is a man.

But now, the relative differences to what designates man, is a verb. Jack is a man, Noun equals noun + verb, and man is a verb. How do I add such particular relative differences to Jack such that I do not change the relative difference of Jack as measured by the standard of man? If I say Jack is red, I can draw that too.

And I am simply creating a synonym, the complexion of Jack is an example of the color red. The problem, currently with common grammar is that generally common grammar consists of terms which are not well defined but contextually defined. What is clear, there is no inference between the two logical statements as shown by the analogic. What should be clear, the first example is arithmetic, while the second geometric. Geometric, as we can see by the second example, is a proportion. Thus, in recursion, we can keep track of both.

Now, if one has read Plato's Theaetetus, then one should have learnt that these two parts of a thing are also called a thing's elements and that the limit is not the relative, nor the relative a limit. Stop, go, stop, and stop is not go, and go is not stop. Although every child is assumed to have learnt this, most PhD

holders demonstrate that they have not otherwise they would know that it is not possible to have more than two parts of speech as Plato tried to teach them, and as our own biology tries to teach us. For me, public education was very scary. Grown-ups, one and all, teaching gibberish, obviously mad. The fact of the matter is, in order to get an education, in a safe environment, one has to study in their own home. When one finds they are living in a world of mentally dysfunctional people, people they have to rely on, then one has to learn to anchor themselves. I was taught how to do that in a dream. I was shown that I had the ability to do it, even though it is hard. Fear is the mind killer, but then so is any emotion when not comprehended. You have to change people's mind and that is done by education, but first one must become educated.

As exampled by the computer, which can also process all information using binary recursion and produce any product with that ability, Grammar is binary, as Plato noted, composed of only relatives and correlatives, nouns and verbs, etc. So, we need to use that to come to an understanding of grammar systems. Given that grammar systems are things, then it is composed of just two parts, relatives and correlatives. Which part is correlative and which part relative?

Reviewing, the correlative part of grammar systems are its symbol sets. The relative part of grammar systems is the methods by which we use the symbols to effect binary recursion. In common grammar, also, we have many behaviors of the hand to construct the symbols and we have many behaviors by which to assign those symbols to name things.

In arithmetic, we have also, many symbols but the behavior imposed upon them is rooted in place value notation. Names are ordered. However, in arithmetic the subject of any particular computation is one thing; often not even named.

Algebra combines common grammar and arithmetic, taking symbols from both so that the grammar can effectively denote many interacting things.

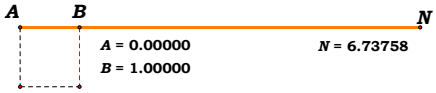
Geometry on the other hand is established by a one-to-one correspondence between relative and correlative. It is also the only grammar for, as Plato noted, producing universal perceptible paradigms, working examples.

And therefore, two multiplied by two, produces for us a Grammar Matrix: Common Grammar, Arithmetic, Algebra and Geometry, put into Biblical metaphor, the four corners of the earth, or four methods of effecting standards of judgment. With this knowledge, we should be able to figure out, because it is generally unsorted, that Common Grammar requires the most knowledge and work to comprehend, and therefore our preliminary work will be learning Arithmetic, Algebra and Geometry first, as Arithmetic and Algebra are much easier, because simpler, than Common Grammar. Many people believe that it is just the opposite because they have become habitualized into not thinking about what they are saying, parroting is not thought. One cannot judge the difficulty of a thing, when they factually are ignorant of it.

It is not possible to be literate in Common Grammar, but not in the other members of our Grammar Matrix. The operations performable upon things are

identical in every member of our Grammar Matrix, and if we do not know them, we cannot apply them, we are not rational because we can glue words together independent of factual operational mechanics. Every system of sentence parsing that I have seen to date, indicates that those who constructed them were simple minded and illiterate. Everything that they learnt in arithmetic, algebra and geometry was simply not used, but they deferred to the mist in their own mind. So, let us learn the paring of Arithmetic, Algebra, and Geometry to help us anchor our mind upon the ground so that even if we were one of these hot air balloons, we would never let the winds take us away.

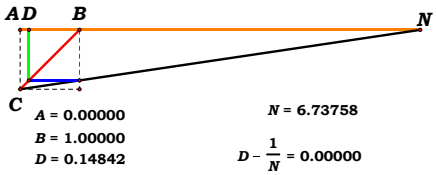
I will show the Four Corners of the intelligible earth, a way to manage information using geometry.



AB is the One, while AN is the Many. Notice the equation between Algebra and Geometry, keeping in mind that Algebra is a combination of Common Grammar and Arithmetic, which means that we have a grammar matrix visualizable in geometry. It is not a number line, but a Grammar Matrix and I will be showing, in the following pages, how to use it for Arithmetic and Geometric recursion of our binary. In geometry, the computations are always exact, and there is no processing time, i.e., output is concurrent with input, meaning faster than light travel as it is independent of speed.

So, how do we map the equality between the unit, AB and the result AN? AB is the One, while AN is the many. AC is our method of showing the relationship between them.

And we map the relationship like this:



For centuries, Geometer's have tried to comprehend this relationship between the one and the many, and have failed. Yet, take a look at it, it really is simple, is it not? With a single line, BC, we show the relationship over the whole of grammar, the idea is simple, its application is never ending. Therefore, the intelligible is not complicated, it is using it for each particular application which is often difficult.

So, consider, in computation, that AB is a correlative, while AN is a relative. AB is a Noun, while AN is a Verb, therefore, even in arithmetic, the numerator is the noun, and the denominator, our verb. We have two, and only two, categories

of names, and we must take a moment to identify which is which, when we think. Our Universal Binary, denotes this division between a limit, and the material difference within those limits. Think of it as a stick, and taking up another, we are rubbing them together to make fire, to turn on the light.

Recursion

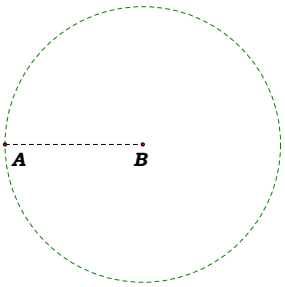
Our first recursion, known as complete induction and deduction, of our binary actually takes place by our ability to put it into one system of grammar, the grammar of our own behavior with our own hand: Geometry.

Recursion is afforded by Complete Induction and Deduction.

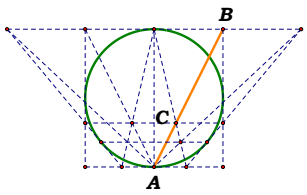
Starting with Geometry, we noted our second example of recursion, writing our First Principle Thing realizing that it is a memory stick of the application of correlatives:

Recursion is two kinds, explicit, for example, we can always make our FPT. The second is implicit, i.e., we could choose to have made the FPT any size we wished, i.e., we could have stopped our hand anywhere in construction, and we can place any number of correlatives within that same difference. The first method, is also called induction, establishing the perceptible. The second is called Deduction, using what we have to proceed with reasoning, again, managing the given data. Let us examine more sophisticated methods of recursion using our FPT.

What I am going to write are possible methods of recursion, both effecting deduction and induction. One of the items a compass affords us is memory not only of our FPT and immediate recursion from either correlative.



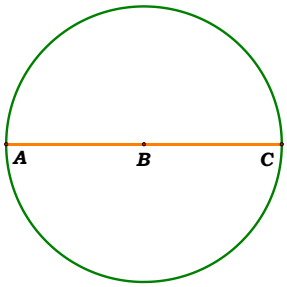
We can also produce what is called the circle at least two different ways using just lines, and writing it as a function of lines. One way is with a unit line, the other is produced with an infinite line, as a reciprocal. One can say that beyond small and great, is the circle. A circle is the reciprocal of the line. Or again, the verb is the reciprocal of the noun, and the noun is the reciprocal of the verb.



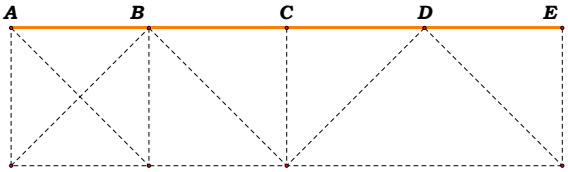
So, no, a compass is not just for drawing circles, nor are straight lines the only possible loci for points. And since the equation for the figure can also produce an ellipse, so no, an ellipse is not produced by cutting through an ice

cream cone even if it is the right flavor or not. And as you can see, all of geometry is effected by binary recursion, not the tools which aid us in writing in the grammar. I need a break. Do I say I squared a circle, or bounced around a square cage? At any rate, do not show stupidity claiming that a geometric construction is Euclidean, unless by choice, or the product of two tools made optional by a digital environment. Or, I can say the circle exists a priori in the realm of the unenlightened flat screen; if I had a tire pump to inflate myself. Or provably, I walked straight on forever, and eventually found myself back home again.

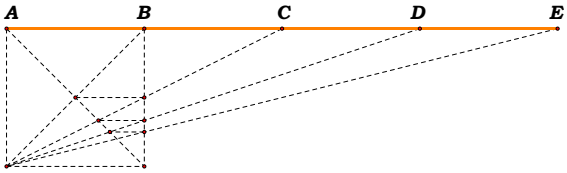
Another way to produce recursion, using the compass is in this wise. This produces arithmetic induction.



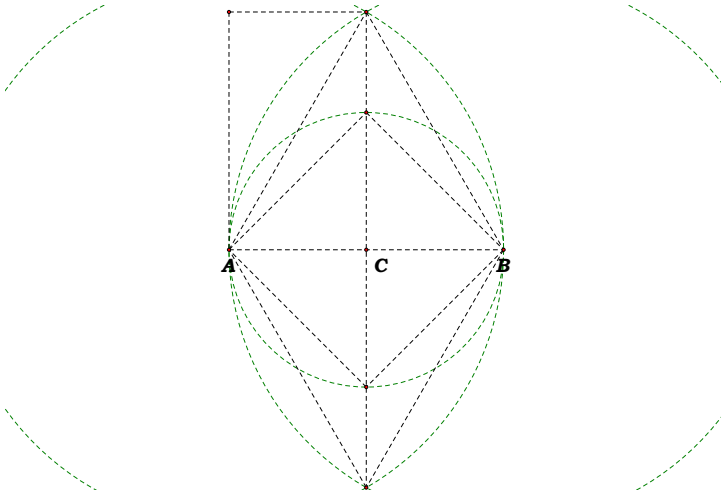
Demonstrated with just a straightedge



An example of using straight lines and proportion

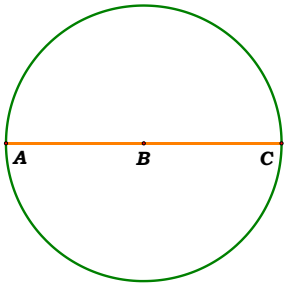


Recursion by deduction leads to an interesting figure:

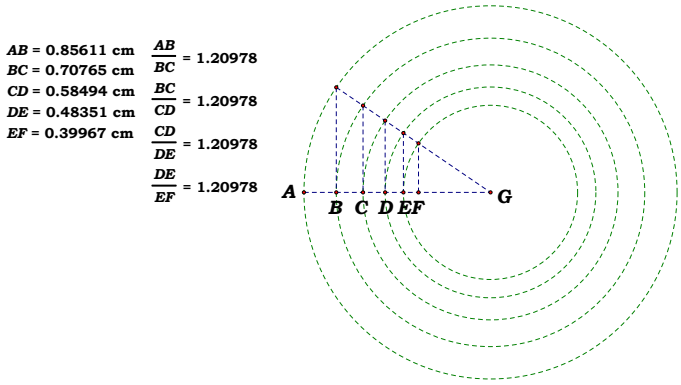


The figure presents us with all the fundamental topics of Euclid's presentation of the elements. However, something is missing, and apparently has been missing since the dawn of geometry, for when I once read an algebra book, the author claimed that exponential notation was not geometrically demonstrable, that it was a pure conceptual abstraction, which, frankly, is impossible, at least to me, for early on in my investigations, I was demonstrating it.

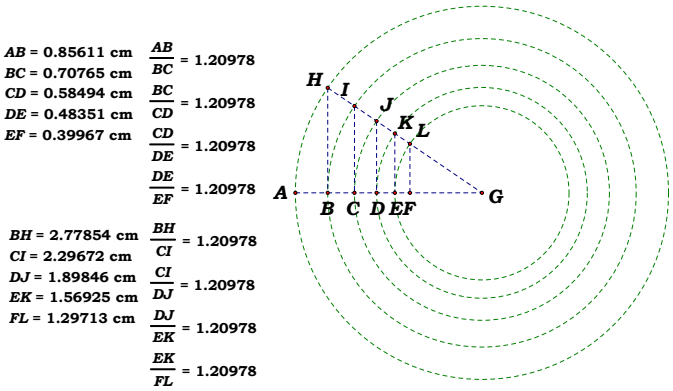
So far, in the above forms of recursion, we find one missing, for all of the results are actually arithmetic recursion. What is the mistake? How do we show the geometric recursion of the circle?



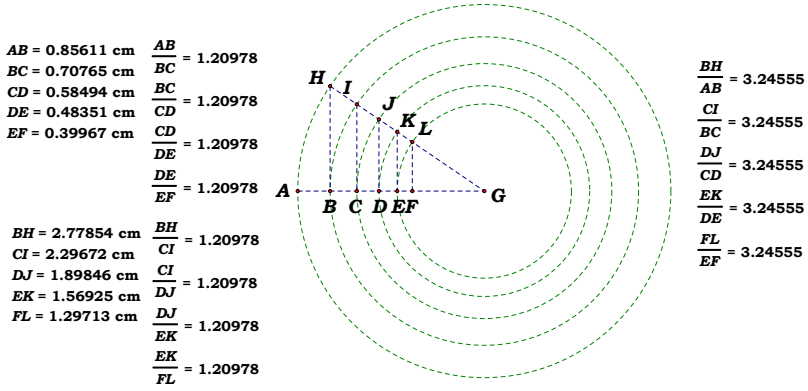
The above figure shows how to use the circle for arithmetic induction producing an arithmetic progression, and we can recursively apply the circle in that manner, but what constitutes the recursion of the circle deductively, geometrically? And since it has apparently been a thorn in geometer's side for so long, let me show it step by step where, given a circle we are going to add another line within the circle and learn geometric recursion.



We find we have another kind of unit called a geometric ratio, where in primitive times it was called an angle, and it was in the opening book of Euclid, defined as a ratio, i.e. geometric, not arithmetic, which readers quickly forgot. In the Elements of Euclid, it is defined as an inclination between two lines, i.e. a ratio which is not what people consider an angle to be today, where it is something mystical emanating from a not thing, a single and lonely correlative called a vertex or point. Current so-called teaching, does not account for the hole in the idea, the missing second correlative.

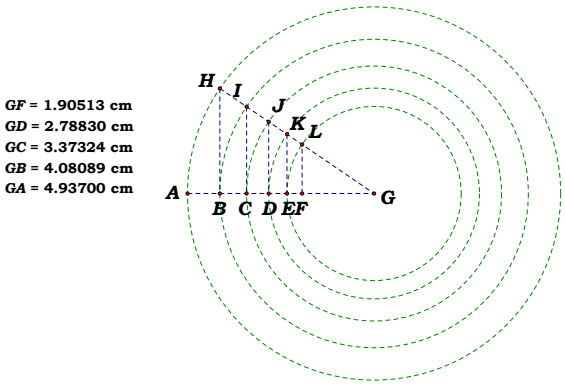


And, if we compare the lines vertically, they are in the exact same ratio.

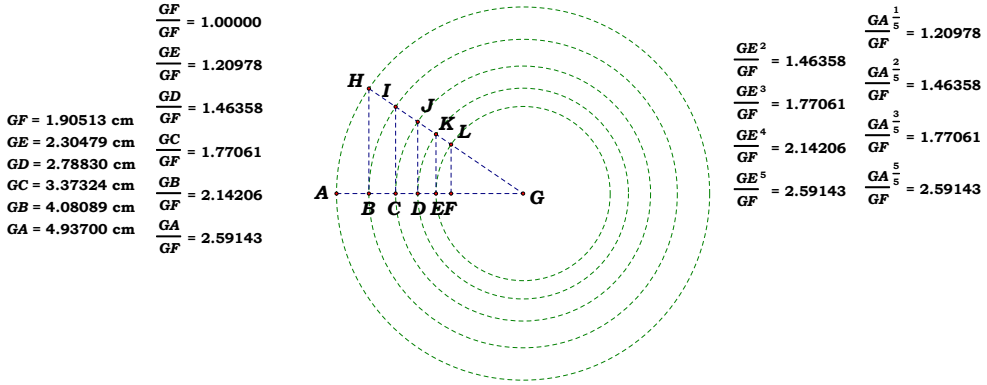


The inclination actually turns out to be 3.24555 in this example and it has absolutely nothing to do with the antiquated concept of degree which is just a an agreed upon number by illiterates. Once we have our Geometric unit of

progression, we can pair it to an arithmetic progression. Let us take what remains, FG and call it a unit.

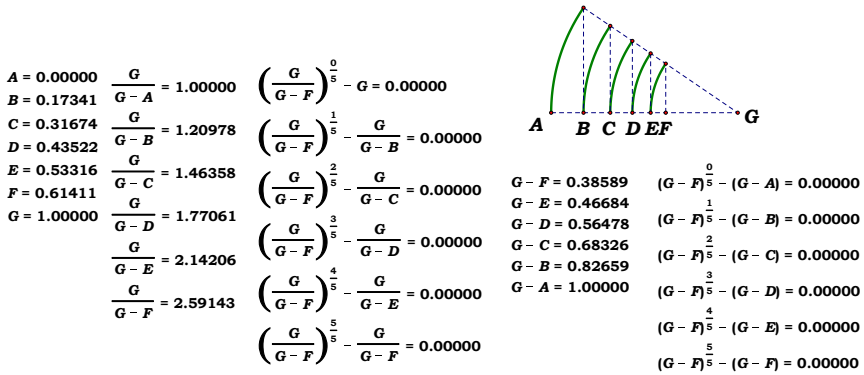


Using any system of measure, we desire; we note the differences. If GF is our unit, we convert the series into that unit like this.



Thus, the only difference between powers and roots, is the direction one is reading from. So, all the properties of exponents are easily demonstrable in geometry, the only question which is at the top of the list is, how does one pair an arithmetic progression with a geometric progression, by way of each's unit?

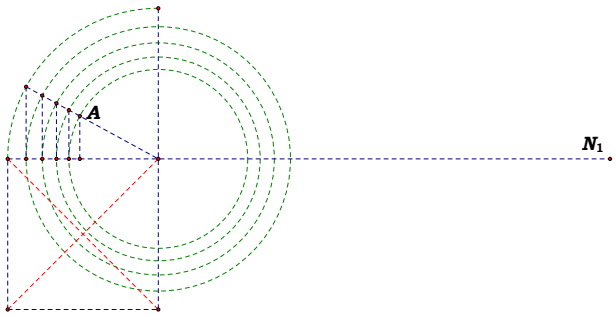
Lastly we conclude:



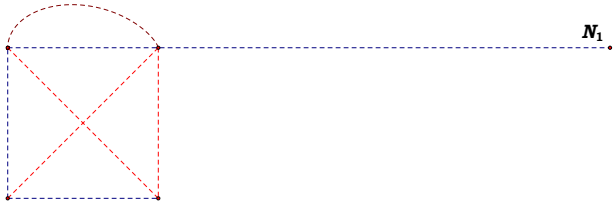
You find today, in elementary Algebra, the explicit, or implicit statements that exponential manipulation is not possible in geometry, that it is a product of inexplicable human genius. So, what does it mean when scientist all over the world believe that they can predict the birth, life and death of the Universe, but

cannot predict the behavior of a simple straightedge and compass, the product of a simple line and circle? It is called schizophrenia, even megalomania. Parroting, mixed with mythology, is symptomatic of a dysfunctional mind. Am I Superman, waiting for his outfit to be returned from the cleaners, so I can go out and buy a ticket to board a flying rock to other dimensions? Or am I just another ape scratching his head?

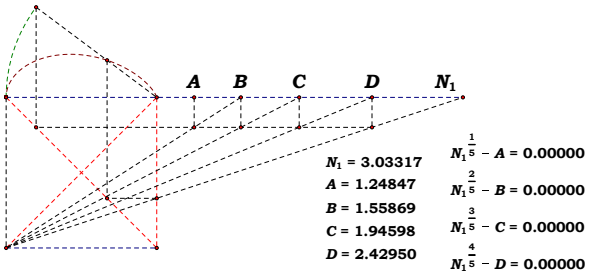
As geometry is that grammar by which one can demonstrate Total Recall by Total Memory, how do we produce that memory for exponential progressions?



I first decide how many geometric units I want to record; I will stay with five; the only point I need to record then is A.

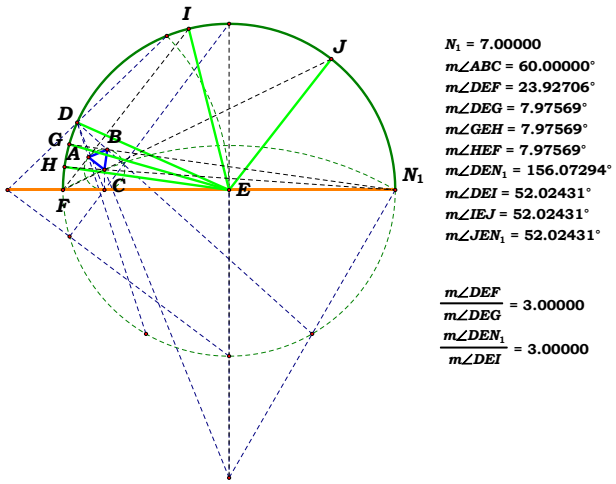


Ignoring everything else but the result, we can now use it to form a fifth exponential series with any N as such. Notice it is not a power or a root, but a fifth in a series.



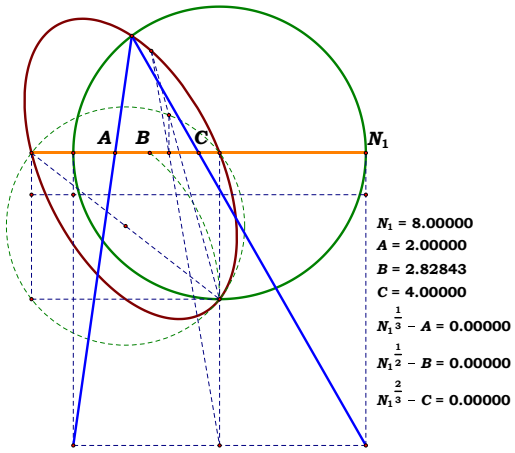
Thus, any geometric series can be applied to any arithmetic. Any claim that I can record and recall or some correlatives by some custom, making a thing different from itself, is just absurd.

In all of this, I am not saying that there are no methods of applying the division of the circumference arithmetically: It actually can be done if one is willing to take the time to work out each figure. However, again, what you find in literature is the simple minded conchoid, not the relationship between actual induction and deduction of the unit, as in the following figure.



However, it too depends upon proportion, but not a good figure for explaining elementary recursion. One can, however, also use memory to find 1 degree, or a portion of it. What use it has, I can not say but one can spend a lot of time slicing and dicing circles.

And I have also shown a rude solution to the Delian problem which again, is not elementary:



Nor perhaps is it very useful as a topic of Metaphysics at this time.

Now, if one is able to wrap their brain around the elementary ideas of Metaphysics, actually demonstrated, the next step is rather mundane, demonstrating simple mathematical operations with our Grammar Matrix.

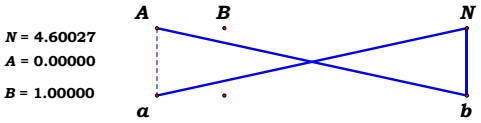
Grammatical Operations

Are derived from Common Notions afforded by sense perception

The Reciprocal

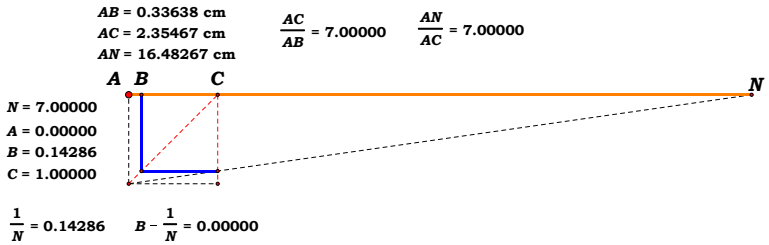
June 3, 2023

Nomenclature

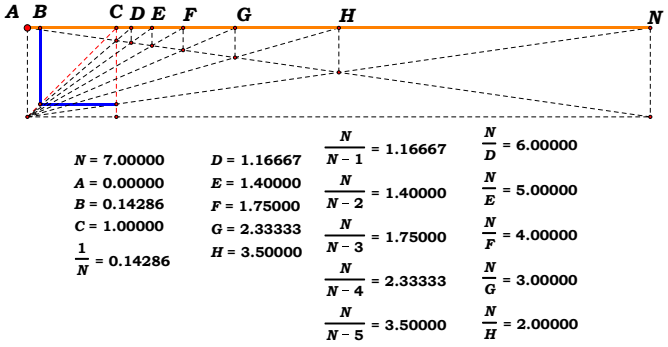


If we are working with any correlative N, we have immediately at our disposal, what I call operational tails, aN, Ab, and Nb which we use to perform operations with. As we go along, we will put them to use. Notice that two operational tails, Ab, and aN, are useful in the box, unit box, and Nb is outside the box. So, we can think in the box, or outside of the box. Ain't that just yuppy? When using any correlative, there are ways of indicating its influence by the use of operational tails. Each correlative actually has a constructable web, which we eventually call upon in our demonstrations. So, every correlative is not only a point, it is either a starting point or a point of termination. Learning is using the in-betweens to communicate with. However, we cannot just assert a random point and claim some operation by it. It has to follow by using the givens.

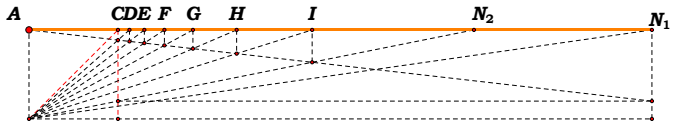
Let us now use our operational tails starting with the creation of a reciprocal. Given any-thing, show its reciprocal.



AC : AB :: AN : AC, which is another way of denoting a reciprocal and it is one way of asserting what the unit is in any situation where the points are not marked. Logically, If AC is to AB as AN is to AC, then AC is the unit relative to A, B, C and N. The reciprocal is commonly called a fraction. Notice, when you use the figure, that the result is concurrent with any change in N. Let us add a tail and with it effect recursion of our result.



Suddenly we enter the realm of Algebraic operations. Now, how would I apply this result using that recursion to divide any other difference whatsoever? Let us draft N_2 upgrading N to N_1.



$N_1 = 7.00000$
$N_2 = 5.00000$
$A = 0.00000$
$C = 1.00000$
$N_1 \cdot N_2 = 35.00000$
$N_2 - 1 = 4.00000$

$D = 1.12903$
$E = 1.29630$
$F = 1.52174$
$G = 1.84211$
$H = 2.33333$
$I = 3.18182$

$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - (N_2 - 1)} = 1.12903$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)} = 1.29630$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)} = 1.52174$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)} = 1.84211$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)} = 2.33333$
$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)} = 3.18182$

$\frac{N_1}{D} = 6.20000$
$\frac{N_1}{E} = 5.40000$
$\frac{N_1}{F} = 4.60000$
$\frac{N_1}{G} = 3.80000$
$\frac{N_1}{H} = 3.00000$
$\frac{N_1}{I} = 2.20000$

$\frac{(N_1 \cdot N_2) - (N_2 - 1)}{N_2} = 6.20000$
$\frac{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)}{N_2} = 5.40000$
$\frac{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)}{N_2} = 4.60000$
$\frac{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)}{N_2} = 3.80000$
$\frac{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)}{N_2} = 3.00000$
$\frac{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)}{N_2} = 2.20000$

$\frac{N_2}{D} = 4.42857$
$\frac{N_2}{E} = 3.85714$
$\frac{N_2}{F} = 3.28571$
$\frac{N_2}{G} = 2.71429$
$\frac{N_2}{H} = 2.14286$
$\frac{N_2}{I} = 1.57143$

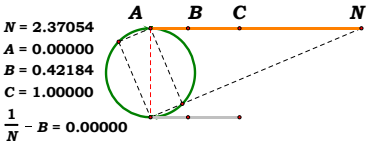
$\frac{(N_1 \cdot N_2) - (N_2 - 1)}{N_1} = 4.42857$
$\frac{(N_1 \cdot N_2) - 2 \cdot (N_2 - 1)}{N_1} = 3.85714$
$\frac{(N_1 \cdot N_2) - 3 \cdot (N_2 - 1)}{N_1} = 3.28571$
$\frac{(N_1 \cdot N_2) - 4 \cdot (N_2 - 1)}{N_1} = 2.71429$
$\frac{(N_1 \cdot N_2) - 5 \cdot (N_2 - 1)}{N_1} = 2.14286$
$\frac{(N_1 \cdot N_2) - 6 \cdot (N_2 - 1)}{N_1} = 1.57143$

We see that we can proportion any result of N_1 by using its unit tail set in proportion by N_2. The result is, a lot more information to consider than we had with the simple reciprocal.

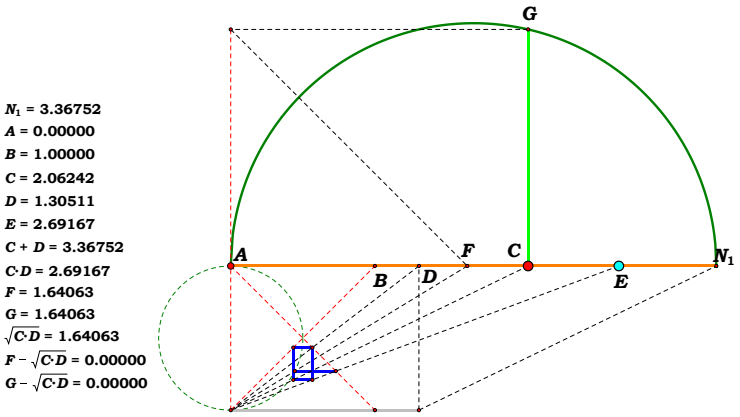
Well, just the reciprocal puts our little Grammar Matrix, to the test. With a couple of basic moves, we are deep into Algebraic series and we are not limited at all by what is considered irrational in Arithmetic, in fact, irrational only means the inability to apply an exact name in that convention of naming. Let us examine the simple operations in of themselves as it is clear, our grammar matrix tells us they can be performed.

The Circle

The easiest way to draw a circle with a straight line is, of course with the reciprocal like so:



In order to make any size circle, it is probably a good idea to know the equation for a circle, which one can acquire from the Elements, or one can get the wrong equation from the Internet.



Therefore, one can draw a unit circle with straight lines and produce a proportion of it to any desired size simply as a function of a reciprocal, a reciprocal of the entire range of possible magnitudes; i.e., from the infinitely small to the infinitely large, all compacted in the unit also known as Complete Induction and Deduction. The phrase Complete Induction and Deduction confuses people as they interpret complete in terms of something perceptible, when it infers the intelligible, meaning it can always, under any circumstance, be demonstrated for any-thing. Would not want anyone floating off into the ozone layer of so-called Transfinite Mysticism.

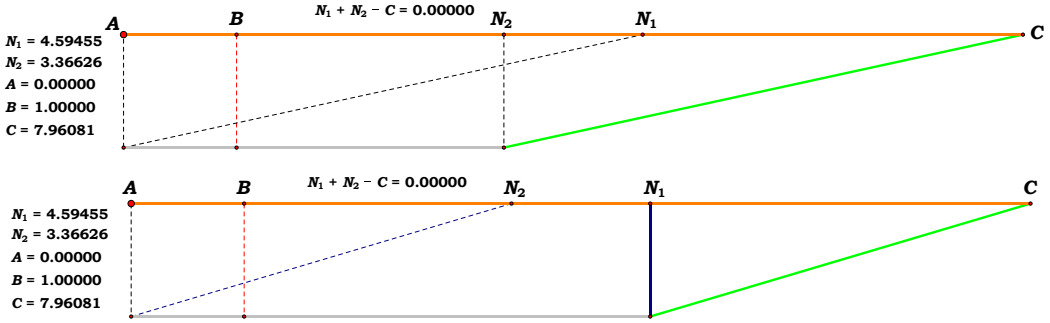
The circle, which became producible by the compass, has now been demonstrated to be a function of Complete Induction and Deduction of binary recursion as the reciprocal of a unit. The circle is not the only product of binary recursion, so it is a sign of feeble-mindedness to claim that geometry does not function completely with all curves.

Addition

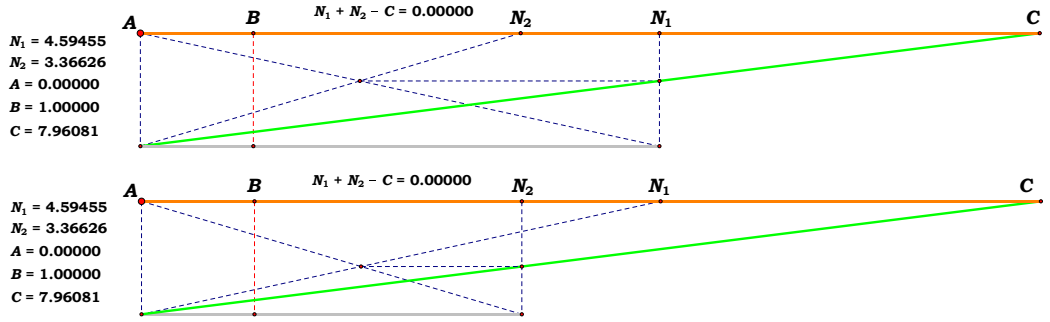
Tuesday, June 6, 2023

We effectively have two operational tails, one is used arithmetically and the other geometrically, thus we can add any two values using each one, and the result will be identical. Because the result is identical, we tend to think of Addition as a single operation, when it is in fact performed in two different concepts of a unit, principally as a noun, or principally as a verb.

I will, therefore example the arithmetic method of addition first, the unit will be constant. Arithmetic addition requires both operational tails, and to demonstrate how plastic the figure is, I will demonstrate the two possible configurations for them.



The following two figures will demonstrate addition using proportion, or geometrically. Here the main operational tails form a ratio which is projected to the unit tail and from there projected to acquire the result.

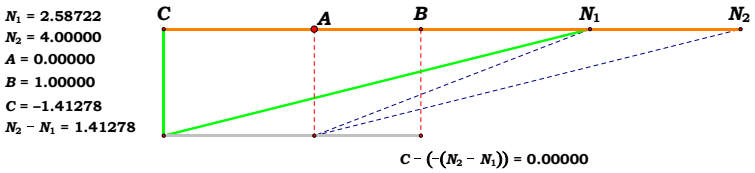


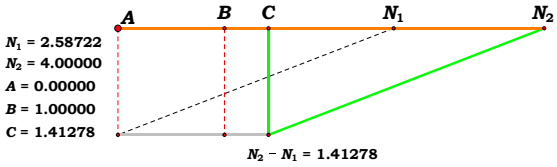
In addition, the order to the terms is irrelevant. This is not true of subtraction.

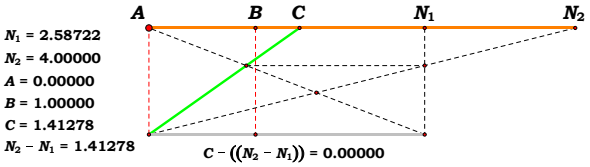
Subtraction

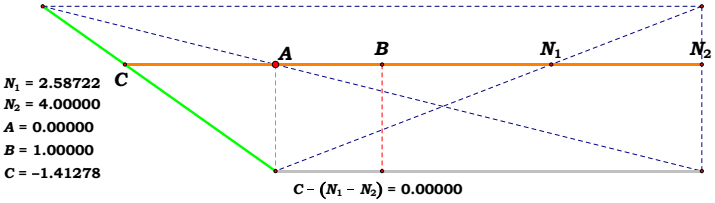
Wednesday, June 7, 2023

Again, the first two equations are Arithmetic and the second two Geometric.





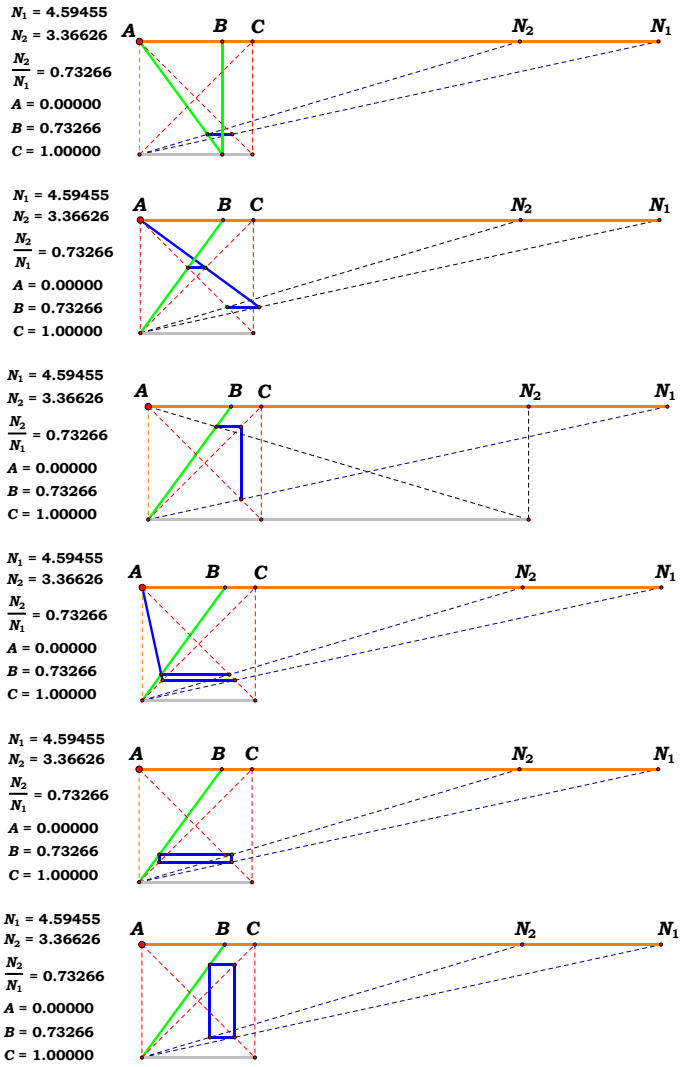




Division

Wednesday, June 7, 2023

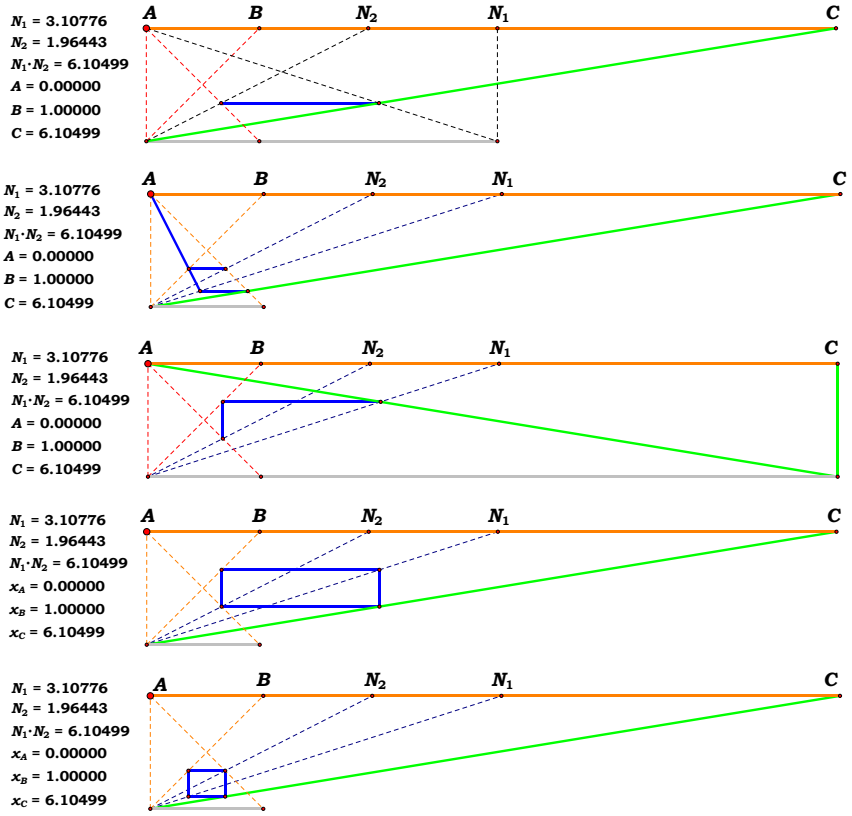
For division, I rounded up six plates. I imagine them in pairs, but as all these plates, I do not imagine that I have discovered all that is doable.



Multiplication

Thursday, June 8, 2023

I only keep five methods of multiplication handy. The reason for only five, is I believe one plate actually turns out to be redundant. The retention of the plate is then just too boring to tolerate.

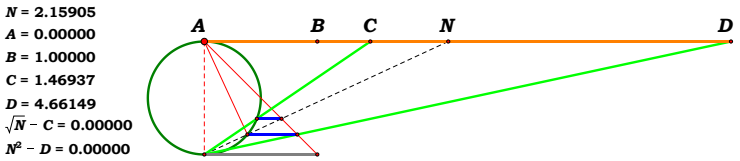


Of these four mathematical operations, one can choose which method for any particular is their fav, or keep their work spicy.

Root and Square

Thursday, June 8, 2023

The root and square are one of the workhorses in computation and they are really simple in geometry.



All of the elements of writing grammar using a Grammar Matrix have been set before you, the one thing they cannot do, is make the abstractions for you. Learning how to process information, our biologically determined job, which, if we can master it is the salvation of man, starts by finding the best grammar teachers available to you; unfortunately, one cannot depend on most people, who do not mind being illiterate, can help you find.

Plato was a grammar teacher; dialectic does not mean the current myths about the name, but it means grammar in accordance with the two parts of a thing, i.e., binary recursion. To date, I have read absolutely no commentary on it or any particular dialog where the writing was not simply stupid. You read Plato to learn about Dialectic, binary recursion and how to use it.

Euclid. When I started my study of geometry, I decided that since the poverty of books on geometry, many simply trying to reword Euclid and others hell bent on spinning yarns, or claiming that a particular use of it was cause to construct mainly, a new, and ridiculous vocabulary, while others, so called non-Euclidean Geometries, are simply pure and provable fictions, a unit is not different from itself, so that I would allow the straightedge and compass to teach me. The result is more than an encyclopedic work which contains many things traditionally said to be impossible in geometry. Never believe the rumors of stupid people. One can view the Elements of Euclid as a very old rowboat with many barnacles needing scraped off of it.

Therefore, I present my work which can be used as study material. To date, corporate publishes have refused to publish any of it, fact is, one never gets a return answer, or some lame excuse that I have no academic credentials.

So, in reading one should start with Euclid, then to my work called the Delian Quest; the point of which is to learn and exercise the mind into seeing the proportions and how they work in a given figure. This will lead one up to Basic Analog Grammar. I do not write, as is commonly done, ridiculous theories, all of them garbage, I just show what can actually be done.

The volume of work I did in such a short time, means that I will spend quite some time in continuing to edit BAG and the BAG Sample Dictionary.

Mansion of the Mind

Saturday, June 10, 2023

Our mind is a container of our experience, that experience, is fleeting and is quickly invisible to all. However, I do construct sign-posts, memories of it through grammar. When the inner sight catches the relationships between things, it becomes quite a lot of furniture in our mind, and there is so much of it for some of us, we never have time to examine and sort it all out. But it cannot be passed on, unless in its last recorded form.

When it is said, that the Temple of God is within, it is also written, that it can be proven by two, or more, reliable witnesses. The only reliable witness I know of is the Rectified Name and that is a biological and provable, fact. It is also a provable fact, that I have lived my life among the poor, but the poor do not beg, they steal your life, because for them, they have no other choice. Eventually, you have nothing left to defend yourself with, which means… what it means. It is a hard thing, when you freely share your supper with the world, and they then rob you of your life. There can only be justice in the world, when that very same world knows judgment. So, the only forgiveness one can ever understand, is when the evil has learnt to be good. And the only justification for what life one has had, is what they have done to make it so.